Interest Rates in Trade Credit Markets

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Abstract

There is evidence that suppliers have private information about their customers' credit risk. Yet, interest rates in trade credit markets usually do not vary with borrowers' risk. Why? We demonstrate that if the demand for intermediate goods is inelastic, then suppliers have no incentive to undercut uninformed banks that ask high interest rates from safe firms. In turn, competition with banks does not let suppliers set higher interest rates to risky firms. Hence, the invariance is at the banking rate when the demand for intermediate goods is inelastic. When the demand is elastic, suppliers have incentives to subsidize interest rates, possibly inducing them to waive interest, as often happens in the U.S.

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1 Introduction

Trade credit is one of the most important sources of short-term external financing for firms in the G7 countries (Canada, France, Germany, Italy, Japan, U.K., and the U.S.).[1] Smith (1987), Mian and Smith (1992) and Biais and Gollier (1997) argue that such prominence is due to an informational advantage: The sales effort of suppliers gives them comparative advantage in assessing their customers' credit risk. Accordingly, Petersen and Rajan (1997) show that, vis-à-vis banks, suppliers extend more credit to firms with current losses and positive growth of sales; a finding that they interpret as evidence that suppliers have comparative advantage in identifying firms with growth potential.

Yet, a supplier's informational advantage is, at first glance, difficult to reconcile with some standard practices in the trade credit markets. Ng, Smith and Smith (1999) and Petersen and Rajan (1994) show that the terms of trade credit in the U.S. are industry-not-firm specific. They usually waive interest in loans that require full payment 30 days after the invoice date (net 30 loans), or combine a 30 day maturity with a two percent discount for early payment within 10 days of the invoice (2-10 net 30 loans). But if suppliers are informed lenders, why don't they charge interest rates that reflect variations in the borrowers' risk?

This paper demonstrates that the pattern of trade-credit rates in the U.S. is an optimal response to suppliers' private information. In a nutshell, waiving interest is optimal if the demand for intermediate goods is elastic at a zero interest rate and suppliers have a sufficiently large advantage (relative to banks) in repossessing assets of financially distressed firms. In contrast, suppliers do not have incentives to undercut uninformed banks that ask a high interest rate from safe firms, if the demand is inelastic at the banking rate. Since competition with banks does not let suppliers finance risky firms at an interest rate higher than the banking rate, the interest rates in the trade credit markets converge to the banking rate, when the demand is inelastic. The interest-elasticity of the demand for the suppliers' products, therefore, determines the level of the invariant rate in the trade credit markets.

To understand the main ideas of the paper, consider an industry whose firms require financing to purchase inputs from a single supplier. In a fraction f of these firms – the safe firms – the investment in the input will be paid back with probability one. In the remaining firms, fraction $1-f$, the investment in the input may fail. We call these latter firms risky.

[1] It is actually the most important source of short-term financing in Canada, France, Germany, and the U.S. See, Rajan and Zingales (1995).

To finance the purchase of inputs, firms can borrow from banks or ask for trade credit. We consider, therefore, firms that, albeit possibly risky, are not credit constrained. In the model, banks act competitively (i.e., interest rates imply that the expected return on a loan equals the cost of funds), but they cannot distinguish between safe and risky firms. Hence, banks set the same interest rate r_B for all firms in the industry.

In contrast to the banks, the supplier knows whether a firm is safe or risky. Thanks to this informational advantage, the supplier can ask different interest rates from the two types of firms. For instance, the supplier may want to finance risky firms at an interest rate that is higher than the banking rate r_B. Nonetheless, risky firms would respond to higher interest rates in the trade credit market by borrowing from banks only. Competition with banks thus prevents suppliers from raising interest rates beyond the banking rate, regardless of the firm's type.

Is it in the supplier's interest to offer interest rates that are lower than the banking rate? We will show that this won't happen if the demand for the supplier's products (i.e., the inputs) is inelastic with respect to the financing cost. An inelastic demand induces the supplier to raise interest rates as much as possible, that is, the optimal trade-credit rate is the banking rate r_B.

It then follows that, if the demand for inputs is inelastic, the banking rate is a natural candidate for an equilibrium in which the trade-credit rate does not vary with the borrowers' risk. After all, while the inelastic demand does not elicit incentives for the supplier to undercut the banking rate, competition prevents suppliers from asking an interest rate higher than the banking rate. One problem remains for this candidate to be legitimate, though. Informed suppliers may be unwilling to lend to risky firms at an interest rate that is set by uninformed banks that fiercely compete with each other. This won't happen if, as Frank and Maksimovic (1998) argue, suppliers are significantly more efficient than banks in salvaging value from assets of financially distressed firms. In this event, suppliers get a higher return than banks when a borrower becomes financially distressed, making it in the suppliers' interest to lend to risky firms at the banking rate r_B. The invariance of interest rates in the trade credit markets thus obtains.

What happens if the demand for inputs is inelastic but suppliers are not more efficient than banks in rescuing assets of distressed firms? The equilibrium with invariant rates breaks down. This result yields a testable implication for our model if we can identify industries in which suppliers are more likely to be more efficient than banks in lending to risky firms.

Petersen and Rajan (1997) provide one way to identify these industries by showing that suppliers extend more credit to industries that keep a lower fraction of finished goods in inventory. They interpret this finding as evidence that it is easier for suppliers to transform repossessed inputs into liquid assets than finished goods. Accordingly, our model predicts that suppliers are more likely to offer standardized interest rates in industries that keep a low fraction of finished goods in their inventory.

Our model builds on two recent papers: Biais and Gollier (1997) and Burkart and Ellingsen (2002). These articles explain why suppliers are willing to lend to firms that have exhausted their debt capacity with banks. In Biais and Gollier, suppliers can identify firms whose credit risks are overestimated by banks. Knowing that a firm's credit line is unduly low, suppliers are willing to extend trade credit. In Burkart and Ellingsen, financially constrained firms have access to trade credit because – as in our paper – suppliers are able to transform collateral into liquid assets more efficiently than banks. There is no invariance of interest rates, however, in either Biais and Gollier's or Burkart and Ellingsen's models. In both models, it is optimal for suppliers to vary interest rates with borrowers' credit risk.

Brennan, Maksimovic and Zechner (1988) is another related work. These authors build a model in which a monopolist finances the sale of its products, setting the price of the good and the interest rate in the vendor financing to maximize expected profits. Brennan, Maksimovic and Zechner argue that the monopolist will use the interest rate to discriminate the demand for the good, if regulators force the monopolist to set the same price to all customers. In this setting, they show that an elastic demand induces the monopolist to offer vendor financing at subsidized rates. Unlike in our paper, the optimal interest rate varies with customers' characteristics in their model.

As in Brennan, Maksimovic and Zechner (1988), our supplier internalizes the impact of financial costs in the demand, possibly offering subsidized rates to their customers. However, unlike in Brennan, Maksimovic and Zechner, a sufficiently elastic demand for inputs may induce our supplier to reduce the trade-credit rates as much as possible, that is, to zero. In this case, low financing costs are so important to sales that it is optimal for the supplier to waive interest in loans to both types of firms.

It then follows that, in our paper, the interest-elasticity of demand determines not only the invariance of interest rates but also its level. If the demand for inputs is elastic at the zero rate, the invariance is at a zero interest rate; if the demand for inputs is inelastic at the banking rate, the invariance is at the interest rate available in bank loans. This result allows for the following interpretation of common practices of trade credit in the U.S. While the scale of production implies an optimal level of inventory that is unlikely to be too sensitive to interest rates, a temporary increase in the level of inventory probably is. Such temporary increases in inventory are good candidates to be financed by trade credit with very short maturity. Our model thus predicts that suppliers should waive interest rates in trade credit with very early repayments, e.g., 10 days, as often happens in the U.S. In turn, the optimal interest rates of trade credit with longer maturity should collapse to the banking rate.

Perhaps more interestingly, characterizing the two candidates for invariance of the terms of credit links these terms to whether customers are credit constrained or not. Since waiving interest attracts all types of firms, credit constraint cannot be the driving force for trade credit in zero-interest loans. In turn, the invariance at high interest rates (e.g., the annual rate of 44 percent that is implicit in the 2-10 net 30 loans) obtains only in trade credit to firms that are not credit constrained. After all, while setting interest rates to credit constrained firms, suppliers are not constrained by competition with banks. Hence, the optimal terms of trade credit will depend solely on firm-specific characteristics: elasticity of demand, probability of default, etc. Another prediction of our model is thus that the invariance of the terms of credit at positive rates should be more pervasive in industries whose firms are not likely to be credit constrained.

The remainder of the paper is organized as follows. Section 2 presents the model and section 3 characterizes an equilibrium with invariance of interest rates in trade credit markets. In section 4, we discuss the empirical implications, the robustness of our results, and we exhibit sufficient conditions for uniqueness of the equilibrium with invariance of interest rates. Section 5 then concludes. Proofs of the propositions that are not in the text can be found in the appendix.

2 The Model

Consider an economy with two dates, $t = 0$ and $t = 1$, and an industry with three risk-neutral agents: firms, banks, and a supplier of the firms' inputs. At $t = 0$, firms require financing to purchase inputs. Banks are always willing to finance the purchase of inputs at an interest rate that covers the cost of funds. Firms, however, may have a second source of financing: trade credit. With an exogenous probability x, the supplier has enough funds to finance their customers. Upon the purchase of inputs at $t = 0$, production takes place and firms sell the economy's single consumption good at $t = 1$. At this time, firms repay the debt and distribute any remaining cash flow to shareholders.

Below we describe the simplest technology and information structure that obtain invariance for trade credit rates in a relevant context, that is, when a supplier has superior information on the firms' credit worthiness.

2.1 Firms

There are two types of firms: safe and risky. The safe firms represent a fraction f of the population and have a deterministic production function. With this safe technology, investing I in inputs at $t = 0$ obtains $Q(I)$ at $t = 1$. We assume that $Q(I)$ is an increasing and concave function, with $Q(0) = 0$ and satisfying the following conditions: there exist $\underline{I} < \overline{I}$ such that $Q'(\underline{I}) > (1 + r)/f$ and $Q'(\overline{I}) < 1$, where r is the riskless interest rate. These harmless technical assumptions on the marginal productivity of investment simply assure that firms buy a positive level of input. Without loss of generality, we assume that the input is the unit of account. Hence, $Q(I)$ is the value of the firms' output in units of inputs.

Risky firms are endowed with a stochastic production function. With this technology, purchasing I units of input at $t = 0$ yields $\tilde{Q}(I)$ at $t = 1$, where:

$$\tilde{Q}(I) = \begin{cases} Q(I), & \text{with probability } \pi \\ \delta I, & \text{with probability } 1 - \pi, \text{ and } \delta \in (0, 1). \end{cases}$$

Note that, with probability π, the risky technology is as profitable as the safe technology. But, with probability $1 - \pi$, the risky technology gets into troubles; the fraction $1 - \delta$ of purchased inputs is lost and the only return on the investment is an amount δI of inputs that remained unused. We assume that both $Q(I)$ and $\tilde{Q}(I)$ are verifiable. As such, firms can write debt contracts that are contingent on the realization of outputs.

2.2 Banks

In the model, banks can neither distinguish between firms of different types nor observe terms of trade credit. Banks know only the proportion of safe and risky firms, and the amount of inputs I that firms purchase. Since banks are risk neutral and operate in a competitive market, they will set an interest, r_B, that yields their opportunity cost, which, given risk neutrality, is the riskless interest rate r.

In a loan for a risky firm that fails, the bank captures the firm's output, $\tilde{Q}(I)$, which, as explained in the previous section, is a fraction δ of the input I originally purchased. It is unlikely, nonetheless, that banks can costlessly transform δI into liquid assets. In fact, one of the key assumptions of our paper is that banks are not as efficient as suppliers in transforming inputs into liquid assets. To emphasize this difference between banks and suppliers, and to facilitate the analysis, we assume that neither the banks nor the firms can rescue the unused inputs, δI, if the technology fails.

2.3 The supplier

In our model, the supplier is a monopolist in the market for inputs.[2] Yet, as in Brennan, Maksimovic and Zechner (1988), we assume that the supplier cannot use the price of the input to discriminate the demand, and, to simplify the analysis we set the price of input equal to one.[3] In doing so, we can focus the analysis on the interest rates.

A common view in the trade-credit literature is that suppliers have comparative advantages over banks in financing purchases of inputs. Biais and Gollier (1997) argue, for instance, that an ongoing sales effort makes it easier for suppliers evaluate their customers' credit risk; an argument that Petersen and Rajan (1997) find evidence for. Accordingly, we assume that, unlike the banks, our supplier knows whether a firm is risky or safe.

Ability to evaluate risk of credit is not the only reason for the existence of trade credit, though. Petersen and Rajan (1997) also find evidence that suppliers are more efficient than banks in transforming collateral into liquid assets. To model this comparative advantage, we follow Frank and Maksimovic (1998) and Burkart and Ellingsen (2002) and assume that, unlike the banks, the supplier can costlessly resell inputs that they capture from bankrupted firms. Hence, when a risky investment of I units of input fails, the supplier captures the unused inputs, δI, assuring some return for their trade credit. Since we assume that banks cannot rescue unused inputs, we can interpret δ as a measure of suppliers' comparative advantage over banks in financing purchases of inputs.

But, as Mian and Smith (1992) show, some suppliers do not have access to funds that

[2] As we shall argue in section 4, assuming a single supplier of inputs is not essential. The driving force of our results is the informational advantage of suppliers, which gives them monopoly power in the financing of firms.

[3] As Petersen and Rajan (1997) point out, anti-trust laws can precent the supplier from using prices to discriminate the demand.

can be used to provide trade credit. We model this potential constraint as follows. With a probability x in the interval $(0,1)$, our supplier has access to funds at the same cost of banks, r. In this event, the supplier can offer trade credit. With probability $1-x$, however, the supplier has no access to funds, preventing trade credit. Firms will then have to secure bank loans to purchase inputs. The supplier's stochastic cost of funds thus assures an active role for banking credit, despite the supplier's potential advantage as a lender. Finally, to simplify the analysis, we assume that the supplier's cost of production is zero.

2.4 The game in the extensive form

Figure 1 describes the extensive form of the game. Nature acts first, determining the type of the firm (safe or risky) and whether the supplier can provide trade credit. If the supplier cannot offer trade credit, an event with probability $1-x$, firms borrow from banks at a rate r_B before purchasing I_B from the supplier. If, instead, trade credit is available, firms choose the source of financing, that is, the supplier or the banks.

The dotted lines in the tree tell us that banks do not know either the type of a firm that asks a loan (dotted line in the lower part of the tree) or the availability of trade credit (dotted line in the upper part). In contrast, the supplier knows whether it can lend and the firm's type. Accordingly, we let the supplier tailor the interest rate to the type of firm, asking interest r_T^R from risky firms and r_T^S from safe firms. These interest rates determine firms' returns on the purchase of inputs, inducing risky firms to invest I_T^R and safe firms to invest I_T^S.

In this game there may be two types of equilibrium in pure strategies. In the first one, the supplier lends to only one of the two types of firms, when trade credit is available. In the second type of equilibrium, suppliers lend to both types of firms, whenever possible. Of course, the invariance of interest rates is meaningless in the first type of equilibrium: All borrowers are in the same class of risk if, in equilibrium, the supplier offers trade credit only to one type of firm. Accordingly, our focus in on the equilibrium in which the supplier lends to both types of firm, when trade credit is available. After characterizing this equilibrium in section 3, we exhibit conditions for it to be unique in section 4.

3 Equilibrium with Invariance of Interest Rates

3.1 Banking credit

In the equilibrium that we look for, the supplier finances purchases of inputs of both types of firms, when trade credit is available. When trade credit is not available, an event with probability $1 - x$, firms finance purchases of inputs by borrowing from banks. Let us then start our analysis by computing the demand for inputs of a safe firm that borrows from banks at an interest rate r_B.

By assumption, safe firms can always repay loans that are used to finance inputs.[4] As a result, a safe firm's optimal investment in inputs solves:

$$\underset{I}{max} \frac{Q(I) - (1 + r_B)\, I}{(1 + r)} \tag{1}$$

Program (1) looks for the investment that maximizes the present value of a safe firm's profit. By investing I at $t = 0$, a safe firm obtains $Q(I)$ at $t = 1$ with probability 1, from which $(1 + r_B)\, I$ will be used to pay principal plus interest to the bank (also at $t = 1$). Since the investment of a safe firm is riskless, its payoff is discounted at the riskless interest rate r. For an interest rate r_B, the first order conditions, which are also sufficient, yield the demand for inputs of the safe firm, $I_B^S\,(r_B)$, by setting the marginal product of input equal to the cost of financing, that is:

$$Q'(I_B^S) = (1 + r_B)\,. \tag{2}$$

Consider now a risky firm that borrows I to purchase inputs. With probability π, the investment will yield the same return $Q\,(I)$ of the safe firms. With probability $1-\pi$, however, the investment will fail, leaving only δI units of inputs at $t = 1$. Regardless of the lending banks' ability to transform the residual inputs into liquid assets, a failure of the risky technology implies that the risky firm loses any rights on the residual inputs. Given the assumption of risk neutrality, the demand for inputs of a risky firm, $I_B^R\,(r_B)$, maximizes the present value of the expected payoffs using the riskless interest rate as the discount rate, that is,

$$\underset{I}{max} \frac{\pi\,[Q(I) - (1 + r_B)\, I]}{(1 + r)}. \tag{3}$$

[4] For any finite interest rate, our assumptions on the marginal productivity of investment (see section 2.1) assure that a small purchase of inputs will more than offset the costs of servicing the debt, leaving a positive profit for the safe firm. Hence, an optimal choice of inputs must imply a positive profit as well. In the absence of uncertainty, a positive profit implies that any debt will be repaid with probability 1.

Like the safe firms, a risky firm's demand for inputs sets the marginal product of investment equal to cost of financing, that is, $Q'(I_B^R) = (1 + r_B)$. It then follows that the demand schedules of safe and risky firms are equal, that is, $I_B^S = I_B^R = I_B$. Indeed, had the demand for loans varied across firms of different types, banks would have been able to infer the type of a firm that requests a loan. Banks and suppliers would then end up with the same information structure, and our model would not be fit to explain why interest rates in trade credit do not seem to reflect suppliers' private information about their customers.

Of course, a request of a bank loan may convey information even if safe and risky firms have identical demands for inputs. For instance, in an equilibrium in which the supplier finances only safe firms, banks should expect that most of their loans go to risky firms. (Banks should not expect all firms to be risky because lack of trade credit might lead safe firms to look for banking credit.) In the equilibrium that we look for, though, the supplier finances both types of firms, whenever trade credit is available. Hence, banks know that lack of funds for trade is the only reason for firms asking for bank loans. Accordingly, requests of loans do not convey information, and banks do not update their priors about firms' types.

Provided that requests for loans do not convey information, we can easily compute the equilibrium interest rate r_B. Since the technologies of both types of firms are common knowledge, the banks know that safe firms will pay principal plus interest with probability 1, while risky firms will honor the debt contract with probability π. (Risky firms do not pay anything with probability $1 - \pi$.) Therefore, the banks will collect principal plus interest at $t = 1$ if the firm is safe, probability f, or if the firm is risky but the technology does not fail, probability $\pi(1 - f)$. In other words, the probability that a bank is paid at $t = 1$ is $f + \pi(1 - f)$. And the expected return of a bank that lends at a rate r_B is $(1 + r_B)(f + \pi(1 - f))$.

Competition among banks drives the expected returns on banking loans to their opportunity cost, which, under our assumption of risk neutrality is the riskless interest rate r. Thus, the interest rate that assures banks their opportunity cost is

$$r_B = \frac{1 + r}{f + \pi(1 - f)} - 1. \tag{4}$$

Having characterized the equilibrium banking rate and the demand for inputs of firms that borrow from banks, our next task is to introduce trade credit. Two questions then naturally arise. Is it optimal for the supplier to finance purchases of inputs? If so, what is the optimal interest rate of the trade credit market. Answering these questions requires solving for the investment decision of a firm that has the option of using trade credit to finance purchases of inputs.

As it turns out, trade credit does not fundamentally change firms' investment decisions. Whether a firm borrows from banks or the supplier, all that matters is the cost of financing. It then follows that the investment decisions of safe and risk firms are still characterized by, respectively, programs (1) and (3), once we substitute the minimum cost of financing for the banking rate. We can, therefore, define an investment function that, for both types of firms, is implicitly defined by the equality of the marginal productivity of investment and the cost of financing, as follows:

$$Q'(I(s)) = 1 + s, \tag{5}$$

where s is the lowest between the banking rate and the trade credit rate. Under our assumptions, the investment function decreases with the cost of financing. Moreover, the investment function is concave on the interest rate if $Q'''(I) < 0$.[5]

Equipped with the demand schedule, the next two sections characterize the supplier's optimal strategies, starting with the optimal terms of trade credit of a supplier that faces a safe customer.

3.2 The supply of trade credit to safe firms

In our model, the supplier is not obliged to finance the purchase of inputs; it can let banks finance firms. Remember, however, that since banks cannot distinguish between safe and risky firms, the competitive rate r_B embeds a cross-subsidy between the two types of firms. In other words, banks profit from safe firms to cover losses with risky firms. This means that, by financing the purchase of inputs of safe firms at a rate r_B, the supplier fetches not only an operational profit (the sale of inputs), but also a financial profit. It then follows that, whenever possible, the supplier will undercut the banking rate (possibly by a small amount ϵ that we will ignore in the analysis) to finance a safe firm's purchase of inputs.

The question then is what term of credit should the supplier offer to the safe firm. On the one hand, a low interest rate decreases the supplier's financial profit. On the other hand, a low interest rate increases the demand for inputs, enhancing the operational profits. The

[5] To prove that $Q'''(\cdot) < 0$ implies that $I\ (\cdot)$ is concave on the interest rate, apply the implicit function to $Q'(I) = (1 + r)$ to obtain $I' = \frac{1}{Q''(I)}$, which is negative because, by assumption, $Q''(I) < 0$. Assuming $Q''' < 0$ and applying the implicit function theorem a second time yields $I'' = -\frac{Q'''(I)}{Q''(I)^2} I' < 0$.

optimal rate r_T^S weighs these two forces by solving the following program:

$$\max_{r_T^S} \frac{\left(1+r_T^S\right) I\left(r_T^S\right)}{(1+r)} \tag{6}$$
$$\text{subject to} \quad 0 \leq r_T^S \leq r_B.$$

The objective function in program (6) is the present value of the supplier's profits. These profits are cashed at $t=1$ and consist of two components: the operational revenue $I\left(r_T^S\right)$ and the financial revenue $r_T^S I\left(r_T^S\right)$. While the operational revenue is always associated to a profit (remember that, for simplicity, we assume that production cost is zero), the financial revenue will result in a financial loss if the interest rate r_T^S is lower than the discount rate r (the supplier's cost of funds). The constraint of the program takes into account two restrictions. First, it rules out negative interest rates, which, being equivalent to a combination of zero interest and a discount in the price of input, amounts to assuring that the supplier cannot use prices to discriminate the demand. The second restriction takes into account that the bank loans are outside options for the financing of the inputs. The interest rate in the trade credit market, therefore, cannot be larger than the banking rate.[6]

As discussed in the previous section, $Q'''(.) < 0$ is a sufficient condition for the investment function to be concave. In this case, Program (6) is concave and its first order condition yields a very simple characterization of the optimal interest rate for loans to safe firms:

Proposition 1 - *Assume that the investment function is concave on the interest rate. Thus, the optimal interest rate for trade credit to safe firms, r_T^S, is equal to the banking rate, r_B, if the elasticity of demand is less than or equal to one at the interest rate r_B. Otherwise, the optimal rate is less than the banking rate. In particular, r_T^S is equal to zero if and only if the elasticity is bigger than one at r_B and bigger than or equal to one at the rate zero.*

The intuition for Proposition 1 is quite simple. While the demand for inputs is inelastic, monopoly power gives the supplier incentives to increase the interest rate. In particular, an inelastic demand at the banking rate r_B implies that a higher interest rate would have increased profits, had not it been for the firm's outside option of borrowing from banks. Hence, an inelastic demand at the banking rate r_B implies that the supplier will increase the

[6] The supplier can fetch an interest rate higher than r_B by denying inputs to firms that do not use trade credit. The analysis in the paper, therefore, ignores distortions in the trade credit markets that are driven by these types of bundling strategies.

interest rate until it reaches the upper bound r_B, beyond which firms prefer to borrow from banks.

What happens if the demand is elastic at r_B? The optimal trade rate is thus lower than r_B, because the financial loss from a lower rate is more than offset by the operational gains from an increase in sales that a lower rate induces. In particular, the supplier will lower the interest while it remains in the elastic portion of the demand, reaching an optimal trade rate of zero when the demand is elastic at its lowest point of elasticity, that is, at the interest rate $r_T^S = 0$.

From Proposition 1, we can derive the supplier's profit function:

$$\Pi_T^S(r_B) = \frac{\left(1 + r_T^S(r_B)\right) I\left(r_T^S(r_B)\right)}{(1+r)}, \tag{7}$$

where $r_T^S(r_B)$ is the optimal trade rate. Defining the interest-elasticity of demand as $\epsilon^D(r) = -\frac{(1+r)I(r)}{I'(r)}$, we can write the optimal trade credit rate as follows:

$$r_T^S(r_B) = \begin{cases} r_B, \text{ if } \epsilon^D(r_B) \leq 1; \\ r_T^S \in (0, r_B), \text{ if } \epsilon^D(0) < 1 \text{ and } \epsilon^D(r_B) > 1; \\ 0, \text{ if } \epsilon^D(0) \geq 1. \end{cases} \tag{8}$$

Equation (8) gives us the optimal interest rate as a function of the elasticity of demand and the banking rate. In section 3.4, we will use this characterization to obtain conditions on the elasticity of demand that assure that the supplier will choose the same interest rate for safe and risky firms. But first we must derive the supply of trade credit to risky firms.

3.3 The Supply of Trade Credit to Risky Firms

Let us now move to the risky firms. In general, riskier firms should pay higher interest rates. Yet, in our model, banks cannot distinguish between risky and safe firms, what makes it impossible for the banking rate to vary with the firm's type. As it turns out, this friction in the market of banking loans has consequences to the trade credit market. Since risky firms can borrow from uninformed banks at the interest rate r_B, an informed supplier cannot ask for a higher interest rate, even if informed banks would have required a higher interest rate from risky firms. It then follows that the interest rate of trade credit to risky firms, r_T^R, must satisfy:

$$r_T^R \leq r_B. \tag{9}$$

The inequality (9) may induce suppliers to forego loans to risky firms. Had banks known that a firm is risky, they wouldn't lend at an interest rate r_B; at such rate, the expected return of lending to risky firms does not cover banks' cost of funds. Banks are willing to lend at r_B because they do not know the firm's type and they count on profiting from loans to safe firms. But then, why should an informed supplier lend to risky firms at an interest rate that, due to competition with uninformed banks, cannot surpass r_B?

We have already discussed one reason for suppliers to lend to risky firms at the banking rate r_B. An elastic demand may induce the supplier to accept a loss in the financing of the input to increase sales. While this first reason for lending to risky firms is relevant for loans to safe firms as well, there is a second reason that does not apply to safe customers. As you may recall, there is evidence that suppliers have a comparative advantage over banks in transforming collateral into liquid assets. If this advantage is sufficiently larger, lending to risky firms at the interest rate r_B may impose an expected loss to banks and yet assure an expected profit to the suppliers.

Of course, if the demand for inputs is too inelastic and the suppliers' advantage over banks is small, then it may be optimal for the supplier to forego loans to risky firms. In this case, the equilibrium in which suppliers lend to both types of firms breaks down. And the invariance of interest rates in the trade credit markets would be meaningless: Any equilibrium would comprise trade credit to firms in the same class of risk. As is often the case in game theoretical models, therefore, the equilibrium that we look for – one that the supplier lends to both types of firms at the same interest rate – obtains only in a certain range of parameters.

To characterize the supply of trade credit to risky firms, we will divide the analysis in two steps. In the first step, we obtain the interest rates that maximize the supplier's expected profits, under the assumption that it is optimal for the supplier to lend to safe and risky firms at an interest rate that is bounded by the banking rate. Then in the second step, we exhibit conditions under which it is indeed optimal for the supplier to lend to both types of firms at the interest rate that the first step characterizes.

3.3.1 Lending to risky firms

Assuming that it is optimal for the supplier to lend to risky firms, the optimal interest rate solves:

$$\max_{r_T^R} \pi \left[\frac{\left(1+r_T^R\right) I\left(r_T^R\right)}{(1+r)}\right] + (1-\pi)\left[\frac{\delta I\left(r_T^R\right)}{(1+r)}\right] \tag{10}$$
$$\text{subject to} \quad 0 \leq r_T^R \leq r_B.$$

In program (10), the objective function is the present value of expected profits, which, from the risky-neutrality assumption, is discounted at the riskless interest rate. With probability π, the risky firm succeeds, allowing the repayment of the debt at $t = 1$, as in loans to safe firms. In this event, the risky firm pays the principal $I\left(r_T^R\right)$, which corresponds to the demand for inputs at the interest rate r_T^R, and interest, $r_T^R I\left(r_T^R\right)$. But, with probability $1-\pi$, the risky firm fails and the supplier rescues the unused inventory of inputs, $\delta I\left(r_T^R\right)$. In maximizing expected profits, the supplier faces two constraints. The interest rate of the trade credit cannot be larger than the banking rate (or else the firm borrows from a bank) and the interest rate cannot be negative.

As in program (6), $Q'''(.) < 0$ is a sufficient condition for both the investment function and program (10) to be concave, in which case the first order condition yields a very simple characterization of the optimal interest rate for loans to risky firms.

Proposition 2 - *Assume that the investment function is concave on the interest rate. Thus, the optimal interest rate for trade credit to risky firms, r_T^R, is equal to the banking rate, r_B,* if the elasticity of demand is less than or equal to $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$ *at r_B. Otherwise, the optimal rate is less than the banking rate. In particular, r_T^R is equal to zero if and only if the elasticity is bigger than $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$ at r_B and bigger than or equal to $\frac{\pi}{\pi+(1-\pi)\delta}$ at the rate zero.*

In loans to safe firms (see equation (8)), an inelastic demand induces the supplier to increase the interest rate as much as possible. In contrast, the supplier will offer trade credit to risky firms at a lower interest rate if the demand is inelastic at the banking rate r_B but higher than $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta} < 1$. Likewise, while the optimal interest rate to safe firms reaches zero if the demand is elastic at zero, the optimal rate to risky firms reaches zero if the demand at the rate zero is larger than $\frac{\pi}{\pi+(1-\pi)\delta} < 1$.

The incentives to offer lower interest rates to risky firms follow from the assumption of zero costs to produce inputs. When the risky firm fails, the contracted interest is irrelevant to the supplier; the return on the loan then amounts to the unused inventory δI. The unused inventory gives a positive profit to the supplier if the cost of producing inputs is zero. As such, the risk of default makes the supplier place a bigger weight on the operational profits and a lower importance to interest payments. Hence, there are stronger incentives to reduce interest rates in trade credit to risky firms.

Of course, these incentives do not hold if default is costly to the supplier. Since costly default reduces the supplier's expected benefits from an increase in sales, interest payments become more important. In particular, if the costs of production are sufficiently large, the optimal interest rate to risky firms may be the banking rate, if the demand for inputs is elastic at r_B. It can be shown, however, that, even in this case, the optimal interest rate will be zero if the demand for inputs is either sufficiently elastic and r_B if the demand is sufficiently inelastic. This is all we need to obtain invariance of interest rates in the trade credit markets.

From Proposition 2, we can derive the supplier's profit function:

$$\Pi_T^R\left(r_B\right)=\pi\left[\frac{\left(1+r_T^R\left(r_B\right)\right)I\left(r_T^R\left(\cdot\right)\right)}{\left(1+r\right)}\right]+\left(1-\pi\right)\left[\frac{\delta I\left(r_T^R\left(r_B\right)\right)}{\left(1+r\right)}\right], \tag{11}$$

where $r_T^R(.)$ is the optimal interest of trade credit to risky firms. Defining the interest-elasticity of demand as $\epsilon^D(r)=-\frac{(1+r)I(r)}{I'(r)}$, we can write the optimal trade credit rate as follows:

$$r_T^R\left(r_B\right)=\begin{cases} r_B,\ \text{if}\ \ \epsilon^D\left(r_B\right)\leq\left[\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}\right]; \\ r_T^R\in\left(0,r_B\right),\ \text{if}\ \epsilon^D\left(0\right)<\left[\frac{\pi}{\pi+(1-\pi)\delta}\right]\ \text{and}\ \epsilon^D\left(r_B\right)>\left[\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}\right]; \\ 0,\ \text{if}\ \ \epsilon^D\left(0\right)\geq\left[\frac{\pi}{\pi+(1-\pi)\delta}\right]. \end{cases} \tag{12}$$

Equation (12) gives us the optimal interest rate – under the assumption that it is optimal to lend to risky firms – as a function of the elasticity of demand and the banking rate. Next, we obtain a necessary and sufficient condition for lending to risky firms to be optimal.

3.3.2 Should suppliers lend to risky firms?

It may be optimal for the supplier to finance risky firms, for two reasons. First, the supplier's comparative advantage in rescuing unused inputs may make the banking rate profitable, even if it imposes an expected loss on banks. Second, as Schwartz and Whitcomb (1997) and Brennan, Maksimovic and Zechner (1998) point out, financing risky firms at a low interest rate may increase sales to a point that the increase in operational profits more than offsets an expected loss in the loan contract. These two effects are summarized in condition (13) below.

$$\frac{\pi\left(1+r_T^R\left(r_B\right)\right)I\left(r_T^R\left(r_B\right)\right)+\left(1-\pi\right)\delta I\left(r_T^R\left(r_B\right)\right)}{\left(1+r\right)}\geq I\left(r_B\right). \tag{13}$$

Condition (13) is intuitive. It simply requires that financing the risky firm increases expected profits. More precisely, the left-hand-side of the condition (13) is the present value of expected profits when the supplier finances the risky firm, under the optimal interest rate $r_T^R\left(r_B\right)$ (see equation (12)). Lending to risky firms is optimal if and only if this present value is larger than $I\left(r_B\right)$, which is the supplier's profits when the risky firm borrows from banks at the rate r_B to pay for the inputs at $t=0$.

In the next section, we combine Propositions 1 and 2 with condition (13) to characterize an equilibrium in which the suppliers offers trade credit to both types of firms at the same interest rate.

3.4 The invariance of interest rates in trade credit markets

An equilibrium with invariance of interest rates in trade credit markets has two main ingredients. First, the supplier must have incentives to finance the two types of firms, or else the invariance of interest rates is meaningless because trade credit would be limited to firms in the same class of risk. The condition (13) summarizes this first ingredient. Second, the invariant interest rate must satisfy the necessary and sufficient conditions for loans to safe and risky firms, as summarized in equations (8) and (12).

Assume for a while that it is optimal for the supplier to finance both types of firms (i.e., condition (13) is satisfied). A quick inspection of the optimal interest rates (see equations (8) and (12)) shows two natural candidates for an invariant interest rate: the banking rate r_B and a zero interest rate. From equation (8), the optimal interest rate of loans to safe firms is the banking rate if, at r_B, the demand for inputs is inelastic. In turn, equation (12) shows that the banking rate is optimal for trade credit to risky firms if, at r_B, the elasticity of demand is less than or equal to $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}<1$. It then follows from the necessary and sufficient conditions of the suppliers that the banking rate is the only candidate for an invariant interest rate, if the elasticity of demand at r_B is less than or equal to $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$.

But is it in the interest of the supplier to finance risky firms at the banking rate when the elasticity of demand at r_B is less than or equal to $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$? Substituting r_B for $r_T^R\left(r_B\right)$ in left-hand side of condition (13) and using $\frac{1+r}{f+(1-f)\pi}-1=r_B$ yields

$$\pi\left(1+r_B\right)+(1-\pi)\,\delta \geq (1+r) \Rightarrow \delta \geq \frac{(1+r)f}{f+(1-f)\pi}. \tag{14}$$

It then follows that it is optimal for the supplier to finance risky firms at the banking rate if the fraction of the purchased inputs that the supplier rescues in default, δ, is bigger than or equal to $\frac{(1+r)f}{f+(1-f)\pi}$. Since δ is a fraction in the interval $(0,1)$, this condition can be satisfied only if $\frac{f}{1-f} \leq \frac{\pi}{r}$. In other words, the probability that a risky firm repays the debt, π, must be high relative to the fraction of safe firms in the industry, f. If this necessary condition is satisfied, there is a cut-off for δ beyond which there is an equilibrium with an invariant interest rate in the trade credit market at the banking rate if the elasticity of demand at r_B is less than or equal to $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$. Moreover, since the first order conditions of the supplier's problem are also sufficient (under the technical assumption that assures that the investment function is concave), the equilibrium with invariant interest rate is unique in the class of equilibria in which the supplier finances both types of firms.[7]

Proposition 3 below summarizes the conditions that assure the invariance of interest rates in the trade credit markets at the banking rate.

Proposition 3 - *Suppose that the investment function is concave on the interest rate, that* $\frac{f}{1-f} \leq \frac{\pi}{r}$, *that the fraction of unused inputs that the supplier rescues in case a risk firm fails satisfies* $\delta \geq \frac{(1+r)f}{f+(1-f)\pi}$, *and that the interest-elasticity of the demand for inputs is smaller than or equal to* $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$ *at the rate* $r_B = \frac{1+r}{f+\pi(1-f)} - 1$. *Then there is an equilibrium in which the supplier finances both types of firms at an interest rate that is equal to the banking rate* r_B. *Moreover, this equilibrium is unique in the class in the class of equilibria in which the supplier finances both types of firms.*

From Proposition 3, the invariance at the banking rate requires that the elasticity of demand at r_B be smaller than or equal to $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$. What happens if this condition is not satisfied? It is easy to show that there is no equilibrium with an invariant interest rate if the demand at r_B is in the open interval $(\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}, 1)$. To see why, note first that, if the demand is inelastic at r_B (as it must happen in the interval $(\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}, 1)$), equation (8) implies that the optimal interest rate of trade credit for safe firms is the banking rate r_B. In

[7] The role of the banking rate as an outside option thus determines the equilibrium level of interetest rates in the trade credit markets, even for suppliers that provide all of their customers' financing requirements. Felli and Harris (1996) also explore this role of outside options in a model of investment decisions in human capital. They show that an employee's productivity in a rival firm matters, even when an investment in firm-specific human capital reduces the chances that the employee changes jobs.

contrast, an elasticity of demand bigger than $\frac{(1+r_B)\pi}{(1+r_B)\pi+(1-\pi)\delta}$ implies, from equation (12), that the optimal trade credit rate for risky firms is smaller than the banking rate r_B. Hence, there cannot be an equilibrium with invariant interest rates in the trade credit markets.

Suppose now that the demand is elastic at the banking rate r_B. From equations (8) and (12), the interest rates to safe and risky firms are both smaller than the banking rate r_B. From the first order conditions of programs (6) and (10), one can easily check that, for interior optima (i.e., optimal interest rates in the open interval $(0, r_B)$), the interest rates to safe and risky firms are not equal. Hence, if the demand is elastic at the banking rate, then the only candidate for the invariant rate is zero.

A zero interest rate is optimal for loans to safe firms if the demand is elastic at the rate zero and, for loans to risky firms, if the elasticity at zero is bigger than or equal to $\frac{\pi}{\pi+(1-\pi)\delta} < 1$. Of course, this latter condition is automatically satisfied by an elastic demand at zero, which is the necessary and sufficient condition for the optimal rate for the safe firm to be zero. We thus obtain a necessary condition for an equilibrium with invariant rates at zero: an elastic demand at a zero interest rate. This condition is also sufficient if it is indeed optimal for the supplier to finance the risky firm at a zero interest rate, which will the the case if condition (13) is satisfied at $r_T^R = 0$, that is, if

$$\pi I\left(0\right) + \left(1-\pi\right)\delta I\left(0\right) \geq \left(1+r\right) I\left(r_B\right). \tag{15}$$

Proposition 4 exhibits conditions on our exogenous variables (i.e., the fraction f of risky firms, the probability π that a risky firm succeeds, and the fraction of unused inputs that the supplier rescues in default, δ) that assure that condition (15) is satisfied. In this case, the supplier is willing to finance not only the safe but also risky firms at a zero interest rate. Proposition 4 also shows that if the demand is elastic at zero, then the equilibrium with an invariant rate at zero is unique in the class of equilibria in which the supplier finances both types of firms.

Proposition 4 - *Suppose that the investment function is concave on the interest rate. Then there is an equilibrium in which the supplier finances both types of firms at a zero interest rate, if the fraction of unused inputs that the supplier rescues in default, δ, is bigger than or equal to* $\frac{(1+r)I\left((1+r)(f+(1-f)\pi)^{-1}-1\right)}{(1-\pi)I(0)} - \frac{\pi}{(1-\pi)}$ *and the demand for inputs is elastic at a zero interest rate. Moreover, these two conditions assure that the equilibrium at the zero interest rate is is unique in the class of equilibria in which the supplier finances both types of firms.*

Proposition 4 takes into account that higher operational profits from increased sales may not be enough to induce the supplier to waive interest. If so, the supplier's comparative advantage in rescuing collateral in default must be larger than $\frac{(1+r)I\left((1+r)(f+(1-f)\pi)^{-1}-1\right)}{(1-\pi)I(0)} - \frac{\pi}{(1-\pi)}$. Economic intuition suggests, nonetheless, that a comparative advantage in default should not be necessary if the demand is sufficiently elastic. After all, in this case, a reduction in the cost of financing to zero should induce an increase in operational profits that more than offset the loss of interest. Proposition 5 formalizes this intuition.

Proposition 5 - *Suppose that the investment function is concave on the interest rate and that the interest-elasticity of demand for inputs is greater than* $\left(1-\frac{\pi}{1+r}\right)\frac{f+(1-f)\pi}{1+r-f-(1-f)\pi}$ *at a zero interest rate. Then there is an equilibrium in which the supplier finances both types of firms at a zero interest rate, even if the supplier cannot rescue any unused input in default, that is,* $\delta = 0$.

Using data from a survey of the credit managers of 2,538 firms in COMPUSTAT, Ng, Smith and Smith (1999) report that interest rates in trade credit markets are often standardized. And that suppliers waive interest whenever their customers repay the loans within 10 days. Waiving interest rates is consistent with Propositions 4 and 5 if the demand for inputs is elastic with respect to interest rates of loans that must be repaid in a very short term.

4 Empirical Implications and Discussion

4.1 Do suppliers have incentives to release information truthfully?

Consider an equilibrium with invariant interest rates and suppose that banks request information on the credit standing of a supplier's customer. Is it in the supplier's interest to release the information truthfully? Proposition 6 shows that announcing that the customer is a risky firm is a weakly dominating strategy for a supplier that can offer trade credit.

Proposition 6 - *Consider a supplier that can offer trade credit. In the equilibrium in which the invariant interest rate is at the banking rate, the expected profit of the supplier increases if it can credibly announce that the customer is a risky firm. In the equilibrium in which the supplier waives interest, no announcement changes the supplier's expected profits. Hence, it is a weakly dominating strategy for the supplier to announce that a customer is a risky firm.*

The intuition for Proposition 5 is quite simple. In the equilibrium in which the invariance is at the banking rate, competition from banks is the only reason that prevents the supplier from further increasing the interest rate. If the supplier can convincingly announce that the customer is a risky firm, banks will increase the interest rate accordingly, letting the supplier increase the interest rate as well. The higher interest rate moves the supplier closer to their unconstrained optimal. In turn, the banking interest rate is not relevant to the supplier in the equilibrium in which the supplier waives interests.

Consider now a supplier that cannot offer trade credit. Here, the incentives to release information are reversed. If the supplier can convince the banks that their customers are safe firms, the banking rate will decrease accordingly, and the customer will demand more inputs. It is then a dominating strategy for the supplier to announce that their customers are safe.

To be sure, banks can offer some revelation mechanism to suppliers. For instance, profit-sharing mechanisms between a bank and a supplier should provide incentives for the supplier to credibly reveal private information. Still, we are not aware of any study that documents revelation mechanisms between banks and suppliers in standard trade-credit transactions. It is conceivable, though, that some sort of revelation mechanism is in place in project loans that are typically structured around very complex contracts. In these types of transactions, we do not expect an equilibrium with invariant interest rates.

4.2 Monopoly power and informational advantage

So far, we have assumed that a supplier enjoys monopoly power in the market for inputs. Are our results robust to competition among suppliers? To answer this question, we assume in this section that suppliers have access to a technology with constant returns to scale and a unit cost c of producing inputs.

Competition in the market for inputs drives prices to the marginal cost, that is, $p = c$. In the absence of operational profits, there is no reason for a supplier to finance firms at a subsidized interest rate. The question then is whether a competitive market for inputs drives financial profits to zero as well.

Suppose first that the suppliers are all equally informed. In this case, there is no scope for a supplier to profit by lending to a safe firm. Competition will drive the interest rate of loans to safe firms to the riskless rate r. Note, however, that the equilibrium interest rate in loans to risky firms will not be equal to the riskless rate. In these loans, the supplier takes into account the probability $1-\pi$ that the debt contract will not be honored and that, in default, only δI will be collected. As such, the interest rate r_T^R that equals the expected return to the riskless rate solves $\pi\left(1+r_T^R\right)+(1-\pi)\,\delta=(1+r)$, which yields $r_T^R>r=r_T^S$. We conclude that competition among equally informed suppliers breaks down the equilibrium with invariance of interest rates in trade credit markets.

It is unlikely, nonetheless, that all suppliers of inputs are equally informed about their customers. It should be easier to learn private information about your best customers. As such, although a threat to buy inputs from an alternative supplier may force a competitive price for the inputs, it may not break down the current supplier's informational advantage. In this case, the analysis of the pure monopoly case applies. In particular, the informed supplier has the option of reducing the interest rate of loans to safe firms, while it cannot increase the interest rate beyond the firm's outside option, which may be the interest rate that a rival supplier offers. As before, a sufficiently inelastic demand makes it optimal for the current supplier to offer financing to both types of firms at their outside option, while an elastic demand may induce the supplier to waive interest.

4.3 Invariance of interest rates and efficiency in default

As Proposition 3 demonstrates, an equilibrium with invariance at the banking rate requires that suppliers be more efficient than banks in lending to risky firms. In this paper, we model this relative efficiency as a greater ability to salvage assets of financially distressed firms. This is not the only reason for suppliers to be more efficient lenders to risky firms, though. Cuñat (2002) points out that the threat of stopping the supply of vital intermediate goods may induce firms to prioritize payments to suppliers. Still, a low comparative advantage in salvaging assets of financially distressed firms makes it more difficult for the supplier's threat to be effective enough to assure that lending to risky firms is profitable. As we argue below, a testable implication of our model then follows.

The suppliers' advantage is salvaging assets is likely to be stronger in firms that hold a low fraction of their inventory in finished goods. After all, once firms transform intermediate goods into finished goods, suppliers can no longer use their regular sales force to sell the firms' inventory. Indeed, Petersen and Rajan (1997) show that suppliers offer larger lines of credit to firms with a low fraction of finished goods in the inventory; a finding that they interpret as evidence that suppliers are relatively more efficient than banks in transforming collateral into liquid assets. Accordingly, our model predicts that suppliers are more likely to offer standardized interest rates in industries whose firms keep a low fraction of finished goods in their inventory.

4.4 Credit constraint and insurance against liquidity shocks

In a sample of small firms in the National Survey of Small Business Finances (NSSBF), Petersen and Rajan (1997) find evidence that the demand for trade credit is inelastic. Given the high interest rate implicit in the most standard discounts for early payments (44 percent a year for the 2-10 net 30 loans), Petersen and Rajan argue that trade credit is likely associated with credit constrained firms. Yet, credit constraint cannot account for the invariance of the terms of credit at positive rates. In the absence of alternative sources of financing, suppliers are free to charge the interest rate that maximizes expected profits, which should vary with the borrowers' credit standing. The model thus predicts that, although suppliers commonly publicize a single trade credit policy for all firms, they charge varying interest rates from the credit constrained customers. Standard terms of trade credit should be available mostly for customers that are not credit constrained.

Likewise, the invariance of the terms of credit is not consistent with arguments that relate high discounts in early payments with an insurance against liquidity shocks. For instance, Wilner (2000) points out that the business relationship provides incentives for suppliers to bail-out financially distressed customers. If these incentives were relevant, the insurance should be priced in the trade credit for the riskier firms but, of course, not as much in the trade credit of the safest firms. Whenever the invariance of the terms of trade credit prevails, therefore, the incentives for suppliers to bail out their customers should not be strong.

4.5 Uniqueness of equilibrium with invariant rates

So far, we have restricted attention to equilibria in which, whenever possible, suppliers offer trade credit to both types of firms. In this class, Propositions 3, 4 and 5 show that the equilibrium with invariance is unique if the supplier's advantage of rescuing collateral in default is sufficiently large and if the demand is either sufficiently elastic or inelastic. Moreover, if the conditions on the elasticity of demand are not satisfied, it can be shown that there may be equilibrium with varying interest rates.

The reason for focusing on the class of equilibria in which there is trade credit for both types of firm is quite clear. The invariance of interest rates is meaningful only if suppliers finance firms in different classes of risk. Yet, as often happens in game theoretical models, there exists an equilibrium in which the supplier offers trade credit to safe firms only.[8] In this section, we exhibit conditions under which this alternative equilibrium breaks down while preserving the equilibrium with invariance of interest rates.

To break down equilibria in which the supplier provides trade credit to safe firms only, it suffices to make it optimal for the supplier to offer trade credit to risky firms. As before, the net benefits of lending to risky firms depends on the banking rate. Our first task, therefore, is to characterize the banking rate in the alternative equilibrium.

Assume then that the supplier denies trade credit to risky firms. If so, risky firms borrow from banks to purchase inputs. Understanding the equilibrium strategies, banks update their beliefs on a firm that requests a banking loan as follows. If the supplier can offer trade credit (an event with probability x) the firm that asks a banking loan is certain to be risky, and the loan will be repaid with probability π. If, however, the supplier cannot offer trade credit (an event with probability $1-x$), a firm that requests a banking loan is safe with probability f – and the loan will be repaid with probability one – while it will be risky with probability $1-f$, in which case the loan will be repaid with probability π. The expected return on a banking loan at an interest rate $\tilde{r}_B$ is thus $(1+\tilde{r}_B)\left[(f+(1-f)\pi)(1-x)+\pi x\right]$. And the interest rate that makes the loan's expected return equal to the cost of funds r solves:

$$(1+\tilde{r}_B)\left[(f+(1-f)\pi)(1-x)+\pi x\right]=(1+r)\,. \tag{16}$$

Solving for the banking rate in equation (16) yields

$$\tilde{r}_B=\frac{(1+r)}{\left[\ (f+(1-f)\pi)(1-x)+\pi x\right]}-1. \tag{17}$$

One can easily verify that the interest rate $\tilde{r}_B$ is bigger than the banking rate r_B of the equilibrium in which the supplier finances both types of firms (see equation (4)). Intuitively,

[8] It is easy to show that there is no equilibrium in which the supplier finances only risky firms. Suppose that such equilibrium exists. In this equilibrium, a risky firm will still borrow from banks when trade credit is not available. Hence the equilibrium banking rate must be higher than the riskless rate. But, if competition drives the banking rate down until it equals the bank's cost of funds, it leaves expected profits to suppliers that are more efficient in rescuing inputs in default. It then follows that it is optimal for suppliers to lend to safe firms, breaking down the candidate for equilibrium. A similar argument breaks down equilibria in mixed stratetigies.

the banking rate $\tilde{r}_B$ of the alternative equilibrium must take into account that a larger number of risk firms borrow from banks in the equilibrium in which the supplier always deny trade credit to risky firms.

We now have all the necessary ingredients to break down the equilibrium in which the supplier denies trade credit to risky firms. To break down this equilibrium, we force that it is optimal for the supplier to finance risky firms at the best possible interest rate. This condition is satisfied if

$$\frac{\pi\left(1+r_T^R\left(\tilde{r}_B\right)\right) I\left(r_T^R\left(\tilde{r}_B\right)\right)+(1-\pi)\,\delta I\left(r_T^R\left(\tilde{r}_B\right)\right)}{(1+r)} \geq I_B^R\left(\tilde{r}_B\right). \tag{18}$$

The left-hand-side of condition (18) is the present value of the expected profit of a supplier that finances the risky firm. This expected profit depends on the probability π that the risky firm repays the debt, the supplier's ability to rescue unused assets in default, δ, and the interest rate $r_T^R\left(\tilde{r}_B\right)$ of the loan to the risky firm. The interest rate $r_T^R\left(\tilde{r}_B\right)$ is supposed to maximize the supplier's expected profits conditioned on lending to the risky firm at a rate that is not larger that the firm's outside option, that is, the banking rate $\tilde{r}_B$. As it turns out, the maximization program that yields $r_T^R\left(\tilde{r}_B\right)$ is identical to the program (10) that solves for the optimal rate to risky firms in the equilibrium that the supplier finances both types of firms. Hence, we can characterize $r_T^R\left(\tilde{r}_B\right)$ by equation (12), once we substitute $\tilde{r}_B$ for r_B.

Consider now the right-hand-side of condition (18). $I\left(\tilde{r}_B\right)$ is the expected profit at $t=0$ of a supplier that does not offer trade credit to the risky firm. This expected profit consists of the sale of inputs (recall that the cost of producing the input is zero), which is determined by the demand for inputs of the risky firm at the banking rate $\tilde{r}_B$. Since, in this case, the supplier is paid at $t=0$, we do not need to discount the profit $I\left(\tilde{r}_B\right)$. It then follows that condition (18) assures that financing the risky firm adds value to the supplier. If this condition is satisfied, the equilibrium breaks down because the supplier would have incentives to finance risky firms.

A quick inspection shows that condition (18) is almost identical to condition (13), which assures that the interest rate to risky firms in equation (12) is optimal in the equilibrium in which the supplier finances both types of firms. The sole difference in the two conditions is the banking rate. Since the necessary and sufficient conditions for the equilibrium with invariance assure that condition (13) holds, it follows from Proposition 4 that condition (18) holds when $\tilde{r}_B$ is equal to r_B. In other words, $\tilde{r}_B$ equal to r_B implies that the conditions that assure that the equilibrium with invariant rates exists also assure that the equilibrium in which the supplier offers trade credit to safe firms only breaks down.

But when is $\tilde{r}_B$ equal to r_B? A sufficient condition is that the market of banking loans is strongly competitive in the sense that a supplier's decision to deny trade credit does not change the equilibrium banking rate. A way to model this competitiveness hypothesis is to assume that the investment I consists of purchases of different inputs each of them supplied by a monopolist. Suppose also that at least one of the suppliers is certain to be credit constrained. In this case, asking for banking loans to finance purchase of inputs does not convey information on the firm's type. More formally, the probability x that trade credit satisfies all financing needs becomes arbitrarily close to zero. And, as one can easily check in equation (17), $\tilde{r}_B$ converges to $\frac{1+r}{f+\pi(1-f)} - 1 = r_B$.

5 Conclusions

Using data from a survey of the credit managers of 2,538 firms in COMPUSTAT, Ng, Smith and Smith (1999) report that interest rates in trade credit markets are often standardized. In particular, a usual term of trade credit requires full payment in 30 days, granting a two percent discount for early payments within 10 days of the invoice. This two percent discount implies an annual interest rate of 44 percent for borrowers who pay the loan in the thirtieth day. As Petersen and Rajan (1994) point out, the discount date is not strictly enforced, which implies some variation in the effective interest rates in the trade credit markets. Still, one would expect that suppliers charge interest according to their beliefs on their customers' credit standing, as it happens in banking loans (see Petersen and Rajan (1995)). Since there is evidence that, vis-à-vis banks, suppliers are actually better informed about the economic health of their customers, a low level of variation of interest rates in the trade credit markets is, at first glance, puzzling.

This paper provides a reason for why interest rates in trade credit markets do not vary with the borrowers' credit risk. We argue that if the demand for intermediate products is elastic, waiving interest increases the demand for the suppliers' products, leading to an increase in operational profits that may offset financial losses in the financing of inputs. A high elasticity of demand, therefore, may explain why suppliers in the U.S. usually offer trade credit at no interest when the loan is repaid in up to 10 days, as Ng, Smith and Smith (1999) and Elliehausen and Wolken (1993) report.

In the other extreme, an inelastic demand with respect to interest rates gives suppliers no incentives to undercut banks that mistakenly perceive safe firms as risky. The optimal interest rate of trade credit to these safe firms is then the interest rate available in banking loans. In turn, competition with uninformed banks prevents suppliers from offering financing at high interest rates to risky customers that banks mistakenly perceive as safe. If suppliers have a large advantage over banks in transforming collateral into liquid assets, then they may be willing to lend at the banking rate to risky firms. If so, the interest rates in the trade credit markets collapse to the banking rate. Otherwise, the equilibrium with invariance breaks down, yielding a testable implication for our model: Invariance of interest rates in trade credit markets is more likely in industries whose firms keep a low fraction of finished goods in inventory.

Appendix

Proof of Proposition 1:

We can write the profit of the supplier as $\Phi_T^S\left(r_T^S\right) = (1+r_T^S)I\left(r_T^S\right)$. Differentiating $\Phi_T^S\left(r_T^S\right)$ with respect to r_T^S yields

$$\frac{\partial}{\partial r}\Phi_T^S\left(r_T^S\right) = (1+r_T^S)I'\left(r_T^S\right) + I\left(r_T^S\right), \tag{19}$$

$$\frac{\partial^2}{\partial r^2}\Phi_T^S\left(r_T^S\right) = (1+r_T^S)I''\left(r_T^S\right) + 2I'\left(r_T^S\right). \tag{20}$$

From equation (20), the profit function Φ_T^S is concave on r_T^S because the investment function is concave and decreasing on r_T^S. Concavity of Φ_T^S implies that the first order condition of the supplier's problem is also sufficient for optimality. To complete the proof, we analyze three cases:

Case 1: Suppose first that $\frac{\partial}{\partial r}\Phi_T^S(r_B) \geq 0$. This first case happens when $(1+r_B)I'(r_B) + I(r_B) \geq 0$, which is equivalent to $\frac{1+r_B}{I(r_B)}I'(r_B) \geq -1$. Noting that the absolute value of the left-hand side of this latter inequality is our definition of the interest-elasticity of the demand for inputs at r_B, $\epsilon^D(r_B)$, it follows that $\frac{\partial}{\partial r}\Phi_T^S(r_B) \geq 0$ if and only if $\epsilon^D(r_B) \leq 1$. By concavity of the profit function, the marginal profits decrease with the interest rate. Hence, $\frac{\partial}{\partial r}\Phi_T^S(r_B) \geq 0$ implies that profits increase with interest in the interval $0 \leq r_T^S \leq r_B$. We thus conclude that $\epsilon^D(r_B) \leq 1$ if and only if it is optimal for the supplier to set $r_T^S = r_B$.

Case 2: Suppose now that $\frac{\partial}{\partial r}\Phi_T^S(0) \leq \mathbf{0}$, which, from the arguments of the previous case, is equivalent to $\epsilon^D(0) \geq 1$. Since marginal profits decrease with the interest rate, $\frac{\partial}{\partial r}\Phi_T^S(0) \leq 0$ implies that the supplier's profit decreases with interest in the interval $0 \leq r_T^S \leq r_B$. Thus, it is optimal for the supplier to set $r_T^S = 0$. And we conclude that $\epsilon^D(0) \geq 1$ if and only if it is optimal for the supplier to set $r_T^S = 0$.

Case 3: Finally, suppose that $\frac{\partial}{\partial r}\Phi_T^S(0) > 0$ and $\frac{\partial}{\partial r}\Phi_T^S(r_B) < 0$. In this case, the supplier's profit reaches its maximum in the interval $0 < r_T^S < r_B$. As a result, it is optimal for the supplier to set $r_T^S \in (0, r_B)$. This third case happens when $I'(0) + I(0) > 0$ and $I'(r_B) + I(r_B) < 0$. Writing the above inequalities in terms of elasticity we obtain, $\epsilon^D(0) < 1$ and $\epsilon^D(r_B) > 1$, which are necessary and sufficient conditions for $r_T^S \in (0, r_B)$, as we wanted to prove. ■

Proof of Proposition 2: We can write the expected profit of the supplier as $\Phi_T^R(r_T^R) = \pi\left[(1+r_T^R)I\left(r_T^R\right)\right] + (1-\pi)\left[\delta I\left(r_T^R\right)\right]$.The first and second derivatives of the expected profit with respect to the interest rate r_T^R are:

$$\frac{\partial}{\partial r}\Phi_T^R\left(r_T^R\right) = \pi\left[(1+r_T^R)I'\left(r_T^R\right) + I\left(r_T^R\right)\right] + (1-\pi)\,\delta I'\left(r_T^R\right) \tag{21}$$

$$\frac{\partial^2}{\partial r^2}\Phi_T^R\left(r_T^R\right) = \pi\left[(1+r_T^R)I''\left(r_T^R\right) + 2I'\left(r_T^R\right)\right] + (1-\pi)\,\delta I''\left(r_T^R\right) \tag{22}$$

The expected profit Φ_T^R is concave on r_T^S, because the investment function is concave and decreasing on r_T^S. Concavity of Φ_T^R then implies that the first order condition of the supplier's problem is also sufficient for optimality. Rearranging terms in $\frac{\partial}{\partial r}\Phi_T^R\left(r_T^R\right)$ as in the proof of Proposition 1 yields:

Case 1: $\epsilon^D\left(r_B\right) \leq \frac{(1+r_B)\pi}{1+r_B\pi+(1-\pi)\delta}$. The expected profits increase with interest rates $\left(\frac{\partial}{\partial r}\Phi_T^R \geq 0\right)$ in the interval $[0, r_B]$. Hence, r_B is the optimal interest rate.

Case 2: $\epsilon^D\left(r_B\right) > \frac{(1+r_B)\pi}{1+r_B\pi+(1-\pi)\delta}$ and $\epsilon^D\left(0\right) < \frac{\pi}{\pi+(1-\pi)\delta}$. From the first inequality, the expected profit decreases with the interest rate at r_B. From the second inequality, the expected profit increases with the interest rate at the rate zero, reaching its maximum in the open interval $(0, r_B)$.

Case 3: $\epsilon^D\left(0\right) \geq \frac{\pi}{\pi+1-\pi\delta}$. The expected profit decreases with the interest rate $\left(\frac{\partial}{\partial r}\Phi_T^R \leq 0\right)$ in $[0, r_B]$, reaching its maximum at $r_T^R = 0$. ■

Proof of Proposition 4: We showed in section 3.4 that suppliers are willing to finance risky firms at a zero interest rate if $\pi I\left(0\right) + \left(1-\pi\right)\delta I\left(0\right) \geq \left(1+r\right) I\left(r_B\right)$. This condition can be rewritten as $\delta \geq \frac{(1+r)I(r_B)}{(1-\pi)I(0)} - \frac{\pi}{(1-\pi)}$. Since the demand for investment decreases with the interest rate, $I'\left(\cdot\right) < 0$, $r_B > 0$ implies that $I\left(r_B\right) < I\left(0\right)$. Define $\frac{I(r_B)}{I(0)} = \beta(r_B) \in (0,1)$. Since r_B is a function of π and r, so is $\beta(r_B)$ and we can write $\beta(\pi, r)$. Hence, we can once more rewrite the condition on δ as $\delta \geq \beta(\pi, r)\frac{(1+r)}{(1-\pi)} - \frac{\pi}{(1-\pi)}$.

Call $\psi\left(\pi\right) = \beta(\pi, r)\frac{(1+r)-\pi}{(1-\pi)} - \frac{\pi}{(1-\pi)}$. Since $\psi(0) = \beta(0, r)(1+r)$, there is an equilibrium with an invariant rate at zero for values of π sufficiently close to zero if the elasticity of demand at zero satisfies the condition in the statement of the proposition, and if $\beta(0, r)(1+r) < 1$, which assures that there is $\delta \in (0,1)$ satisfying $\delta \geq \beta(\pi, r)\frac{(1+r)-\pi}{(1-\pi)} - \frac{\pi}{(1-\pi)}$. We claim that the set of parameters that satisfy $\beta(0, r)(1+r) < 1$ is not empty. Plugging $r_B = \frac{1+r}{f+\pi(1-f)} - 1$ into the definition of $\beta(\pi, r)$ and making $\pi = 0$ in $\beta(0, r)(1+r) < 1$ yields $I\left(\frac{1+r}{f} - 1\right)(1+r) < I(0)$. For $r = 0$, this inequality holds because $I(\cdot)$ decreases with the interest rate and $f < 1$.

To prove that the inequality holds for any $r \geq 0$, define the left-hand side of the inequality as a function of r: $\varphi(r)$. Thus, it suffices to show that φ decreases with r. Differentiating φ obtains $\varphi'(r) = I(r_B) + I'(r_B)\frac{1+r}{f}$. Hence, after some algebra, $\varphi'(r) \leq 0$ if and only if $\epsilon^D(r_B) \geq 1$; an inequality that holds trivially because, by assumption of the proposition, the demand is elastic at r_B.

Let us now show that the equilibrium at zero interest rate is unique in the class of equilibria in which the supplier finances both types of firms. Equations (8) and (12) imply that the supplier will necessarily set a zero interest for loans to risky and safe firms, if $\epsilon^D(0) \geq 1$. If $\epsilon^D(0) \in \left(\frac{\pi}{\pi+(1-\pi)\delta}, 1\right)$, the supplier will set a zero interest rate for the risky firm and an interest rate in the interval $(0, r_B)$ to the safe firm. Hence, there is no equilibrium with an invariant rate. Finally, if $\epsilon^D(0) < \frac{\pi}{\pi+(1-\pi)\delta}$, then, from equations (8) and (12), the supplier sets interest rates in the interval $(0, r_B)$ for safe and risky firms. A quick inspection of the first order conditions of programs (6) and (10) shows that the optimal interest rates of the two types of firms cannot be equal if $\delta > 0$. As a result, there cannot be another equilibrium with invariant interest rate in this case as well. We conclude that there will be a unique equilibrium with invariant rates (at zero) if $\delta \geq \frac{(1+r)I(r_B)}{(1-\pi)I(0)} - \frac{\pi}{(1-\pi)}$ and $\epsilon^D(0) \geq 1$. ■

Proof of Proposition 5: In Proposition 4, the equilibrium with invariance at a zero interest rate obtains if the supplier's advantage in default satisfies $\delta \geq \frac{(1+r)I(r_B)}{(1-\pi)I(0)} - \frac{\pi}{(1-\pi)}$. This restriction is irrelevant if $\frac{(1+r)I(r_B)}{(1-\pi)I(0)} - \frac{\pi}{(1-\pi)} \leq 0$, or equivalently,

$$(1+r)\frac{I\left((1+r)(f+(1+f)\pi)^{-1}\right)}{I(0)} \leq \pi. \tag{23}$$

By the concavity of $I(.)$, any $x \geq 0$ implies that $I(x) \leq I(0) + I'(0)x$, which, at $x = (1+r)(f+(1+f)\pi)^{-1}$ implies equation (23) if $\frac{1+r}{\pi} \leq \frac{1}{1-\epsilon^D(0)(1+r)(f+(1+f)\pi)^{-1}}$. This latter inequality is equivalent to $\epsilon^D(0) \geq \left(1 - \frac{\pi}{1+r}\right)\frac{f+(1+f)\pi}{1+r-f-(1+f)\pi}$, as we wanted to prove. ■

Proof of Proposition 6: In the equilibrium with invariance at the banking rate, let r_B^R be the interest rate that banks would have offered to a known risky firm. Competition among banks implies that the expected return of the loan at the interest rate r_B^R equals the cost of funds r. Hence, $\pi(1 + r_B^R) = (1 + r)$ which implies that $r_B^R = \frac{1+r}{\pi} - 1 > r_B$. This means that the interest rate in banking loans increases if a supplier convinces the banks that the customer is a risky firm. As a result, the constraint in the supplier's program changes from $0 \leq r_T^R \leq r_B$ to $0 \leq r_T^R \leq r_B^R$. Since $r_B^R > r_B$, the constraint is relaxed, implying an increase in expected profits because a concave investment function implies that the supplier's profit function is concave. Finally, the banking rate is irrelevant in the equilibrium in which the supplier waives interests because the constraint $r_T^R \leq r_B$ does not bind when $r_T^R = 0$. ■

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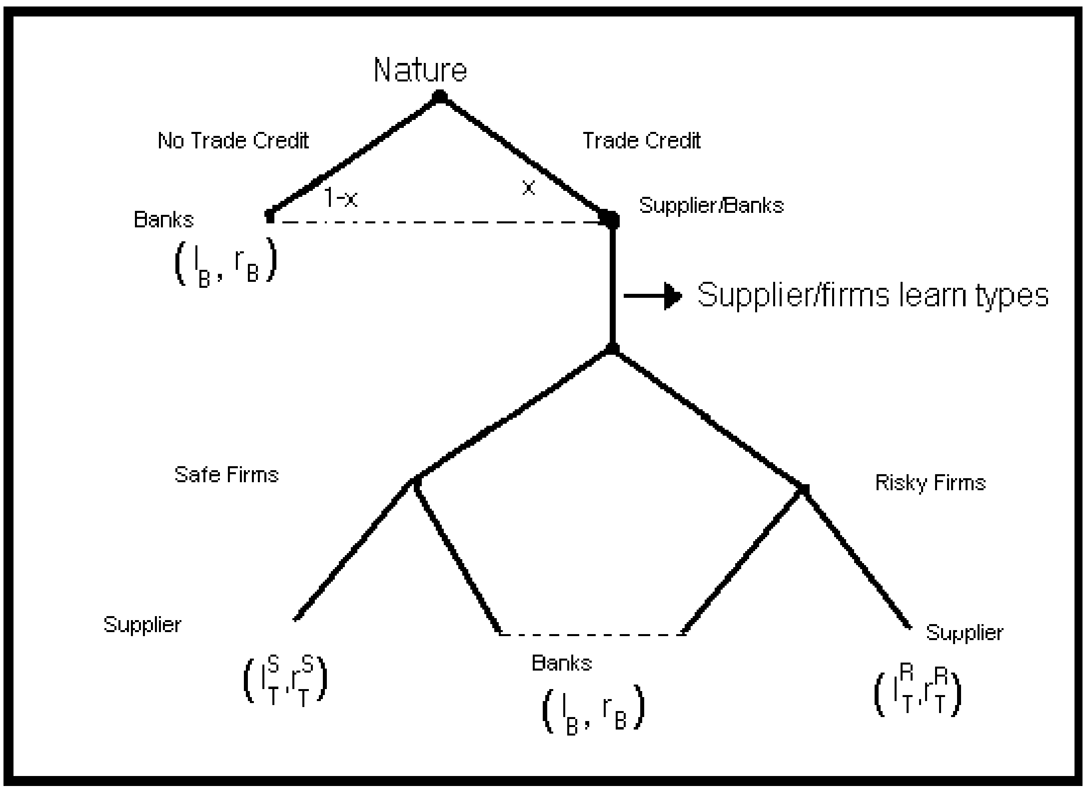

Figure 1: Game in the extensive form