HOW TOUGH SHOULD YOU BE

Inflation targeting, fiscal feedbacks and multiple equilibria

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I - Introduction

The adoption of the inflation targeting regime in Brazil has sparkled debate on the merits and shortcomings of the regime. Many pointed out correctly that the new monetary and exchange rate regime would allow interest rates to drop significantly, which indeed happened, albeit possibly not as much as desired. In fact, real interest rates have been remarkably high compared to other emerging economies, some of which face inflation rates of the same order of magnitude as Brazil.

Frustrated with the persistence of high real interest rates, economists have been searching the reasons for such performance. In recent papers, some as Bresser Pereira and Nakano (2001), and Toledo (2002)[1] have advanced a proposition about the possibility of multiple equilibria in interest rate determination, high interest rates being the consequence of a "bad" equilibrium, in which high interest rates and high probabilities of default interact.

Quite naturally, in such a setting, the possibility of "virtuous" and "vicious" circles related to different equilibria takes quickly the front stage: the Central Bank could, in principle, move from a "bad" to a "good" equilibrium, simply reducing the domestic interest rate. In this note, we develop a model that generates multiple equilibria in an explicit inflation-targeting framework, integrating monetary policy reaction functions to debt sustainability considerations.

While the baseline inflation-targeting model works under the standard small open economy assumption of exogenous offshore rates, in a variation we explore the possibility that domestic interest rates – presumably via a fiscal channel – affect offshore rates through its effects on sovereign spreads. Thus, there is a feedback effect on inflation and interest rates through the exchange rate.

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* Fernanda Senna has provided useful comments to this note and helped me in the efforts to solve the model details. And Márcio Nakane made essential corrections to the model with his usual elegance. Yet, I owe most of this paper's intellectual debt to Pérsio Arida, whose ideas strongly influenced mine. The formulation presented here draws heavily on his and I have benefited from his comments, suggestions, and interpretations more than the single authorship of this paper could honestly suggest. Needless to say, all remaining errors are, of course, my entire responsibility.

[1] Bresser Pereira and Nakano (2001) argue forcefully on the multiple equilibria story, based on a model outlined by David Romer (2001), but in a closed economy context, in which the different equilibria result from default perceptions. Toledo (2002), on the other hand, seems to be arguing in a framework very similar to the one we should develop here. Quoting: "(…) when already high interest rates are increased, growth decreases, requiring a higher primary surplus (…) and the likelihood of maintaining the debt in a sustainable path decreases. This means higher risk and hence a higher offshore interest rate (…) [and] a depreciated real exchange rate. (…) [H]ence, the increase in interest rates leads to a higher depreciation, (…) [therefore] inflation does not decline or, worse, it can even increase". Clearly, Toledo presents a story quite similar to ours, although the justification for multiple equilibria there is nowhere to find

This feedback seems a promising way of introducing multiple equilibria in the story. To be sure, however, we remain agnostic on whether the multiple equilibria story can be really relevant to explain the persistence of high interest rates in Brazil. The possibility of being in the “wrong” equilibrium does not necessary mean that the economy is indeed there, and the properties of a “bad” (high interest rate) equilibrium in our model do not seem to conform observed patterns.

Less conspicuous than the issue of multiple equilibria, but possibly more relevant to policy discussion is the possibility of a “floor” to the inflation target, that is, a constraint on the Central Bank ability to set the inflation target below a critical threshold. This critical level would be determined by variables such as the size of primary surpluses relative to the fiscal debt, international interest rates, and the size of the supply and demand shocks to which the economy is subject. A Central Bank too tough on inflation can generate some nasty dynamics on the determination of interest rates. Again, it is not clear whether the existence of an inflation target floor is the reason for the high real interest rates; nevertheless, it sheds some analytical light on an issue usually treated in an impressionistic manner.

Finally, even if multiple equilibria and an inflation target floor do not seem the reason for the persistence of high real interest rates in Brazil, the model can offer some clues on the subject. The feedback mechanism – even if it does not necessarily throw the economy in perverse equilibria – does imply, in general, higher interest rates than the standard small open economy model. Moreover, large debts do imply higher interest rates related to a higher default probability, indicating that tightening fiscal policy remains a crucial policy variable for obtaining lower real interest rates.

II – An open economy inflation targeting model

The model we present below is a close relative of Arida (2002a) model, consisting of a Phillips curve with inflation inertia, an expression for aggregate demand, and the Central Bank loss function. The Phillips curve below, however, is augmented to include the effects of the nominal exchange rate pass-though on inflation. Ideally, we would have to include the exchange rate in the aggregate demand function as well, but – for the sake of simplification – we do not. As we should argue ahead, this should not affect our conclusions in qualitative terms, while some quantitative differences could arise.

$$p_t = p_{t-1} + a y_t + q\Delta e_t + e_t \quad (1)$$

$$y_t = -b\left(i_t - E_t p_t - \bar{r}\right) + h_t \quad (2)$$

$$L = E_t\left[\sum_{n=0}^{\infty} d^{\,n}\left(p_{\,t+n} - \bar{p}\right)^2\right] \qquad (3)$$

Equation (1) is the augmented Phillips curve, where π stands for inflation, y is the GDP gap, e the nominal exchange rate (so Δe is the currency depreciation), and ε a supply shock (i.i.d., with zero mean and finite variance). Equation (2) explains the GDP gap as a negative function of expected real interest rate differences to the neutral real interest rate, that is, the one that would prevail at full-employment, and η is a demand shock, with the same properties as the supply shock above. Finally, the loss function is given by (3), where δ is the discount factor[2] (smaller than 1), and the inflation target is assumed fully credible, that is, expected inflation is equal to the target.

So far, this is precisely Arida's model, except for the currency depreciation term in (1). Yet, before moving on, we have to determine how the currency should behave. Following the usual modeling of the nominal exchange rate, we assume that uncovered interest parity holds, that is, the difference between domestic and offshore interest rates is exactly equal to the expected (and, in this case, actual) depreciation. That is:

$$\Delta e_{t+1} = i_t - i_t^* \qquad (4)$$

We should also assume, for simplicity, that there is a long-term level for the currency, to which the nominal exchange rate reverts in a single period, that is:

$$e_{t+1} = \bar{e}$$

Hence:

$$e_t = \bar{e} + i_t^* - i_t \qquad (5)$$

and therefore:

$$\Delta e_t = \bar{e} + i_t^* - i_t - e_{t-1} \qquad (6)$$

[2] In what follows, I take $\delta = 0$, that is, the Central Bank here is myopic, so that the intertemporal optimization problem becomes a much simpler, static, one. I have worked out the full intertemporal solution (please refer to Appendix A), which yields essentially the same results and insights that arise from the myopic case, but at the cost of some 3 or 4 pages of algebra. As I show there, in the intertemporal model, stability conditions become somewhat less stringent than in the simpler case.

Simplifying further, assume that the long-term exchange rate holds also in period (t-1), so that the entire play about the nominal exchange rate takes place in period (t) alone.

$$\Delta e_t = i_t^* - i_t \qquad (7)$$

Using (2) and (7) in (1), we can express the inflation rate as:

$$p_t = p_{t-1} - (ab + q)i_t + ab(\overline{p} + \overline{r}) + qi_t^* + ah_t + e_t \qquad (8)$$

The reaction function is therefore:

$$i_t = \frac{ab(\overline{r} + \overline{p}) + qi_t^* + (p_{t-1} - \overline{p})}{ab + q} \qquad (9)^3$$

Equation (9) summarizes the Central Bank behavior: it raises interest rates whenever inflation is higher than the target and lowers it when the opposite occurs. Note that a rise in offshore interest rates, i*, implies an increase in domestic interest rates, but not by the same proportion. Under this rule, inflation would be equal to the target plus the effects of the demand and supply shocks, meaning that expected inflation would be indeed equal to the target, that is:

$$p_t = \overline{p} + ah_t + e_t \qquad (10)$$

The real interest rate, therefore, would be given by (11), also the reduced form for the reaction function:

$$i_t - \overline{p} = \frac{ab\overline{r} + q(i_t^* - \overline{p}) + ah_{t-1} + e_{t-1}}{ab + q} \qquad (11)^4$$

Finally, the GDP gap would be:

[3] In steady state, $\Delta e = 0$, hence the i* = i, and the reaction function would be:

$$i_t = \overline{i} + (ab + q)^{-1}(p_{t-1} - \overline{p}) \qquad (9')$$

[4] Again, in steady state, the real interest rate would be equal to the neutral interest rate, plus the effects of the demand and supply shocks, so expected real interest rate is the neutral rate.

$$i_t - \overline{p} = \overline{r} + (ab + q)^{-1}(ah_{t-1} + e_{t-1})$$

$$y_t = -b\,\frac{q\left[i_t^* - (\bar{p} + \bar{r}) + a\,h_{t-1} + e_{t-1}\right]}{a\,b + q} + h_t \qquad (12)^5$$

Before moving on to the next section, let me just go back to the point briefly touched in note (3). In steady state, all the equations that describe the equilibrium in this model are extremely similar to those in Arida's model, which should be no surprise, given that this model was based on his formulation. Yet, not only lower deviations from the neutral real interest rate are required, but also output fluctuates less than in Arida's model. The reason is essentially the additional exchange rate channel, through which interest rates can have an impact on inflation. In other words, smoother movements of real and nominal interest rates are necessary, implying smoother movements of actual output.

Note that, if we had included the exchange rate on the demand equation, this feature would remain. The appreciation of the exchange rate, induced by higher interest rates, would not only have a direct impact on inflation via pass-though, but an additional indirect impact via aggregate demand. Hence, the qualitative feature of the open economy version of the model would not be affected; in quantitative terms, the "smoothing" properties of the open economy model would be stronger, that is, an even lower effort would be required from monetary policy.

III – Country risk and feedback mechanisms

So far we assumed that the offshore interest rate is exogenous, which is the usual assumption about small open economies, and indeed one should not expect Brazilian domestic economic policy affecting the Fed Funds rate, or anything similar. Yet, in the current context, the relevant offshore rate is not the Fed Funds rate, but rather the return on an asset that presents similar risk to the domestic instruments, for example, Brady or Global bonds.

In order to capture this feature, consider that there are two offshore assets in addition to the domestic one: a risk-free bond that yields an interest rate i^{US}, exogenously determined, and a home country sovereign instrument, with yield i*. Yet, this second asset is not risk-free: there is a default probability (1-λ), which we can relate to the fiscal performance.

[5] Once more, to re-emphasize the similarity between these results and Arida's (2002a), in steady state actual GDP gap would follow the process:

$$y_t = h_t - (ab + q)^{-1}(abh_{t-1} + be_{t-1})$$

Assume that at the beginning of the period[6], the government inherits a debt stock, b_{t-1}, which it must service at the rate i_t, deducted the effects of inflation on the debt. The primary surplus, s, is a random variable, with support $[s_L, s_H]$. If the primary surplus is higher than the real debt service (interest on the debt deducted inflation), the debt is serviced and life goes on. If, however, the primary surplus is insufficient to service the debt, we assume that the government defaults on all its debt[7].

We can define, therefore, two critical interest rates. Let i_L be the level of the interest rate such that all possible realizations of the primary surplus are consistent with maintaining the debt service, that is, $P\left[s \leq s_L \equiv (i_L - \bar{p})b_{t-1}\right] = 0$, where:

$$i_L = \frac{s_L}{b_{t-1}} + \bar{p} \qquad (13a)$$

Conversely, i_H represents the critical level of interest rates for which no possible realization of the primary surplus is sufficient to prevent the default, i.e., $P\left[s \leq s_H \equiv (i_H - \bar{p})b_{t-1}\right] = 1$, hence:

$$i_H = \frac{s_H}{b_{t-1}} + \bar{p} \qquad (13b)$$

Thus, based on the discussion above, we can define the repayment probability function, λ(i), which assigns to any value of i the probability of the debt being serviced, conditioned on the support of the primary surplus distribution.

$$\begin{aligned} & i \leq i_L \Rightarrow l(i) = 1; \\ & i \geq i_H \Rightarrow l(i) = 0; \\ & i_L < i < i_H \Rightarrow l'(i) < 0; l''(i) \leq 0 \end{aligned} \qquad (14)^{8}$$

The chart below summarizes the repayment function λ(i). For interest rates below i_L, the default probability is zero (λ=1), increasing as domestic interest rate increases, either at constant (λ"=0)

[6] Romer (2001) presents a similar model.
[7] That is, there is some sort of cross-default clause, which motivates the arbitrage equation ahead.
[8] A possible function with those properties is the following:

$$l(i) = \left(\frac{i_H - i}{i_H - i_L}\right)^a ; a \leq 1'; i_L \leq i \leq i_H$$
$$l(i) = 1; i \leq i_L$$
$$l(i) = 0; i \geq i_H$$

or increasing rates (λ"<0). For interest rates higher than i_H, the default probability becomes one (λ=0).

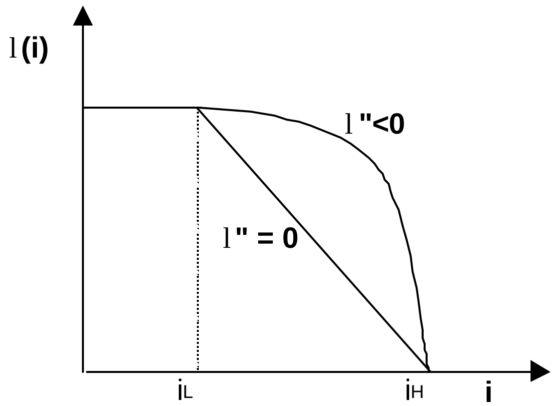

Figure 1: Repayment probability

Under the assumption that investors are risk-neutral, the arbitrage between the offshore assets requires:

$$i^* \, l\,(i) = i^{US} \qquad (15)$$

Equation (13), thus, establishes a relationship between domestic and offshore interest rates, which can be summarized by the following expressions:

$$i = f(i^*) \qquad (16)$$

$$f' = -\frac{l\,(i)}{i^* \, l'(i)} > 0 \qquad (16a)$$

$$f'' = \frac{i^* f'\left[l\,l'' - l'^2\right] + l\,l'}{\left(i^* \, l'\right)^2} < 0 \text{ if } l'' \leq 0 \text{ (sufficient condition)} \qquad (16b)$$

$$\lim_{i \to i_H} i^* = +\infty \qquad (16c)$$

$$i^* = i^{US}; i \leq i_L \qquad (16d)$$

The chart below depicts the relation between domestic and offshore interest rates implied by (16), SS. For domestic interest rates lower than i_L, the home country offshore interest rate is equal to the risk free rate[9]. As domestic interest rate increases, so does the offshore rate, and, as domestic interest rate approaches the level that would lead to certain default, the offshore rate shoots up to infinity.

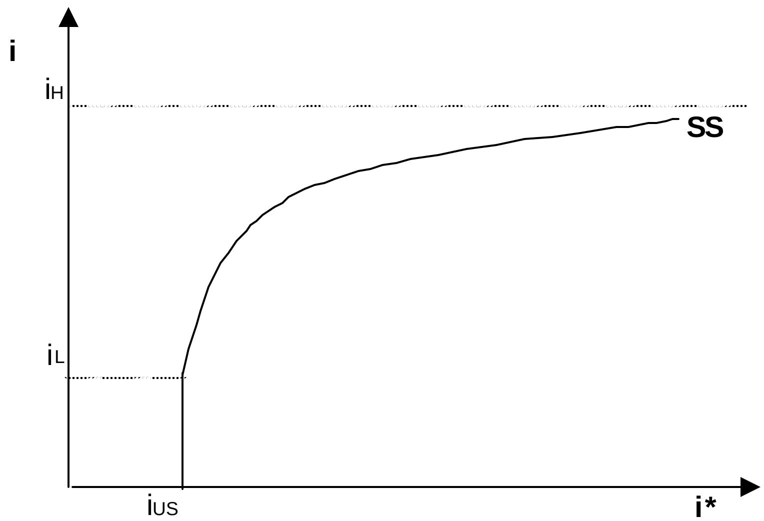

Figure 2: Feedback from domestic interest rates into offshore interest rates

Including the reaction function (11) in the picture, we have the simultaneous determination of domestic and offshore interest rates, as shown in the next chart. The slope of the reaction function line (TR) is $\theta/(\alpha\beta+\theta)$, smaller than 1, and we have drawn the chart below such that there are two equilibria: a stable equilibrium A, which yields a lower interest rate (i_{GOOD}) and an unstable one, B, in which we have a higher interest rate (i_{BAD}). Additionally, as we are not interested in situations in which offshore interest rates are equal to the risk free rate, we have also assured that even the "good" equilibrium takes place with a positive sovereign spread (i^*_{GOOD}-i^{US}).

[9] This needs not be the case in reality, for other forces not modeled here may affect sovereign spreads as well. In the current setting, however, if the default probability is indeed zero, so are sovereign spreads and the offshore rate equals the risk-free rate. We can include, in "ad hoc" fashion, an exogenous risk-premium ψ, insensitive to the domestic interest rate, in order to replicate these effects in this setting, but – as one can see ahead – a reinterpretation of the impacts of the risk free rate yield virtually the same results.

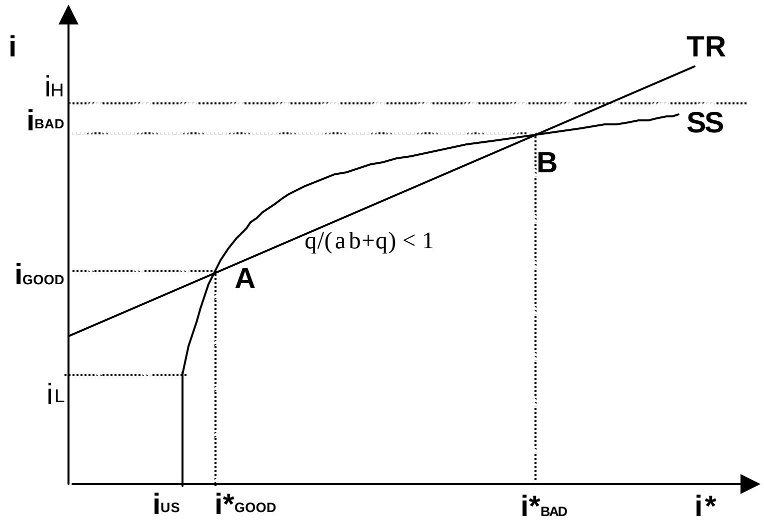

Figure 3: Simultaneous determination of domestic and offshore interest rates

Recall that the Central Bank is nothing more than the reaction function. In the reduced form of the reaction function (11), it raises domestic interest rates whenever offshore rates are higher than the level consistent with the inflation target and reduces rates in the opposite case. Consider now any point over TR to the left of i^*_{GOOD}: the Central Bank is in equilibrium, but not the market. This level of domestic interest rates leads the market to evaluate the default risk such that it requires a higher premium, that is, higher offshore rates. But then, to preserve its own equilibrium, the Central Bank raises rates again, which leads to further reassessment of the default risk and so on.

Considering points to the left of i^*_{GOOD}, it is easy to conclude that this process is stable and leads to equilibrium A. Similarly, points to the right of i^*_{GOOD} (but still to the left of i^*_{BAD}), lead the Central Bank to reduce rates and, once more, approach the equilibrium point A. That is, the "good" equilibrium A is also the stable one.

We have already seen that points to the right of i^*_{GOOD} but to the left of i^*_{BAD} imply a process of declining domestic rates. Consider now points over the reaction function to the right of i^*_{BAD}: the market foresees a higher probability of default, and consequently offshore rates rise. The Central Bank raises domestic rates in response, leading to further reassessments of the default likelihood and the economy is dragged into an explosive interest rate path. That is, the "bad" equilibrium is also an unstable one: small deviations from B would either lead the economy to converge to the "good" equilibrium A, or experience ever increasing interest rates.

Thus, possible feedbacks from the fiscal impacts of domestic interest rates on country risk, and hence on exchange rates, we can generate multiple equilibria and some interesting dynamics. Before moving on to the issue about the existence of an equilibrium, it may be useful to perform some comparative static exercises, for they can shed some light on the existence subject.

IV – Comparative statics and existence

Consider, first, a fiscal expansion, that is, the reduction of the expected primary surplus, in this case through a downward shift in the primary surplus support, from $[s_L, s_H]$ to $[s_L\text{-}k; s_H\text{-}k]$. It is not difficult to see from (13a) and (13b) that both i_L and i_H are now lower than before, that is, the rate at which the probability of default is zero must now be reduced, while lower critical rates are sufficient to cause the certainty of default. Hence, the SS curve shifts downward to SS' (figure 4), and we have two new equilibria at A' and B'.

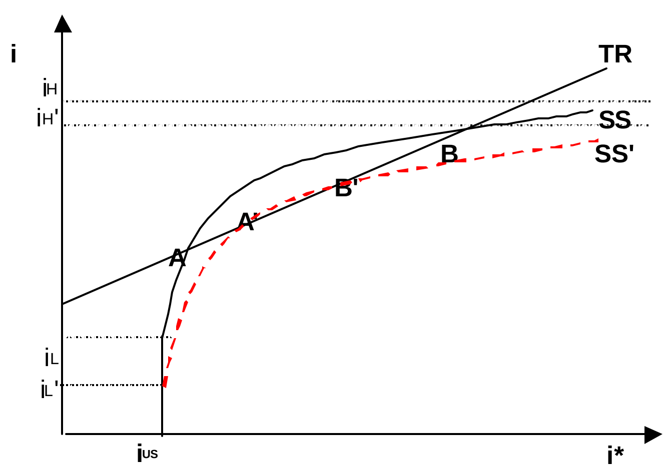

Figure 4: Effects of an expected primary surplus reduction

Hence, in the stable equilibrium, interest rates would increase in response to the fiscal expansion. The intuition behind this result is straightforward: the reduction of the primary surplus would – everything else equal – increase the likelihood of default, for there is now a lower probability mass that the realization of the primary surplus would attend the non-default requirement. Sovereign risk would rise, forcing the offshore interest rate and hence the exchange rate up. In order to offset the impact of higher offshore rates on the exchange rate, the Central Bank must raise the local rate. This would feedback into sovereign risk, but the process is stable, and eventually local rates would converge to the new equilibrium level.

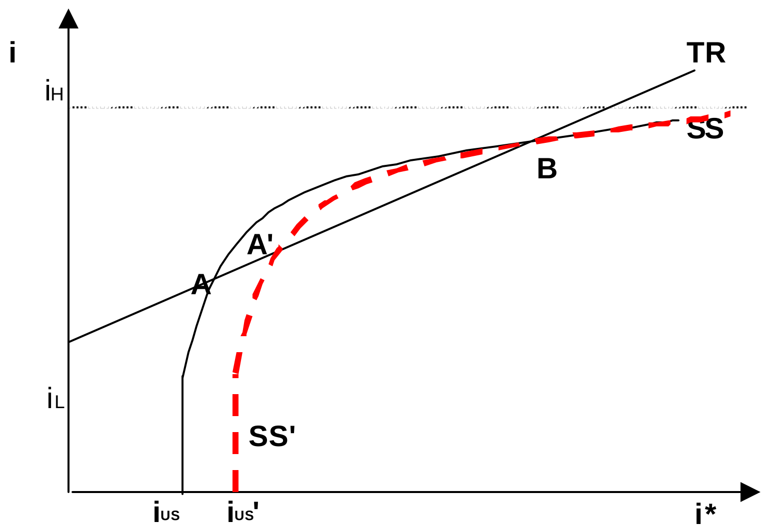

Figure 5: Effects of a rise in the risk-free rate

An increase in the risk-free rate leads to a correspondent adjustment in the local rates in the new stable equilibrium, as shown in the chart above (figure 5). Only note that the shift in the SS curve is not a parallel one, but rather for the "low" values of i*; for "high" values of i*, the increase in i^{US} has a lesser effect, and the schedule continues to approach asymptotically (unchanged) i_H. Note, further, that – if sovereign spreads depend on other factors in addition to the default risk – one can see that a reduction in the willingness to lend would cause a similar shift in the SS schedule.

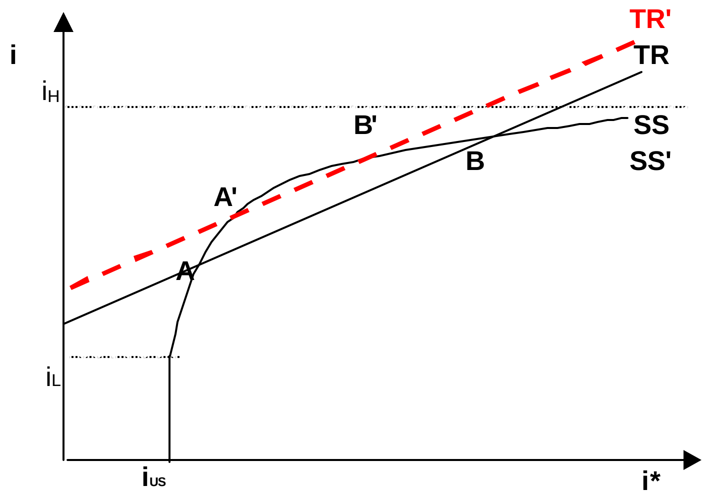

Figure 6: Effects of aggregate demand and supply shocks

Consider now a change in the intercept of the reaction function (figure 6), due to positive shocks to aggregate demand and/or supply. As in the previous cases, the new stable equilibrium would exhibit a higher interest rate. The immediate reaction to the shocks (higher interest rates) would lead to a higher appraisal of country risk and hence the dynamics we briefly described above would apply as well. At the end of the process, both domestic and offshore rates would be not only higher than they were previously, but also – and crucially – higher than they would be if the feedback process were not present. Note that there is some sort of "multiplier" effect taking place due to the links between domestic interest rates and default risk.

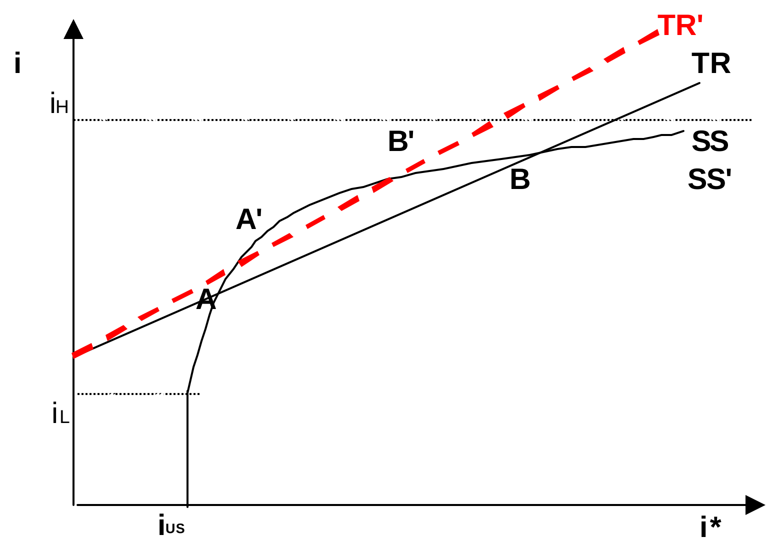

Figure 7: Effects of higher pass-through

There is a similar effect if the slope of the reaction function increases (figure 7), due to a higher pass-through (θ) or a reduction in the sensitivity of aggregate demand to interest rates (β) or finally to a lower sensitivity of inflation to the GDP gap (α). In any case, note that the slope of the reaction function cannot be higher than 1, even if the pass-through coefficient approaches infinity.

From the comparative static exercises above, one can conclude that the conditions for the existence of equilibria should impose some restrictions on the combination of the fiscal policy, the risk-free interest rate, and on the slope (pass-though) and intercept (neutral interest rate, shocks) of the reaction function.

In fact, if we use the particular functional form for $\lambda(i)$ introduced above, under an additional constraint (a=1), it is possible to show[10] that there will be at least one equilibrium if:

$$\left[i_H - \frac{a\,b(\bar{r} + \bar{p}) + a\,h_{t-1} + e_{t-1}}{a\,b + q} \right]^2 \geq 4 \frac{q\left(i_H - i_L\right) i^{US}}{a\,b + q} \qquad (17)$$

or:

$$[i_H - \text{Intercept(TR)}]^2 \geq 4.\text{Slope(TR)}\ (i_H\text{-}i_L)\, i^{US} \qquad (17a)$$

Alternatively, there will be at least an equilibrium as long as $\bar{p} \geq \bar{p}^c$, i.e., if the inflation target is higher than a critical threshold, $\bar{p}^c$, the inflation target for which (17) holds as an equality, that is:

$$\bar{p}^c = 2\sqrt{\frac{(a\,b + q)\left(i_H - i_L\right) i^{US}}{q}} + \frac{a\,b\bar{r} + a\,h_{t-1} + e_{t-1}}{q} - \frac{a\,b + q}{q} \frac{s_H}{b_{t-1}} \qquad (17b)$$

Note that the i_H in the left hand side of (17) rises with the inflation target $\bar{p}$, while the difference between i_H and i_L in the right hand side does not. More to the point, i_H rises 1 to 1 with the inflation target, while the second term in the left hand side of (17) (the intercept of the reaction function) increases by the less than 1, so the entire left hand side rises with the inflation target. Hence, for targets higher than or equal to the threshold defined by (17c), there will be at least an equilibrium, whereas setting the target below the critical level would imply no possible equilibrium.

With that in mind, we note that the threshold $\bar{p}^c$ increases the supply and demand shocks (η,ε), the risk-free rate (i^{US}) and the size of the debt (b_{t-1}), and decreases with the size of the primary surplus. In other words, there are fiscal, "monetary", and real constraints to the inflation target.

On the fiscal side, a large debt reduces the room for a too low inflation target, for it implies a higher debt service and hence a higher probability of default. Higher primary surpluses have the opposite impact, reducing the probability of default. Higher risk-free international rates also impose a higher threshold for the inflation target, as do large domestic supply and demand shocks.

[10] Please refer to Appendix B.

That is, the inflation target is not a completely free variable, which the Central Bank can set at its total discretion. There are constraints related to the economy fundamentals, and pushing the inflation target too low implies a possibility of inconsistency between the target and the market reaction through the arbitrage between domestic and international interest rates.

V – Properties of the unstable equilibrium

Given the existence, in this framework, of two different equilibria, it is just natural to inquire about the possibility of a Central Bank locked in the "bad" equilibrium, that is, setting interest rates far above the necessary to assure consistency with the inflation target. In addition to that, it would be interesting to know the properties of such equilibrium, as there can be implications that would help us in the empirical task of checking the relevance of the claim about the Central Bank caught in a perverse balance.

Note, thus, for one thing, that in the "bad" equilibrium, the currency is weaker than in the "good" one. To see that, note that $i^*_{BAD} > i^*_{GOOD}$, which means, of course, that $i_{BAD} > i_{GOOD}$, but, as the slope of the reaction function is smaller than one – that is, the Central Bank does not offset entirely the increase in offshore interest rates – it must be the case that the interest rate differential is higher in the "bad" equilibrium. Hence, the currency must be weaker in the "bad" equilibrium as well. The intuition is straightforward: this is an equilibrium because the Central Bank needs to raise interest rates to offset the impact of the currency, whereas the market pushes the currency high (via sovereign spreads) because it sees a higher probability of default.

This stands in contrast, for example, to Bresser Pereira and Nakano (2001), who sponsor lower interest rates to – among other things – lead to a weaker currency and therefore an improved trade and current account balances. It turns out that, if the Central Bank were indeed in a perverse equilibrium, a movement towards the good equilibrium would strengthen the currency, once offshore rates would fall faster than local rates. The claim that lower rates are therefore necessary to weaken the currency and improve the external balance is simply inconsistent with the claim that the economy would be locked in an unfavorable equilibrium.

Consider now the reaction to a supply shock. As shown in figure 6 above, a (inflation-rising) supply shock would shift the reaction function upwards, so that the new unstable equilibrium would be B'. To be sure, strictly speaking, this would lead to an ever-increasing interest rate, if we were to take the convergence properties to the letter. If – just for the sake of the argument – an omniscient Central Bank[11] simply adjusts instantaneously to B', the reaction to the supply shock would be lower interest rates and a lower sovereign spread, just the opposite of the observed pattern.

In contrast, if the economy were initially at a point like A and suffered a supply shock, the stable process would lead to higher domestic rates and sovereign spreads, just as one could observe in the aftermath of the supply shocks that hit the Brazilian economy along 2001. The stylized facts seem to fit better the version that highlights the stable equilibrium as the relevant one.

Similarly, if the relevant equilibrium were the unstable one, one would have to conclude that a fiscal tightening would lead to higher sovereign spreads and interest rates, which – again – seems at odds with the data.

To be sure, casual observation is no replacement for a careful empirical analysis, but it appears to us that the properties of the unstable equilibrium do imply patterns for the data that seem difficult to observe. We cannot hand out a definite ruling on the relevance of the unstable equilibrium based simply on the observations above, but there are strong indications to believe that – as a matter of fact – the economy could hardly be in such a position.

Not only the model predictions regarding comparative statics for the unstable equilibrium do not seem to conform the data pattern, but also the implications for the dynamics of interest rate adjustment look rather bizarre. Any shocks would lead either to infinite interest rates or to convergence to the stable equilibrium: we do not observe the first and – if we observe the second – the multiple equilibria problem ceases to be relevant.

Note that – now dropping the ***ad hoc*** assumption about an omniscient Central Bank choosing deliberately the unstable equilibrium – we are not really in a model with saddlepath properties. The Central Bank, in our framework, is actually the reaction function derived in the first part of the model, that is, a rule that links offshore to domestic interest rates, among other things. The Central Bank, in this setting, simply does not know of the existence of the SS schedule, but reacts, mechanically, to the changes it observes in the variables. Hence, whereas the model can offer a rationalization to the existence of multiple equilibria, it cannot offer a reason for the unstable equilibrium being the prevalent one.

[11] Again, the Central Bank in our setting is not omniscient, but rather a simple reaction function. See below.

VI – Concluding remarks

Going back to the issue that motivated this note one might ask: why are interest rates so high in Brazil? We believe the model can offer some hints via the feedback mechanism from interest rates to the default risk. Indeed, contrasting the world in which offshore interest rates are fully exogenous (the standard small open economy assumption) to a world in which sovereign spreads react to considerations about debt sustainability, it is possible to show that interest rates are usually higher in the latter. The size of the debt affects the level of domestic rates, as well as fiscal policy, indicating that the persistence of high interest rates might result from these variables.

Indeed, in addition to variables usually highlighted in the standard open economy models, such as the size of demand and supply shocks (as well as the sensitivity to these shocks) and the pass-though, the model emphasizes the role of fiscal variables, like primary surpluses and the government debt.

For instance, given the size of the primary surpluses, a higher debt leads to higher equilibrium interest rates, and the parallels to the current Brazilian situation are not terribly difficult to draw. Additionally, a too tight inflation target – in light of severe shocks and a high debt – can imply higher interest rates as well.

To be sure, the extent of the feedback effect depends crucially on the sensitivity of the default risk to domestic interest rates, which – again – is an empirical matter. Whereas the model assigns to the default risk the entire premium over the risk-free rate, there are conceivably other factors that could be working together with the fiscal accounts in the determination of risk premia. It goes far beyond the objectives of this note to go deep into the empirical issue of what determines sovereign spreads. I do have, however, a belief – still to be confirmed by hard data and a good econometrician – that the most important portion of sovereign spreads movements under normal circumstances result from global, as opposed to idiosyncratic, factors.

That is, global “risk aversion” seems to me the most important factor in the explanation of sovereign spreads, while domestic policy factors – the size of the current account relative to exports, etc – play a secondary role in the process. The persistence of high sovereign spreads in the Brazilian case, regardless the improvements in fiscal fundamentals does look consistent with that interpretation.

All that said, while interest rates are indeed exceptionally high in Brazil, the claim that lowering interest rates could sparkle a virtuous cycle – lower domestic rates feeding into lower sovereign spreads, which reinforce the case for lower rates – rests on the view that the Central Bank is locked in a perverse equilibrium. The elements in this paper do not allow us to rule out the possibility, for sure, but it does offer hints about the properties of the unstable equilibrium that seem at odds with observation, which is the main reason of our agnosticism about the relevance of that equilibrium.

Appendix A: Reaction function when δ > 0

The results we obtained in the note were derived under the assumption that the discount factor, δ, was zero, that is, the Central Bank was myopic, and hence would minimize the loss function only in the current period, paying no attention to whatever takes place thereafter. This should not be the general case, however, as there are presumably two channels through which current decisions about interest rates affect future inflation: the exchange rate depreciation and inertia. Thus, a forward-looking Central Bank would have to take into account these effects, which in turn would affect the setting of current interest rates. In this appendix, we show that the basic results obtained for the particular case we studied, remain valid under less stringent assumptions.

To see that, consider the Phillips curve for period t+1

$$\pi_{t+1} = \pi_t + \alpha y_{t+1} + \theta \Delta e_{t+1} + \varepsilon_{t+1} \qquad \text{(A1)}$$

Using uncovered interest parity $\Delta e_{t+1} = i_t - i_t^*$ and the expression for current inflation below

$$\pi_t = \pi_{t-1} - (\alpha\beta + \theta) i_t + \alpha\beta(\bar{\pi} + \bar{r}) + \theta i_t^* + \alpha\eta_t + \varepsilon_t \qquad \text{(A2)}$$

we can express inflation in period t+1 as:

$$\pi_{t+1} = \pi_{t-1} - \alpha\beta i_t + \alpha\beta(\bar{r} + \bar{\pi}) + \alpha y_{t+1} + \alpha\eta_t + \varepsilon_t + \varepsilon_{t+1} \qquad \text{(A3)}$$

Before moving on, note that the parameter θ no longer affects inflation in period (t+1). Solving the Phillips Curve (A1) forward and using (A3), we have, in general:

$$\pi_{t+n} = \pi_{t-1} - \alpha\beta i_t + \alpha\beta(\bar{r} + \bar{\pi}) + \alpha\eta_t + \alpha\sum_{j=1}^{n} y_{t+j} + \sum_{j=0}^{n} \varepsilon_{t+j} \qquad \text{(A4)}$$

The loss function becomes therefore:

$$L = E_t\left[\left(\pi_{t-1} - (\alpha\beta + \theta) i_t + \alpha\beta(\bar{\pi} + \bar{r}) + \theta i_t^* + \alpha\eta_t + \varepsilon_t - \bar{\pi}\right)^2\right]$$
$$+ E_t \sum_{n=1}^{\infty} \delta^n \left(\pi_{t-1} - \alpha\beta i_t + \alpha\beta(\bar{r} + \bar{\pi}) + \alpha\eta_t + \alpha\sum_{j=1}^{n} y_{t+j} + \sum_{j=1}^{n} \varepsilon_{t+j} - \bar{\pi}\right)^2 \qquad \text{(A5)}$$

Note that now it is clear that the current interest rate affects the loss function in all periods. Minimizing L with respect to i_t, and playing around with the FOC, one obtains the following reaction function:

$$i_t = \frac{[\alpha\beta + (1-\delta)\theta](\pi_{t-1} - \bar{\pi}) + \alpha\beta[\alpha\beta + (1-\delta)\theta](\bar{r} + \bar{\pi}) + (1-\delta)(\alpha\beta + \theta)\theta i_t^*}{(\alpha\beta)^2 + (1-\delta)\theta(2\alpha\beta + \theta)} \qquad \text{(A6)}$$

In the (i^*, i) plane, (A6) remains a positively sloped line. For δ = 0, (A6) implies the same slope for the reaction function as in the text, while for δ=1, the reaction function becomes horizontal, and equilibrium would ensue as long as the intercept is lower than i_H. Clearly, the full-fledged intertemporal analysis imposes less stringent conditions for the existence of equilibria.

Appendix B: Equilibrium conditions

Assume λ(i) takes the convenient form:

$$
\begin{aligned}
l(i) &= \left(\frac{i_H - i}{i_H - i_L} \right)^a ; a \le 1; i_L \le i \le i_H \\
l(i) &= 1; i \le i_L \\
l(i) &= 0; i \ge i_H
\end{aligned}
\qquad (B1)
$$

Further, consider the very special case, in which a=1. Use (B1) in (15) to find:

$$
i^* = i^{US} \left(\frac{i_H - i_L}{i_H - i} \right) \qquad (B2)
$$

Now replace (B2) in the reaction function (11) and one can find the following second-degree equation:

$$
(ab + q)i^2 - [ab(\bar{r} + \bar{p}) + (ah + e) + (ab + q)i_H]i + [ab(\bar{r} + \bar{p}) + (ah + e)]i_H + q(i_H - i_L)i^{US} = 0
$$

Thus, there will at least an equilibrium if:

$$
\left[i_H - \frac{ab(\bar{r} + \bar{p}) + a h_{t-1} + e_{t-1}}{ab + q} \right]^2 \ge 4 \frac{q(i_H - i_L)i^{US}}{ab + q} \qquad (B3)
$$

as claimed in the text.

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