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Fertility and the Value of Life*

Abstract

The value of life methodology has been recently applied to a wide range of contexts as a means to evaluate welfare gains attributable to mortality reductions and health improvements. Yet, it suffers from an important methodological drawback: it does not incorporate into the analysis child mortality, individuals' decisions regarding fertility, and their altruism towards offspring. Two interrelated dimensions of fertility choice are potentially essential in evaluating life expectancy and health related gains. First, child mortality rates can be very important in determining welfare in a context where individuals choose the number of children they have. Second, if altruism motivates fertility, life expectancy gains at any point in life have a twofold effect: they directly increase utility via increased survival probabilities, and they increase utility via increased welfare of the offspring. We develop a manageable way to deal with value of life valuations when fertility choices are endogenous and individuals are altruistic towards their offspring. We use the methodology developed in the paper to value the reductions in mortality rates experienced by the US between 1965 and 1995. The calculations show that, with a very conservative set of parameters, altruism and fertility can easily double the value of mortality reductions for a young adult, when compared to results obtained using the traditional value of life methodology.

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> "In many parts of the Essay I have dwelt much on the advantage of rearing the requisite population of any country from the smallest number of births. I have stated expressly, that a decrease of mortality at all ages is what we ought chiefly to aim at; and as the best criterion of happiness and good government, instead of the largeness of the proportion of births, which was the usual mode of judging, I have proposed the smallness of the proportion dying under the age of puberty."
>
> Thomas R. Malthus (1826)

1 Introduction

This paper incorporates fertility and altruism into the "value of life" approach. We develop a manageable way to deal with valuations of reductions in mortality rates when fertility choices are endogenous and individuals are altruistic towards their offspring. Our framework, designed to account for non-monetary aspects of welfare gains, falls on the tradition inaugurated by Schelling (1968), and extends previous techniques developed by Usher (1973), Arthur (1981), and Rosen (1988, 1994), among others.

Recently, the value of life methodology has been applied to a wide range of contexts as a means to evaluate welfare gains attributable to mortality reductions and health improvements. Nordhaus (1999), Murphy and Topel (2001), and Garrett (2001) applied this methodology to analyze different aspects of health related gains in welfare in the United States throughout the twentieth century. Becker, Philipson, and Soares (2003) applied an adapted version of this technique to evaluate the evolution of welfare inequality across countries, once improvements in life expectancy are accounted for.

These studies made important contributions to the understanding and measurement of non-monetary aspects of human welfare. Yet, they suffer from an important methodological drawback. They do not incorporate into the analysis child mortality, the fertility decisions of individuals, and their consequences in terms of welfare evaluation. Two interrelated dimensions of fertility choice are essential in evaluating life expectancy and health related gains. First, child mortality rates – which would be irrelevant for the welfare of an adult individual in an egoistic setup – can be very important in determining welfare in a context where individuals choose the number of children they have. Second, if altruism motivates fertility, life expectancy gains have a twofold effect: they directly increase utility via increased survival probabilities, and they increase utility via increased welfare of the offspring.[1]

In the paper, we evaluate the welfare implications of mortality reductions in a setup where individuals choose the number of children they have and are altruistic towards their children. We show that, under these circumstances, the value of adult mortality changes can be decomposed into three factors: the consumption factor from the traditional value of life specification, originally derived by Rosen (1988); a fertility factor, which accounts for the welfare improvements related to the higher probability of having children; and an altruism factor, which accounts for the fact that the mortality reductions will also be enjoyed by all future generations. Additionally, our approach allows us to calculate an adult's willingness to pay for reductions in child mortality. This willingness to pay will generally depend on the effect of child mortality on the final costs of child production, the uncertainty regarding the number of surviving children, and, again, on the fact that these gains will also be experienced by all future generations.

To illustrate the empirical relevance of fertility and altruism in the value of life problem, we apply our methodology to value the reductions in mortality rates experienced by the US between 1965 and 1995. The parameterization of the model poses a series of new questions regarding the calibration of some unusual parameters. We try to deal with these in the most reasonable way presently possible. The calculations show that, with a very conservative set of parameters, altruism and fertility can easily double the value of mortality reductions for a young adult, when compared to results obtained using the traditional value of life methodology.

Most of the issues discussed in the paper touch on the more general question of how to deal with children and future generations in the evaluation of specific policy interventions. Since the value of mortality risks is a completely forward looking measure, egoistic adults who survive childhood place zero value on mortality changes at earlier ages. For adults, childhood mortality is a contingency they already survived and will never face again. As a consequence, in the traditional framework, the contribution of changes in child mortality to the welfare of an adult is null. Naturally, this problem arises because the traditional framework does not account for altruistic behavior towards future generations. Obviously, future generations cannot voice their concerns or reveal their preferences via market behavior. And children, because of their lack of maturity and the consequent dependence on parental care, are incapable of legally deciding. For these same reasons, contractual arrangements involving children, or possibly unborn individuals, cannot directly incorporate life expectancy benefits into current evaluations of specific interventions (in this

[1] Cropper and Sussman (1988) and Rosen (1994) consider the problem of marginal willingness to pay for reductions in mortality rates when individuals leave bequests to a single descendant. Under certain circumstances, this may correspond to the incorporation of altruism into the analysis. But they do not include the choice of number of children in the problem. In addition, a relatively small share of parents leaves significant bequests, and that limits the applicability of their approach even when fertility is held constant.

regard, see Becker and Murphy (1988)). As an alternative, we allow for endogenous fertility and altruism in order to transfer the benefits of certain policies across different generations.

Altruism in human societies is certainly not restricted to the direct family or, particularly, to direct descendants. Nevertheless, the case for incorporating parent's altruism towards children in public policy evaluations seems far stronger than any other. Not only does this approach incorporates preferences of a significant share of the population who is not allowed to decide (children), but it also establishes an intergenerational link that ultimately accounts for the benefits accrued by all future generations. Additionally, the task of assigning values to welfare gains experienced by children is transferred to those who are actually their legal guardians, and already decide in their names in all relevant dimensions of life, namely, the parents.

The structure of the remainder of the paper may be outlined as follows. Section 2 presents a simple model illustrating the main implications of the incorporation of fertility and altruism into value of life calculations. Section 3 develops the general version of the model, and derives a formula for the social value of changes in survival functions. Section 4 illustrates the empirical relevance of our approach, by calculating the value of the mortality reductions experienced by the US between 1965 and 1995. The final section concludes the paper.

2 A Simple Model of Fertility and the Value of Life

This section illustrates the consequences of altruism and endogenous fertility for the valuation of life expectancy changes. We construct a simple example, analogous to Rosen's (1988, Section 1), to highlight the dimensions added to the problem and to compare our results to the previous literature.

Consider individuals who live for two periods: childhood and adulthood. Individuals face a probability p_c of surviving birth. If they survive birth, they become adults and face a probability p_a of survival into adulthood. Decisions are made just before adult death is realized. Adults value consumption, the number of surviving children they have, and the utility that each child will enjoy as an adult. Adults are responsible for all decisions in the economy.

Adults at period t receive a given endowment w_t. They decide on consumption and number of births (fertility), before the event with probability p_a is realized. Actuarially fair insurance is available for every good consumed by parents; n_t represents the number of births, b the goods cost of having a child, and e the goods cost of raising a surviving child. Given these assumptions, we write the budget constraint as:

$$w_t = p_a(c_t + bn_t + p_c n_t e). \tag{1}$$

The costs of having and raising children are fixed. To keep things simple and comparable to previous results, we assume no margin in which parents can invest or transfer additional resources to their children.[2] Following Becker and Barro (1988), we assume that the discount rate applied to the future utility of children increases with the number of children in a strictly concave fashion. We define $N_t \leq n_t$ as the number of surviving children, and the discount rate applied to the utility of each surviving child as αN_t^{ϕ}. In the valuation of survivors, ϕ represents the degree of risk aversion of the parents with $\phi < 1$.

By the dynastic interaction, the problem proposed has a recursive structure. The expected utility of an adult individual at time t satisfies the following value function

$$V_t = \max_{\{c_t, n_t\}} \left\{ E\left[u(c_t) + \alpha N_t^{\phi} V_{t+1} \right] \right\}, \tag{2}$$

subject to (1).

Note that there are two different dimensions of uncertainty in the problem. One is related to whether the individual will die before being able to realize the consumption and fertility plans, while the other is related to the number of children who survive (N_t) out of the total number of children born (n_t). The value function is defined as the expected utility of an adult individual. Therefore, there is no uncertainty regarding the value of V_{t+1} itself if the series $\{w_t, w_{t+1}, w_{t+2}, ...\}$ is known. We maintain this assumption and, as Rosen (1988), normalize the utility in the event of death to zero in order to write

$$V_t = \max_{\{c_t, n_t\}} \left\{ p_a \left[u(c_t) + \alpha E\left(N_t^{\phi}\right) V_{t+1} \right] \right\}, \tag{3}$$

subject to

$$w_t = p_a(c_t + bn_t + p_c e n_t).$$

This problem is equivalent to the one discussed in Rosen (1988, Section 1) once fertility and altruism are incorporated into the analysis, although we should note that fertility introduces a new, nontrivial, dimension of uncertainty in the discussion. Since one of our main motivations is to explore child mortality in a context of value of life calculations, we deal explicitly with this uncertainty. To study child mortality, we follow Sah (1991), and assume that the number of surviving children (N_t) follows a binomial distribution with density function

[2] Note that our assumptions eliminate the traditional quantity-quality trade-off in terms of the analytitcal results of the model. Nevertheless, when the model is applied to data to evaluate different situations, this trade-off will be reflected in different values for the parameters b and e. As will be discussed, these parameters will determine the monetary values of welfare gains related to fertility.

$$f(N_t; n_t, p_c) = \begin{pmatrix} n_t \\ N_t \end{pmatrix} p_c^{N_t}(1-p_c)^{n_t-N_t}, \text{for } N_t = 0, 1, 2, ..., n_t. \tag{4}$$

Therefore, by definition, we can write

$$E\left(N_t^{\phi}\right) = \sum_{N_t=0}^{n_t} N_t^{\phi} f(N_t; n_t, p_c).$$

As in Kalemli-Ozcan (2002), we approximate the function $E\left(N_t^{\phi}\right)$ around the expected number of survivors $E\left(N_t\right) = p_c n_t$ by the delta method. This strategy allows us to deal with fertility as a continuous variable and still account for the effects associated with the risk regarding the number of surviving children, via the explicit consideration of the second moment of the distribution. A second order approximation to $E\left(N_t^{\phi}\right)$ leads to

$$E_t\left(N_t^{\phi}\right) \simeq (p_c n_t)^{\phi} - \frac{\phi(1-\phi)}{2}(1-p_c)\left(p_c n_t\right)^{\phi-1}, \tag{5}$$

since $E_t\left(N_t - E_t(N_t)\right) = 0$ and the variance for the binomial distribution satisfies $Var(N_t) = E_t\left[N_t - (p_c n_t)\right]^2 = n_t p_c(1-p_c)$. Using this result, the individual problem becomes

$$V_t = \max_{\{c_t, n_t\}} p_a \left\{u(c_t) + \alpha\left[(p_c n_t)^{\phi} - \frac{\phi(1-\phi)(1-p_c)(p_c n_t)^{\phi-1}}{2}\right] V_{t+1}\right\}, \tag{6}$$

subject to (1). The first order conditions determining the optimal consumption and fertility decisions are given by the following expressions, plus the budget constraint:

$$p_a u'(c_t) = p_a \lambda_t, \quad \text{and}$$

$$p_a \alpha \left[\phi(p_c n_t)^{\phi-1} p_c + \frac{\phi(1-\phi)^2(1-p_c)(p_c n_t)^{\phi-2} p_c}{2}\right] V_{t+1} = p_a(b + e p_c)\lambda_t,$$

where λ_t is the multiplier on the budget constraint.

For future reference, note that the second expression in the first order conditions can be rewritten in a simpler form as

$$p_a \alpha \frac{\partial E(N_t^{\phi})}{\partial n_t} V_{t+1} = p_a(b + e p_c)\lambda_t.$$

Changes in Adult Survival

The value of changes in life expectancy can be expressed as the marginal willingness to pay for increases in the probabilities p_a and p_c. For p_a, we define MWP_t^a as:

$$MWP_t^a = \frac{\partial V_t}{\partial p_a}\frac{1}{\lambda_t}.$$

This marginal willingness to pay can be interpreted as the monetary value of the marginal utility from increased survival probability, or, alternatively, as the marginal rate of substitution between the survival probability and income. From the envelope theorem, this expression can be written as

$$MWP_t^a = \frac{u(c_t) + \alpha E(N_t^\phi)V_{t+1}}{\lambda_t} + \frac{p_a \alpha E(N_t^\phi)}{\lambda_t}\frac{\partial V_{t+1}}{\partial p_a} - c_t - bn_t - p_c e n_t.$$

Using the first order conditions, we obtain:

$$MWP_t^a = \left(\frac{1}{\varepsilon_c} - 1\right) c_t + (b + ep_c)\left(\frac{1}{\varepsilon_n} - 1\right) n_t + p_a \alpha E(N_t^\phi)\frac{\partial V_{t+1}}{\partial p_a}\frac{1}{\lambda_t},$$

where ε_c and ε_n denote the elasticities of the consumption and fertility sub-utility functions in relation to their respective arguments:[3]

$$\varepsilon_c = \frac{\partial u(c_t)}{\partial c_t}\frac{c_t}{u(c_t)}, \qquad \text{and} \qquad \varepsilon_n = \frac{\partial E\left(N_t^\phi\right)}{\partial n_t}\frac{n_t}{E\left(N_t^\phi\right)}.$$

In a stationary environment, where $w_t = w_{t+1} = w_{t+2} = ... = w$, we have that $\frac{\partial V_t}{\partial p_a} = \frac{\partial V_{t+1}}{\partial p_a} = \frac{\partial V}{\partial p_a}$ and $\lambda_t = \lambda_{t+1} = \lambda$. In this case, we can write

$$MWP^a = \frac{1}{1 - p_a \alpha E(N^\phi)}\left[\left(\frac{1}{\varepsilon_c} - 1\right) c + (b + ep_c)\left(\frac{1}{\varepsilon_n} - 1\right) n\right]. \tag{9}$$

Equation (9) can be immediately compared to the results from the original value of life literature. It highlights the insights gained by incorporating fertility and altruism into the analysis. Without fertility decisions and altruism, (9) would be reduced to $(\varepsilon_c^{-1} - 1)c$, which is exactly the result presented by Rosen (1988, p.287). This result summarizes the fact that increases in life expectancy will be more valuable for higher levels of consumption and a lower elasticity of the sub-utility function $u(c_t)$. However, because higher survival probabilities increase the cost of the

[3] Fertility will increase the traditional valuation formula if $\varepsilon_n < 1$. This condition corresponds to

$$\phi\frac{1 - p_c}{p_c}\frac{2 - \phi}{2} < n_t,$$

which we assume from now on. This will always be true for "reasonable" values of p_c and n_t. For example, for any child survival rate above 0.5, the left-hand side of the expression is smaller than 0.5 for all ϕ between 0 and 1. Since we have a single-sex model, n_t can be thought of as half the fertility rate. This means that the inequality above would hold for any values of fertility and child mortality currently observed in the world.

actuarially fair insurance, reductions in mortality lower consumption in case of survival.[4] This is precisely the sense in which Rosen (1994) identifies a trade-off between the quantity and the quality of life.

The second term inside brackets represents an analogous effect in relation to fertility. The term $(b + ep_c)$ converts the value of fertility into monetary (consumption) units, while the rest of the expression is completely analogous to the case of consumption. The new expression states that increases in life expectancy will be more valuable in high fertility societies and for low values of the elasticity of the fertility sub-utility in relation to its argument. Again, this last effect derives from the fact that increases in survival probabilities increase the cost of survivors, therefore reducing fertility in case of survival.

Finally, the term outside brackets adjusts the value of life for the fact that not only will these changes affect the current generation, but they will also benefit all future generations. MWP_a accounts for the present discounted value of the welfare gains $[(\varepsilon_c^{-1} - 1)c + (b + ep_c)(\varepsilon_n^{-1} - 1)n]$ per generation, discounted at a rate $p_a \alpha E(N^{\phi})$.[5] In order to write the present value of welfare gains for all future generations in an expression as simple as the one above, we will assume a stationary environment. So, when applied to the data, our methodology will be able to evaluate the welfare gains of certain changes in mortality rates if current conditions were maintained indefinitely into the future. In other words, our framework will tell us what the value of certain life expectancy improvements would be in a stationary world where the conditions observed in the data persisted forever. In any case, to the extent that economies display long-run growth, it will be an underestimation of the true welfare improvements brought about by reductions in mortality rates.

The results discussed in this section neatly illustrate the features incorporated into the valuation of adult mortality changes once fertility and altruism are taken into account. Apart from the usual channels, a permanent decline in adult mortality will benefit the current and future generations due to altruistic links, and to the higher probability that adults will live long enough to have children. But, besides these gains, there are also important aspects of changes in child mortality and their interactions with fertility that are ignored in the traditional approach. By adding another uncertainty margin to the problem, these aspects reveal a new dimension in the valuation of life expectancy changes.

[4] A more straightforward interpretation would take p_a as the deterministic adult lifetime. In this case, increases in adult longevity would increase the period over which a fixed amount of resources has to be spread out, reducing consumption in each period of life.

[5] For this to be the case, we must have $p_a \alpha E(N^{\phi}) < 1$, which is precisely the condition required for the recursive problem described before to be well defined.

Changes in Child Survival

We define MWP_t^c as the marginal willingness to pay of adults at time t for increases in p_c. As we stated before, parents, not children, express their concern for survivors and their willingness to pay for mortality changes. From the envelope theorem, and using the first order conditions, we can write MWP_t^c as:

$$MWP_t^c = \frac{\partial V_t}{\partial p_c}\frac{1}{\lambda_t} = (b + p_c e)\frac{p_a\phi(p_c n_t)^{\phi-1}n_t + p_a\frac{\phi(1-\phi)^2(1-p_c)(p_c n_t)^{\phi-2}n_t + \phi(1-\phi)(p_c n_t)^{\phi-1}}{2}}{\phi(p_c n_t)^{\phi-1}p_c + \frac{\phi(1-\phi)^2(1-p_c)(p_c n_t)^{\phi-2}p_c}{2}}$$
$$+ \frac{p_a\alpha\left[(p_c n_t)^{\phi} - \frac{\phi(1-\phi)(1-p_c)(p_c n_t)^{\phi-1}}{2}\right]}{\lambda_t}\frac{\partial V_{t+1}}{\partial p_c} - p_a e n_t,$$

or more compactly as:

$$MWP_t^c = \frac{p_a}{p_c}\left\{bn_t + \frac{(1-\phi)}{2+(1-\phi)^2\mu^2}(b + p_c e)\right\} + p_a\alpha E(N_t^{\phi})\frac{\partial V_{t+1}}{\partial p_c}\frac{1}{\lambda_t},$$

with $\mu = \frac{\sqrt{Var(N_t)}}{E(N_t)}$ as the coefficient of variation.

Again, in a stationary environment we can write:

$$MWP^c = \frac{1}{1 - p_a\alpha E(N^{\phi})}\frac{p_a}{p_c}\left[bn + \frac{(1-\phi)}{2+(1-\phi)^2\mu^2}(b + p_c e)\right]. \tag{10}$$

The discount term multiplies the expression as in the marginal willingness to pay for changes in adult survival rates. Changes in child mortality are assumed to be permanent, and, therefore, they will not only be enjoyed by this generation, but also by all future generations.

In the case of child mortality, two elements compose the valuation of mortality changes for any given generation (the expression multiplying the discount factor). The first term inside the square brackets represents savings for the households due to lower costs in the acquisition of survivors, while the second derives exclusively from uncertainty considerations.

Two extreme examples help to clarify the economic forces at work here. With risk neutrality ($\phi = 1$), the second term inside brackets disappears. In this case, the goods cost per child born determines the marginal willingness to pay for reductions in child mortality. On the other extreme, when $\phi < 1$ and $b = 0$, the goods cost of children is defined only for survivors, and so the value of changes in child mortality depends only on the risk premium due to uncertainty. So, even without additional economic costs, the gain in mortality may improve welfare since increased chances of child survival reduce parent's uncertainty.[6]

[6] Proposition 3 in Sah (1991) considers a similar discussion, although he frames the problem in terms of discrete number of births and the indirect utility function.

With no risk aversion and no cost per child born ($b = 0$ and $\phi = 1$), the gains of changes in child mortality disappear because the number of births adjusts on a one-to-one basis to changes in child mortality, so as to keep constant the expected number of survivors. As parents are risk neutral and there is no cost wedge between children born and children surviving, parents have a target number of expected survivors that is maintained irrespective of the child mortality rate.[7]

In summary, two elements compose the valuation of reductions in child mortality: costs of non-surviving children and reductions in the uncertainty associated with fertility. These gains are completely different from the traditional valuations in Arthur (1981) and Rosen (1988) and deserve particular attention in an attempt to attach social values to observed health improvements. In this case, child mortality, and the fact that individuals understand that their offsprings will also enjoy the current benefits, seem to be central issues.[8]

3 General Model

This section generalizes the model presented before. Our goal is to obtain expressions that allow the valuation of specific mortality changes for an adult individual at any given age. With that in hand, the model will be able to determine the total social value of any given change in survival probability functions.

As will be clear later on, a critical variable for the calculation of the social value of mortality changes is the value of these changes for an individual entering adulthood. Therefore, we start by considering the expected discounted utility for a representative individual entering adulthood at age a:

$$V_T^a = \int_a^{\infty} u(c(t))S(t,a)dt + S(\tau,a)\alpha E\left[N_T{}^{\phi}\right] V_{T+1}^a, \tag{11}$$

with $S(t,a)$ as the discounted survivor function, or the function describing the probability that the agent survives from ages a to t, discounted at the rate of time preference.[9] At age a, individuals decide their profile of consumption, and at age $\tau > a$, parents realize their fertility plans. All children are born at the same time τ. V_{T+1} reflects the children's utility once they reach adulthood at age a. Since the value function refers to a different generation, T identifies generations. N_T represents the number of children surviving to age a, out of a total number of

[7] Households already incorporate the decline in mortality in their optimal programs in a fully insured way; see, for example, the discussion in Becker and Barro (1988).

[8] The quantitative importance of these factors is probably greater for developing countries which are still experiencing huge reductions in child mortality rates.

[9] If $S^*(t,a)$ is the survival function, the discounted survival function is given by $S(t,a) = e^{-\rho(t-a)}S^*(t,a)$, where ρ is the subjective discount rate. We use the discounted survivor function to save on notation.

births n_T, from parents belonging to generation T. The difference between survivors and births corresponds to the effects of mortality on children.[10]

For the sake of simplicity, we abstract from the dynamic nature of the child rearing process. In line with the formulation of the previous section, we assume that children are born, face all the risks related to child and pre-adult mortality at one single moment, and immediately become adults. In terms of the undiscounted survival function $S^*(t,i)$, we only consider the final survival probability between ages 0 and a, $p_c = S^*(a,0)$, where, as before, p_c represents the total survival probability for children. We will revisit this issue when calibrating the model (section 4).

As in the previous section, we consider explicitly the role of uncertainty and assume that the number of surviving children behaves as a random variable with binomial distribution. To approximate the expectation of N_T^{ϕ}, we proceed as before:

$$E\left[N_T^{\phi}\right] \simeq (p_c n_T)^{\phi} - \frac{\phi(1-\phi)}{2}(1-p_c)(p_c n_T)^{\phi-1}.$$

Costs of children are divided in two parts: costs of having children, and costs of raising surviving children. Given the assumptions regarding the way in which child mortality operates, we assume that parents have to pay a fixed cost per child born, and an additional fixed cost per child reaching adulthood. Additionally, we assume that interest rates are equal to subjective discount rates,[11] and maintain the assumption of actuarially fair insurance for every good consumed by the parents. Therefore, for a given endowment W_T received by a member of generation T at age a, households have the following budget constraint:

$$W_T = \int_a^{\infty} c(t)S(t,a)dt + S(\tau,a)\left[bn_T + p_c e n_T\right]. \tag{12}$$

This formulation keeps the basic features discussed before, but adds a couple of new dimensions in terms of the impact of mortality reductions on different age groups. Now we have to keep track of exactly when adult mortality reductions take place. For instance, as it will be clear in the following pages, adult mortality reductions taking place before fertility decisions are realized (τ) have qualitatively different impacts from mortality reductions taking place after that. First order conditions for the individual problem in this case are almost identical to the ones discussed in the simpler version of the model. Therefore, we omit them here.

[10] Strictly, consumption should also be indexed by generation, as in $c_T(t)$, indicating the consumption of generation T at age t. Again, we write $c(t)$ to save on notation.

[11] Since we are not interested in life cycle considerations, and this saves on notation and simplifies the exposition, it seems the best way to proceed. Besides, this assumption does not change any major result of the analysis, and facilitates the empirical implementation of the model.

To evaluate changes in mortality in this context, we follow Murphy and Topel (2001) and think of changes in the survival function in terms of changes in some underlying parameter θ. The variable θ is assumed to shift the survival function in some particular way. In this sense, we define $S_\theta(t,i) = \frac{\partial S(t,i;\theta)}{\partial \theta}$ as the change in the conditional probability of survival from age i to age t brought about by a change in θ.

3.1 The Problem of an Individual Entering Adulthood

Changes in Adult Survival

We start with the problem of an individual entering adulthood, at age a. As we mentioned before, this individual will be key in allowing for the valuation of mortality changes for individuals at all different ages. We label the willingness to pay of generation T at age a for changes in adult survival probabilities MWP_a^A. As before, we express the value of changes in life expectancy as the marginal willingness to pay for increases in the survival probabilities $S(t,a)$. The valuation corresponds to

$$MWP_a^A = \frac{\partial V_T}{\partial \theta}\frac{1}{\lambda_T^a}.$$

Using the envelope theorem, this expression can be written as:

$$MWP_a^A = [\lambda_T^a]^{-1}\left[\int_a^\infty u(c(t))S_\theta(t,a)dt + \alpha S_\theta(\tau,a)E(N_T^\phi)V_{T+1} + \alpha S(\tau,a)E(N_T^\phi)\frac{\partial V_{T+1}}{\partial \theta}\right] - \int_a^\infty c(t)S_\theta(t,a)dt - S_\theta(\tau,a)(b+p_c e)n_T.$$

Substituting for λ_T^a from the first order conditions and reorganizing terms leads to

$$MWP_a^A = \int_a^\infty \left(\frac{1}{\varepsilon_c}-1\right)c(t)S_\theta(t,a)dt + S_\theta(\tau,a)(b+p_c e)\left(\frac{1}{\varepsilon_n}-1\right)n_T + \frac{\alpha S(\tau,a)E(N_T^\phi)}{\lambda_T^a}\frac{\partial V_{T+1}}{\partial \theta}.$$

This expression is completely analogous to the one obtained in the static setting, with the only difference that we are taking into account the dynamic implications of the change in adult survival rates. As before, in a stationary environment, we have $\frac{\partial V_{T+1}}{\partial \theta} = \frac{\partial V_T}{\partial \theta} = \frac{\partial V}{\partial \theta}$, so that:

$$MWP_a^A = \frac{1}{1-\alpha S(\tau,a)E\left(N^\phi\right)}\left[\int_a^\infty \left(\frac{1}{\varepsilon_c}-1\right)c(t)S_\theta(t,a)dt + S_\theta(\tau,a)(b+p_c e)\left(\frac{1}{\varepsilon_n}-1\right)n\right]. \tag{13}$$

The value of $S_\theta(t,a)$ will depend on exactly when the changes in mortality take place. We can identify two benchmark cases: changes in mortality that affect survival rates before fertility decisions are realized, and changes in mortality that affect survival only after fertility decisions are realized. In the first case, we have $S_\theta(t,a) \neq 0$ for some t such that $a \leqslant t \leqslant \tau$, and the expression above cannot be further simplified. In the second case, we have $S_\theta(t,a) = 0$ for every $t < \tau$, so that we can rewrite the expression for the marginal willingness to pay in a simpler form:[12]

$$MWP_a^A = \frac{1}{1-\alpha S(\tau,a)E(N^\phi)} \int_a^\infty \left(\frac{1}{\varepsilon_c} - 1\right) c(t) S_\theta(t,a) dt.$$

Here, since the change will not affect survival before the age of reproduction $S(\tau,a)$, adults value the reduction in mortality independently of their fertility choice (from the envelope theorem, there is no direct first order effect working through fertility). Therefore, the term related to ε_n that was present before disappears. The valuation corresponds simply to the consumption related utility gains equivalent to Rosen (1988) and Murphy and Topel (2001) (numerator), and to the discounted value of these gains for future generations (denominator).

When mortality reductions affect survival probabilities before age τ, individuals also benefit directly from the increased probability of living long enough to have children. Since individuals are altruistic, they are willing to pay to ensure new generations through the realization of fertility decisions. This force is not present when changes in mortality only affect survival rates after age τ.

Changes in Child Survival

The case of changes in child survival is completely analogous to the simpler version of the model. We define MWP_a^C as the marginal willingness to pay of an adult at age a for increases in p_c. Using the envelope theorem and the first order conditions, we write the willingness to pay as:

$$MWP_a^C = \frac{S(\tau,a)}{p_c}\left[bn_T + \frac{(1-\phi)}{2+(1-\phi)^2\mu^2}(b+p_c e)\right]\frac{\partial p_c}{\partial \theta} + S(\tau,a)\alpha E\left(N_T^\phi\right)\frac{\partial V_{T+\tau}^a}{\partial \theta}\frac{1}{\lambda_T^a},$$

As before, in a stationary environment, we can reorganize terms to obtain an expression identical to equation (10):

$$MWP_a^C = \frac{1}{1-\alpha S(\tau,a)E\left(N^\phi\right)}\frac{S(\tau,a)}{p_c}\left[bn + \frac{(1-\phi)}{2+(1-\phi)^2\mu^2}(b+p_c e)\right]\frac{\partial p_c}{\partial \theta}. \tag{14}$$

[12] To be more precise, we could be integrating only from the age t^* that marks the first change in mortality, instead of integrating from a to infinity.

The interpretation of the different terms in the equation above and of the economic forces involved in this valuation was discussed in section 2.

3.2 The Problem of a Young Adult

To apply the model to heterogeneous populations, we have to extend it to deal with the problem of adult individuals at different ages. We define young adults as individuals who have not yet realized their fertility choices. For these individuals at age i, where $a \leqslant i < \tau$, the questions involved are similar to the ones discussed in section 3.1, but for one small detail. Since individuals will not necessarily be at age a, we will not be able to isolate $\frac{\partial V^a}{\partial \theta}$ on the left hand-side in order to obtain a simple expression for the marginal willingness to pay, as we did before. This is exactly why MWP_a^C and MWP_a^A will play such a critical role. We can use the results from section 3.1 to overcome this problem and value changes in mortality at any age between a and τ.

Changes in Adult Survival

We define the marginal willingness to pay of an individual at age i for changes in adult survival rates, brought about by a change in θ, as $MWP_i^A = \dfrac{1}{\lambda_T^i}\dfrac{\partial V_T^i}{\partial \theta}$. Following the same steps as before,

$$MWP_i^A = \int_i^\infty \left(\frac{1}{\varepsilon_c} - 1\right) c(t) S_\theta(t,i)dt + S_\theta(\tau,i)(b + p_c e)\left(\frac{1}{\varepsilon_n} - 1\right) n_T + \frac{\alpha S(\tau,i) E\left(N_T^\phi\right)}{\lambda_T^i}\frac{\partial V_{T+\tau}^a}{\partial \theta}.$$

Note that the marginal willingness to pay refers to an individual at age i, while the derivative of the value function in the right hand side refers to an individual at age a. Therefore, we cannot transfer the derivative of the value function to the left hand-side in order to obtain the discounted value of changes in mortality for all future generations. However, given our results from the previous section, this is not necessary. Since the solution to the individual problem of a young adult is time consistent, the multiplier on the budget constrain will always have the same value in a stationary environment, irrespective of generation or age. We can see that from the first order conditions and the budget constraint. Consider a young adult from generation T that solves the maximization problem at age i and follows its solution between ages i and j (with $j > i$), and later decides to reoptimize at age j to check whether the initial optimal plan still applies. Equality between the interest rate and the subjective rate of time preference implies constant consumption throughout the remaining life. Call c^i the optimal consumption implied by the previous calculations, at age i. The first order conditions and budget constraint imply that the optimal plan at age j is to choose n_T and c^j to solve the following system of non-linear equations:

$$\begin{cases} \dfrac{(b+ep_c)u'(c^j)}{\alpha\left[\phi(p_c n_T)^{\phi-1}p_c + \frac{\phi(1-\phi)^2}{2}(1-p_c)(p_c n_T)^{\phi-2}p_c\right]} = V^a_{T+\tau}, \quad \text{and} \\ \\ \dfrac{W_T}{S(j,i)} - \displaystyle\int_i^j \frac{c^i}{S(j,t)}dt = \int_j^\infty c^j S(t,j)dt + S(\tau,j)\left[bn_T + p_c e n_T\right]. \end{cases}$$

The new budget constraint takes into account the evolution of wealth implied by the insurance policy and the interest rate (incorporated into the survival function). Note that, by definition, $S(j,i) = S(j,t)S(t,i)$ as long as $i < t < j$, so that we can rewrite the resources side of the budget constraint as $\frac{W_T}{S(j,i)} - \frac{1}{S(j,i)}\int_i^j c^i S(t,i)dt$. Rearranging terms in the budget constraint, and using the definition of survival functions, we can write:

$$W_T = \int_j^\infty c^j S(t,i)dt + \int_i^j c^i S(t,i)dt + S(\tau,i)\left[bn_T + p_c e n_T\right].$$

The $V^a_{T+\tau}$ in the right-hand side of the first equation in the system does not depend on the age of the individual and, therefore, any pair (c, n_T) that satisfies the first equation at age i, will also satisfy it at age j. In relation to the budget constraint, the modified form expressed above makes it clear that, if the pair (c^i, n_T) was feasible at age i, it will still be feasible for individuals who survive up to age j. These two results together imply that the plan derived by the optimization problem of an adult individual at age a is indeed time consistent, and individuals will stick to it throughout their adult lives. Consumption will therefore be constant at the level chosen at age a (c^a), implying a constant value for the multiplier on the budget constraint (from $u'(c^a) = \lambda^i_T$).

In a stationary environment, this means that the multiplier will be constant through time and generations, so that we can write:

$$\frac{1}{\lambda^i_T}\frac{\partial V^a_{T+\tau}}{\partial\theta} = \frac{1}{\lambda^a_{T+\tau}}\frac{\partial V^a_{T+\tau}}{\partial\theta} = \frac{1}{\lambda^a_T}\frac{\partial V^a_T}{\partial\theta} = MWP^A_a.$$

This result yields:

$$MWP^A_i = \int_i^\infty \left(\frac{1}{\varepsilon_c} - 1\right)c(t)S_\theta(t,i)dt + S_\theta(\tau,i)(b+p_c e)\left(\frac{1}{\varepsilon_n} - 1\right)n_T + \alpha S(\tau,i)E(N^\phi_T)MWP^A_a.$$

As this expression makes clear, MWP^A_a, or the marginal willingness to pay of an individual at age a, is a key concept in the problem. It allows the incorporation of the valuation of future generations via the altruism of the parents to be. In this case, the willingness to pay of parents at age i is composed, as before, by the consumption and fertility factors, plus the discounted value of these gains for their children. In turn, children will become adults when their parents are τ

years old. The meaning of this expression is exactly the same of equation (13), the only difference being that the marginal willingness to pay of parents will not be exactly the same as the marginal willingness to pay of their children. Therefore, it is not possible to express MWP_i^A directly as the simple discounted value of the sum of the consumption and fertility factors.

Once more, the value of $S_\theta(t,i)$ will depend on the moment when mortality reductions take place. Consider three benchmark cases. First, changes that take place in ages already surpassed by the individuals will only affect parents through their altruism towards children. In this case, $S_\theta(t,i)=0$ for all $t>i$, but the individual will not attach zero value to these changes because of the altruism motive. In this case, we can simplify MWP_i^A to

$$MWP_i^A = \alpha S(\tau,i)E(N_T^\phi)MWP_a^A.$$

The second possibility is that changes in mortality involve survival probabilities that directly affect parents, and that take place before fertility decisions are realized. In this case, we are back to the original expression for MWP_i^A derived above.

Finally, mortality reductions may not affect survival probabilities up to age τ, but affect survival probabilities thereafter. In this case, $S_\theta(t,i)=0$ for every $t<\tau$, and we can simplify MWP_i^A to

$$MWP_i^A = \int_i^\infty \left(\frac{1}{\varepsilon_c}-1\right)c(t)S_\theta(t,i)dt + \alpha S(\tau,i)E(N_T^\phi)MWP_a^A. \tag{15}$$

This is the case that we had before, where mortality changes do not affect survival before the age of reproduction (τ) and, therefore, adults value the reduction in mortality independently of their fertility choice. Welfare effects correspond only to the consumption related to utility gains and to the discounted value of these gains for future generations.

Changes in Child Survival

For changes in θ that affect child mortality, the result is also analogous to the expression for a young adult at age a. Define the marginal willingness to pay of an individual at age i for improvements on child survival as $MWP_i^C = \frac{\partial V_T^i}{\partial \theta}\frac{1}{\lambda_T^i}$, such that

$$MWP_i^C = \frac{S(\tau,i)}{p_c}\left[bn_T + \frac{(1-\phi)}{2+(1-\phi)^2\mu^2}(b+p_c e)\right]\frac{\partial p_c}{\partial \theta} + S(\tau,i)\alpha\frac{\alpha E(N_T^\phi)}{\lambda_T^i}\frac{\partial V_{T+\tau}^a}{\partial \theta},$$

As before, in a stationary environment, time consistency of the individual's problem allows us to write

$$MWP_i^C = \frac{S(\tau, i)}{p_c}\left\{bn + \frac{(1-\phi)}{2+(1-\phi)^2\mu^2}(b + p_c e)\right\}\frac{\partial p_c}{\partial \theta} + \alpha S(\tau, i)E(N^{\phi})MWP_a^C. \tag{16}$$

This expression has the same interpretation of equation (14). The only difference is that altruism for future generations is expressed via the discounted value of the marginal willingness to pay of individuals entering adulthood (MWP_a^C), and not of individuals at the same age of their parent.

3.3 The Problem of an Old Adult

For those individuals at age i, where $i > \tau$, fertility decisions were already realized. As mentioned before, we assume that these individuals ceased to be altruistic towards their offspring. A normative interpretation of this hypothesis, mentioned in the introduction, is that the inclusion of forms of altruism not related to the parent-children relationship in welfare evaluation is less justifiable, and not nearly as important, as the inclusion of the altruism of parents towards young children. The altruism of older parents towards their 40 year old sons and daughters does not seem too different from the altruism of friends towards each other. Few would argue that the latter should be taken too seriously or, in any case, that it would have any major effect in welfare analysis.

With the hypothesis of egoistic old adults, the problem of an individual at some age $i > \tau$ turns into the classical problem of the value of life literature, without altruism and without fertility. If mortality reductions take place at age groups that do not directly benefit the individual, the marginal willingness to pay is zero. If mortality reductions take place at some age after i, we are back to the traditional value of life setup. We define the marginal willingness to pay of an old adult for changes in θ as MWP_i^O, and write:

$$MWP_i^O = \int_i^{\infty}\left(\frac{1}{\varepsilon_c} - 1\right)c(t)S_{\theta}(t, a)dt. \tag{17}$$

This corresponds to what we called the consumption factor. It represents the goods trade-off between quantity and quality of life identified by Rosen (1994).

3.4 Social Value of Mortality Reductions

With the previous results in hand, we are able to determine the social value of any given change in the survival function. The social value will be function of the age distribution of the population, and of the valuation attached to the changes by each specific age group. As in Murphy and Topel (2001), the valuation of each age group will ultimately depend on the moment and extent of the reductions in mortality. We already developed valuations of mortality changes for all relevant age groups.

Suppose that a population of P individuals is distributed across ages according to the density function $f(\cdot)$. The social value of a given change in mortality rates, brought about by a shift in the parameter θ, is given by

$$Social\ MWP = P\left[\int_{a}^{\tau}(MWP_i^C + MWP_i^A)f(i)di + \int_{\tau}^{\infty} MWP_i^O f(i)di\right].$$

Or, defining

$$MWP_i = \begin{cases} MWP_i^C + MWP_i^A, \text{ if } i \leqslant \tau \text{ and} \\ MWP_i^O, \text{ if } i > \tau \end{cases}$$

we can write it in a simpler form as:

$$Social\ MWP = P\int_{a}^{\infty} MWP_i f(i)di. \tag{18}$$

The social value is simply the weighted sum of the value of the mortality changes across the different age groups, where the weights are given by the number of individuals in the population that belong to each specific group ($Pf(i)$). It is immediate to see that the age distribution may have significant impacts on the social value of a specific change in survival probabilities. A society with a high proportion of old individuals will value less reductions in child or young adult mortalities, and value more extensions in old age life expectancy. This adds to the point that the social value of recent child mortality reductions in developing countries may be quite significant, given that these societies typically have very young populations.

4 Parametrization and Empirical Implications

We use the model developed in the previous section to give monetary values to the mortality reductions experienced by the United States population between 1965 and 1995. This is a case that has been studied in the literature, and, therefore, constitutes a good initial example. Our goal is to assess the empirical relevance of the new dimensions introduced in the value of life calculation by incorporating fertility and altruism.

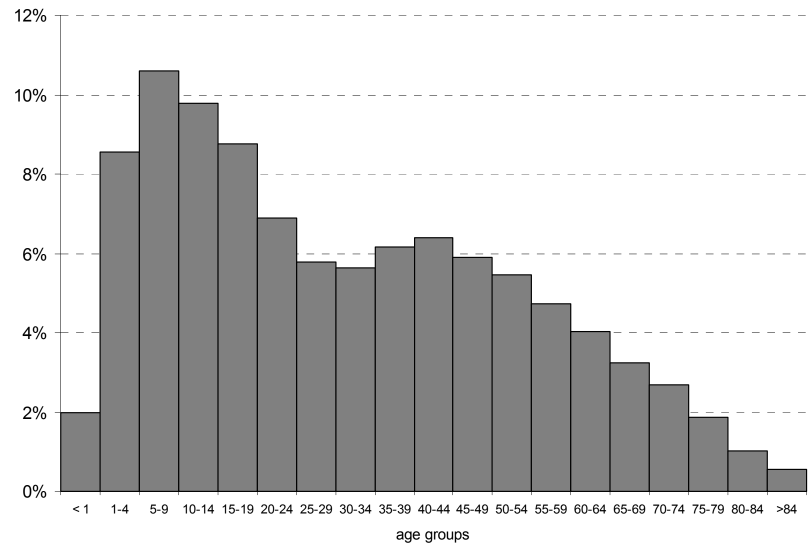
We look at the problem from the perspective of individuals living in 1965, and ask how much they would be willing to pay for the gains in survival probabilities that actually took place in the following 30 years. As discussed previously, the numbers generated by the methodology assume that individuals consider the economic conditions prevalent in 1965 as the ones that would persist indefinitely into the future. To proceed, we need values for a series of parameters and variables determining initial conditions (c, n_T, ε_c, b, e, ϕ, α, and the interest rate), the survival functions ($S^*(t,a)$), and changes in the survival functions ($S^*_\theta(t,a)$).

Figure 1: Survival Probabilities for the US Population (from age 0 to t), 1965 and 1995

The survival functions for 1965 and 1995 are calculated from age specific number of deaths and population, obtained from the World Health Organization Mortality Database. The database contains information broken down by 5 year age intervals, so we assume constant mortality rates within each interval. We calculate the 1965 numbers as averages for the period between 1960 and 1969, and the 1995 numbers as averages between 1990 and 1999. Figure 1 illustrates the shift in the (undiscounted) survival function for the US population between 1965 and 1995. This shift corresponds to a change in overall life expectancy at birth from 70 to 76 years. Note that the US experienced relatively modest gains in child mortality during this period, mainly because of the already extremely low starting point (mortality before 5 years went from 2.84 to 0.99 percent). Also, the age distribution of the American population in 1965, portrayed in Figure 2, had a relatively high share of old adults when compared to young adults. If anything, it seems that the particular example picked will tend to minimize the importance of the forces that we are trying to highlight.

Of the parameters and variables needed, some are easily obtainable and common in the value

Figure 2: Age Distribution of the US Population, 1965



of life literature. First, interest rates are assumed to be 3 percent per year, and this number is used to calculate the discounted survival functions ($S(t,a)$). As mentioned, the assumption of subjective rate of time preference equal to the interest rate implies constant consumption throughout life. This allows the consumption term in the willingness to pay expressions to be factored as $\left(\varepsilon_c^{-1} - 1\right) c \int_i^\infty S_\theta(t,i)dt$. Consumption is set to the 1965 value of total per capita consumption (private plus government, in 1996 US$), obtained from the Federal Reserve Board Economic Database ($13,835). The value of ε_c can be estimated from compensating differentials for occupational mortality risks. Murphy and Topel (2001), using numbers from the literature on occupational risks, estimate ε_c to be 0.346.

For fertility (n_T), the survival probability of children (p_c), and the discounted survival probability of adults up to the age when fertility is realized ($S(\tau,a)$), we use the 1965 values of, respectively, the total fertility rate (2.9, or $n_T = 1.45$), the survival probability between ages zero and 18 (0.96), and the discounted survival probability between ages 18 and 40 (0.50). We take p_c to be the survival probability from zero to 18 because 18 years of age seem to be the natural benchmark for legal and financial independence of children. Besides, this timing matches well the data on the costs of raising children, which will be used to calibrate some of the remaining parameters. In relation to the age when fertility is realized, we choose $\tau = 40$. Since we are consolidating the whole child rearing process in a single moment, we choose the middle point of the parenthood process as such moment. If we think that parents usually start having children when they are 25 or 30, and take care of children in one way or another until they are roughly 50 or 55, the middle point would be around age 40.

The remaining parameters needed – b, e, ϕ, and α – are somewhat uncommon in applied work, so some exploratory calibration effort is required.

Estimates of b and e can be obtained from surveys of expenditures on children, such as the ones undertaken by the US Department of Agriculture (USDA). Lino (2001) presents the 1960 estimates of the USDA for the average costs of raising a child up to age 18, in 2000 US\$. We use the CPI to deflate this value to 1996 US\$. The USDA estimates imply that roughly 5 percent of the expenditures on children take place before age 2, while 95 percent take place between ages 2 and 18.[13] We assume that costs up to age 2 are related to the costs of having a child (b), and costs between ages 2 and 18 are related to costs of raising a surviving child (e). This yields $b = \$6,956$ and $e = \$126,812$. Again, if anything, these estimates seem to underestimate the value of b in comparison to e, and this will tend to minimize the importance of child mortality in welfare evaluations.

Finally, we need estimates of α and ϕ. Identification of these parameters requires somewhat subtler information. It requires knowledge of how much individuals value their own utility vis-à-vis their children's utility, and by how much this relative valuation changes as their number of children changes. We know of no direct study trying to estimate these relations. Therefore, to calibrate the values of α and ϕ, we use the evidence presented by Cropper, Aydede, and Portney (1992) regarding the rate at which people implicitly discount future lives saved. They estimate the average discount rate applied to lives saved 25 years in the future to be 0.087, and the increase in this discount per additional child below 18 in the household to be 0.0145. We make the heroic assumption that these estimates correspond to, respectively, $\alpha S(\tau, a) E(N_T^{\phi})$ and $\alpha S(\tau, a) \frac{\partial E(N_T^{\phi})}{\partial n_T}$, evaluated at the average fertility level.[14] These expressions define two non-linear equations in α and ϕ, that can be numerically solved. For the case in question, we obtain $\alpha = 0.16$ and $\phi = 0.24$.

There are various problems with this identification of α and ϕ. First, in the hypothetical exercise used in the interviews that entail Cropper, Aydede, and Portney's (1992) estimates, there are no personal relations between the individuals being considered. Second, even in this context, it

[13] These shares are roughly constant across income levels and time. Also, the growth in expenditures on children through time seem smaller than one might expect. In 1960, total expenditures per child up to age 18 were \$133,768, while in 2000 this number was \$150,899 (both in 1996 dollars).

[14] Suppose that the value of a given reduction in mortality could be expressed as a function of the "number of lives saved" (L). Consider a variable θ_1 which reduces mortality today by some amount, and a variable θ_2 which reduces mortality for future generations by another. An individual will be indifferent between the changes in mortality brought about by θ_1 and θ_2 if and only if $\frac{\partial L}{\partial \theta_1} = \alpha S(\tau, a) E(N_T^{\phi}) \frac{\partial L}{\partial \theta_2}$. This is the very loose sense in which we assume that $\alpha S(\tau, a) E(N_T^{\phi})$ corresponds to the number estimated by Cropper, Aydede, and Portney (1992).

is not clear whether their estimates correspond exactly to the theoretical concepts that we would ideally want. Changes in survival probabilities cannot be summarized in terms of number of lives lost, with the same values for any individual at any given age. Additionally, the ideal exercise to estimate α and ϕ would consider permanent changes in mortality taking place at the present against permanent changes in mortality taking place at some future point, benefiting relatively more future generations. The survey used by the authors consider trade-offs regarding a one-shot reduction in the number of deaths in the present, against a one-shot reduction in the number of deaths in the future. Finally, Cropper, Aydede, and Portney (1992) themselves identify a series of signals indicating that their results probably overestimate the actual discount rate applied to future lives saved.[15]

Despite all these problems, we think of these numbers as an initial benchmark to can guide our discussion. We take Cropper, Aydede, and Portney's (1992) 0.087 number to be a lower bound estimate of the intergenerational discount factor. In our view, and given all the caveats discussed in the last paragraph, the intergenerational discount applied between members of the same family is likely to be larger than this.

We calibrate the model using the parameters generated by this benchmark specification and then change the specification by varying the intergenerational discount rate. First, we consider the value suggested by Arrow (1995). Arrow suggests that the pure social discount rate across generation should be roughly 4%. Discounting this value from the perspective of an individual entering adulthood, we obtain $\alpha S(\tau, a)E(N_T^{\phi}) = 0.484$, which is very close to one-fourth of the discount rate estimated in Cropper, Aydede, and Portney (1992) (thinking in terms of rates, we have $\sqrt[4]{0.087} = 0.543$). This yields $\phi = 0.04$ and $\alpha = 0.95$.

Following, we consider the intergenerational discount rate suggested by the evolutionary biology literature. According to this view, the altruism between two individuals can be genetically motivated only if it is given by the "coefficient of relationship" between them, or , in other words, by the pool of common genes that the two individuals share (see discussion in Bergstrom (1996)). This would give a pure intergenerational discount rate betweens parents and children equal to 50%, which, discounted from the perspective of an individual entering adulthood, yields $\alpha S(\tau, a)E(N_T^{\phi}) = 0.252$. This in an intermediary case, close to one-half of the discount rate estimated by Cropper, Aydede, and Portney (1992) (again, in terms of rates, we have $\sqrt{0.087} = 0.293$). This number generates $\phi = 0.08$ and $\alpha = 0.49$.

[15] One of the reasons is related to evidence suggesting that individuals interviewed did not fully understand the trade-offs posed to them in the questions. For example, individuals usually justified a choice strongly biased towards current lives arguing things of the type "there may be enough time to figure out how to save these other lives in the future anyway," as if the trade-offs being suggested were not the "real" ones.

Introspection suggests that all three cases are rather conservative estimates. It is worth remembering that $\alpha S(\tau, a)E(N_T^{\phi}) = 0.087$ implies that parents values their own consumption more than 10 times more than the utility of each child. In any case, we do not think that this is the ideal way to calibrate the values of α and ϕ. Instead, we consider our tentative approach an important first step in a direction that requires further research.

Table 1 presents the values of the main parameters and initial period variables used in the benchmark specification. Table 2 presents the marginal willingness to pay of an individual entering adulthood (age a) for the three sets of parameters, together with the values of α and ϕ implied by the different parameterizations, and the increase in the calculated welfare gains due to the incorporation of fertility and altruism. Table 3 presents the total social value of the gains in survival rates for the same set of parameters, together with the increase in the calculated social gains due to the new dimensions considered.

Table 1: Values of Variables in Benchmark Parameterization

Base Year: 1965

p_c	0.96
$S(\tau, a)$	0.50
c	13,835
n_T	1.5
ϕ	0.24
α	0.16
b	6,956
e	126,812
ε_c	0.35
$E(N_T^{\phi})$	1.09
$\frac{\partial E(N_T^{\phi})}{\partial n_T}$	0.181
μ	0.147
$\alpha S(\tau, a)E(N_T^{\phi})$	0.087
ε_n	0.25

Table 2 shows that even the most conservative set of parameters yields a significant increase in the calculated marginal willingness to pay of an 18 year old individual. The results imply that, for $\alpha S(\tau, a)E(N_T^{\phi}) = 0.087$, an 18 year old individual in 1965 would be willing to pay US$27,035 for the reductions in mortality that were observed between 1965 and 1995. This number is 20 percent higher than what would be obtained had fertility and altruism been ignored. As we increase the weight that parents give to their children, this difference increases exponentially. If we assume that $S(\tau, a)E(N_T^\phi) = 0.252$, the willingness to pay of an 18 year old individual in 1965 becomes US$38,470, or 71 percent higher than in the traditional setup. In the last case, when we use the discount rate suggested by Arrow (1995), the calculated willingness to pay of a young adult is US$66,773, or 197 percent higher than what would be obtained in the traditional setup.

Table 2: Value of Mortality Reductions in the US between 1965 and 1995 for an 18 Year Old Individual, Different Sets of Parameters

parameterization	$\alpha S(\tau, a)E(N_T^\phi)$	ϕ	α	MWP_a (1996 $)	Increase from Traditional
Cropper et al. (1992)	0.087	0.24	0.16	27,035	20%
Evolutionary Biology	0.252	0.08	0.49	38,470	71%
Arrow (1995)	0.484	0.04	0.95	66,773	197%

When we move to the analysis of the social value of mortality reductions, these numbers decline. This is because a large fraction of the population is composed by old adults, who already surpassed the assumed child rearing age ($\tau = 40$).[16] Table 3 shows that, in this case, the most conservative set of estimates ($\alpha S(\tau, a)E(N_T^\phi) = 0.087$) implies that the social value obtained using our methodology is 6 percent higher than what would be obtained otherwise. But, again, this number increases exponentially with increases in the weight given to children by their parents. With the intermediate set of estimates ($S(\tau, a)E(N_T^\phi) = 0.252$), the calculated social value of the reductions in mortality is 22 percent higher than what it would be in the traditional approach. With parents discounting children at the rate suggested by Arrow (1995) ($\alpha S(\tau, a)E(N_T^\phi) = 0.484$), the calculated social value becomes 65 percent higher than what would be obtained ignoring fertility and altruism.

[16] In 1965, the US population aged 40 and above was 28 percent larger than the population between ages 18 and 40.

Table 3: Social Value of Mortality Reduction in the US between 1965 and 1995, Different Sets of Parameters

parameterization	$\alpha S(\tau, a)E(N_T^\phi)$	ϕ	α	*Social MWP* (Bill., 1996 $)	Increase from Traditional
Cropper et al. (1992)	0.087	0.24	0.16	4,920	6%
Evolutionary Biology	0.252	0.08	0.49	5,659	22%
Arrow (1995)	0.484	0.04	0.95	7,620	65%

In terms of values, the intermediate set of estimates ($S(\tau, a)E(N_T^\phi) = 0.252$) implies that the gains in life expectancy experienced in this thirty-year period had a social value of 5.66 trillion dollars (in 1996 US$). This value corresponds to almost 2 times the value of the US GDP in 1965. And 1 trillion dollars come directly from the effects of fertility and altruism in the valuation of changes in mortality.

Though the results are quantitatively significative, and indicate the relevance of fertility and altruism in value of life-type calculations, we must repeat that several dimensions of our empirical implementation tend to underestimate the factors that we are trying to highlight. Indeed, we see the evidence as an extremely strong case in favor of the incorporation of fertility and altruism into the analysis. In our subjective evaluation, a more precise estimation of the parameters b, e, α, and ϕ, and the application of the methodology to countries or periods with larger changes in child mortality, would both tend to increase the relative importance of fertility and altruism.

5 Final Comments

This paper developed and applied a methodology that incorporates fertility decisions and altruism into the framework of the value of life literature. We showed that it is possible to evaluate the welfare implications of mortality reductions in a setup where individuals choose the number of children they have and are altruistic towards their children. Under these circumstances, the value of adult mortality changes can be generally decomposed into three factors: the consumption factor from the traditional value of life specification; a fertility factor, which accounts for the welfare improvements related to the higher probability of having children; and an altruism factor, which accounts for the fact that mortality reductions will also be enjoyed by all future generations. Additionally, our approach allows the calculation of an adult's willingness to pay for reductions in child mortality. This value will generally depend on the effect of child mortality reductions on the final costs of child production, on the uncertainty regarding the number of surviving children, and, again, on the fact that these gains will also be experienced by all future generations.

To illustrate the empirical relevance of fertility and altruism in the value of life problem, we used our methodology to value the reductions in mortality rates experienced by the US between 1965 and 1995. Our results show that altruism and fertility can easily double the value of mortality reductions for a young adult, when compared to results obtained using the traditional value of life methodology.

Yet, some additional effort on the estimation of "new" parameters is necessary, in order to generate a more precise estimate of the relative importance of fertility and altruism in the problem. These parameters are related to the costs of having and raising children, and to the way in which parents discount their children's utility when compared to their own. We believe that these issues deserve further attention, and may be important areas for future empirical research.

References

[1] Arrow, Keneth J. (1995). "Intergenerational Equity and the Rate of Discount in Long-term Social Investment." IEA World Congress, December 1995, 23p.

[2] Arthur, W. Brian. (1981). The Economics of Risks of Life. *American Economic Review*, Vol. 71, 54-64.

[3] Becker, Gary and Robert Barro (1988) A Reformulation of the Economic Theory of Fertility. *Quarterly Journal of Economics*, Vol. 103, 1-25.

[4] Becker, Gary and Kevin. Murphy (1988). The Family and the State. *Journal of Law and Economics*, Vol. 31, 1-18.

[5] Becker, Gary, Tomas Philipson, and Rodrigo R. Soares (2003). "Quantity and Quality of Life and the Evolution of World Inequality." Unpublished manuscript, University of Chicago.

[6] Bergstrom, Theodore C. (1996). Economics in a Family Way. *Journal of Economic Literature*, Vol. 34, No. 4. (Dec., 1996), pp.1903-1934.

[7] Cropper, Maureen L., Sema K. Aydede, and Paul R. Portney (1992). Rates of Time Preference for Saving Lives. *American Economic Review Papers and Proceedings*, Vol. 82, 469-472.

[8] Cropper, Maureen L. and Frances G. Sussman (1988). Families and the Economics of Risks to Life. *American Economic Review*, Vol. 78, 255-260.

[9] Garrett, Allison M. (2001). "Health Improvements and the National Income and Product Accounts: 1880 to 1940." Unpublished manuscript, University of Chicago.

[10] Kalemli-Ozcan, Sebnem (2002). Does Mortality Decline Promote Economic Growth? *Journal of Economic Growth*, Vol. 7, 411-439.

[11] Lino, Mark (2001). USDA's Expenditure on Children by Families Project: Uses and Changes Over Time. *Family Economics and Nutrition Review*, Vol.13, No.1, 81-86.

[12] Malthus, Thomas R. (1826). Appendix I, Paragraph 14 in *An Essay on the Principle of Population, A View of its Past and Present Effects on Human Happiness*. Sixth edition, London, John Murray, Albemarle Street.

[13] Murphy, Kevin M. and Robert Topel (2001). "The Economic Value of Medical Research." Unpublished manuscript, University of Chicago.

[14] Nordhaus, William D. (1999). "The Health of Nations: The Contribution of Improved Health to Living Standards." Unpublished manuscript, Yale University.

[15] Rosen, Sherwin (1988). The Value of Changes in Life Expectancy. *Journal of Risk and Uncertainty*, Vol. 1, 285-304.

[16] Rosen, Sherwin (1994). The Quantity and Quality of Life: A Conceptual Framework. In: George Tolley, Donald Kendel, and Robert Fabian (eds.). *Valuating Health for Policy: An Economic Approach.* Chicago, University of Chicago Press, 221-248.

[17] Sah, Raaj (1991). The Effects of Child Mortality Changes on Fertility Choice and Parental Welfare. *Journal of Political Economy*, Vol. 99, 582-606.

[18] Schelling, Thomas C. (1968). The Life You Save May Be Your Own. In: Samuel B. Chase, Jr (editor). *Problems in Public Expenditure* Analysis. Brookings Institution, Washington D.C., 1968, p.127-61.

[19] Usher, Dan (1973). An Imputation of the Measure of Economic Growth for Changes in Life Expectancy. In: Milton Moss (ed.). *The Measurement of Economic and Social Performance, Studies in Income and Wealth*, Volume 38. Conference on Research in Income and Wealth. New York, National Bureau of Economic Research and Columbia University Press, 193-225.