A monetary mechanism for sharing capital: Diamond and Dybvig meet Kiyotaki and Wright*

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Abstract

Historically, payment systems and capital intermediation interact. Friedman (1959), and many observers of bank instabilities, have advocated separating depositary from credit institutions. His proposal meets today an ever-increasing provision of inside money, and a shortage of monetary models of bank intermediation. In this paper, we evaluate the proposal from a new angle, with a model in which isolating a safe payments system from commercial intermediation undermines information complementarities in banking activities. Some features of the environment resemble the models in Diamond and Dybvig (1983), and Kiyotaki and Wright (1989).

1 Introduction

Historically, commercial banks play active roles in both payment systems and capital intermediation. Friedman (1959), expressing widespread views, is concerned that movements in the demand for money interact with credit activities in undesirable ways, constituting a likely cause of many episodes

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of bank failure. He proposes a banking reform forcing a separation between depositary institutions and intermediaries. Friedman's proposal deals with two functions of financial systems that economists typically do not study in the same model. We study both functions with a model in which money complements capital in exchanges, alleviating transaction difficulties related to information. We then show that Friedman's proposal may interfere with an optimal use of information.

We emphasize economies of scope, although Friedman's proposal has attracted attention for other reasons. One claim is that depositary institutions, once separated, can be made more stable with the help of reserve requirements or, perhaps, government-provided deposit insurance. A plausible consideration is that though liquidity problems may have existed in the past, recent innovations in the financial system would enable banks to prevent them without difficulties. The argument is in agreement with recent research in monetary theory, which predicts an increasing role for inside money with the aid of modern information technologies.

When capital intermediation is completely financed by long-term liabilities, there is a consequent reduction in the system illiquidity. Some economists have objected to the idea that this reduction is in itself a desirable goal. We have chosen to discuss Friedman's proposal in a context in which fiat assets can be easily provided, as if the non-bank public is indifferent between deposits and currency, and suspension of their convertibility never becomes a problem. In our context, questions about the illiquidity of the banking sector are replaced by considerations about its ability to reallocate idle resources.

From a theoretical perspective, we focus on inside money that is tailored to individuals in the form of credit lines, and show that there are information frictions allowing money and such a form of credit to coexist. To keep the model tractable, we introduce a very rudimentary form of capital goods, and assume that the opportunities for consumption and production are governed by deterministic cycles and idiosyncratic risk. We combine two simple ideas about the potential roles of fiat and productive assets. First, information frictions supporting inside money lead to incomplete insurance. Desirable allocations have inside money flowing mostly to relatively wealthy individuals, with high propensity to consume. Second, it is also desirable to have capital flowing from spenders to producers when production is decentralized.

In our model, the importance of inside money varies according to the levels of idiosyncratic risk and capital stocks. When risk is nonexistent and the supply of capital is high, the bank sector plays no role on the welfare of the non-bank public. We show however that the use of inside money becomes important when there is risk or capital scarcity. The creation of inside money is trusted to bankers because they are the group that is the easiest to monitor and to punish by society. Information about histories of the non-bank public is however imperfect, and the need to provide incentives for supporting the value of money results in incomplete insurance. The same frictions leading to the essentiality of inside money produce a nondegenerate distribution of individuals, according to their propensity to consume. The bank sector can therefore use this information to reallocate capital under some conditions.

The rest of the paper is organized as follows. The environment is described in section 2. In section 3, we study a benchmark version of the model where the non-bank public uses outside money and capital is not scarce. This set of allocations is helpful for showing that in the limit, when there is no productivity risk, inside money is unnecessary. In section 4 we study how the results of the previous section change when capital is scarce.

In section 5 we weaken the perfect anonymity of the non-bank public by introducing the concept of credit lines. We first ignore capital transfers, and construct a simple inside-money allocation with binary credit records. We derive a necessary and sufficient condition for this allocation to be implementable. If the discount factor is sufficiently high, the constructed credit allocation attains a higher welfare than benchmark allocations. We then argue that when the capital supply is relatively low, credit allocations have the banking sector intermediating capital.

We conclude in section 6 with a discussion of our assumptions and other literature. Although we restrict ourselves to planner problems with a simple binary-state structure, and hence do not fully characterize optimum credit allocations, we discuss why we think our results generalize to these allocations. All the proofs appear in the appendix.

2 The environment

We study planner's problems for intermediation, like Diamond and Dybvig (1983) and Green (1987). Our environment is however designed to give money a medium-of-exchange role, in the spirit of Kiyotaki and Wright (1989). In particular, we build on Cavalcanti and Wallace (1999). Their model emphasizes the information requirements of the banking construct introduced by Cavalcanti, Erosa and Temzelides (1999).

Time is discrete and the horizon is infinite. There is one type of divisible and perishable consumption good per date, but people rank goods in odd dates differently from goods in even dates. There is also a limited supply of a perfectly durable, but indivisible, productive assets, called capital. These assets are non reproducible. There is a $[0,1]$ continuum of each of 2 types of people. Each type is specialized in consumption and production: a type e person consumes even-date goods and produces odd-date goods, and a type d person consumes odd-date goods and produces even-date goods . Each type maximizes expected discounted utility with discount factor $\beta \in (0,1)$. We find it useful to have a notation for the two-period discount factor, $\delta \equiv \beta^2$. We also find it convenient to refer to a type e individual in an even (odd) date, or a type d individual in an odd (even) date, as a *consumer* (*producer*).

Production of goods is decentralized, requires capital, and is subject to idiosyncratic productivity shocks. We assume that shocks are drawn from a fixed distribution with binary support. The probability of the high productivity realization is denoted by π. A person without capital holdings ($a = 0$), or drawing the low realization, cannot produce. A person holding a unit of capital ($a = 1$), and drawing the high realization, can produce any choice of $y \in \mathbb{R}_+$ units of the corresponding date good, at a utility cost normalized to be y itself. Utility in a period is given by $u(y)$ when consuming, or $-y$ when producing, where y is the amount consumed or produced. The function u is defined on $[0,\infty)$, is increasing and twice differentiable, and satisfies $u(0) = 0$, $u'' < 0$, $u'(0) = \infty$ and $u'(\infty) < 1$.

The $[0,1]$ continuum of each of two types is further divided into two groups of equal measure, defined by the amount of information publicly available about their histories. The society is able to keep a public record of the assets, actions and shock realizations of the first group, called bankers. Regarding the other group, the non-bank public, or non bankers, for short, the society can keep a record of announcements that is shared only among bankers. More precisely, at date 0, each non banker $i \in [0,1]$ is assigned a unique password number $s = f(i)$ according to a function f that is known only to bankers. Although each identity i is private information, bankers can record messages from non bankers declaring a pair (i,s) in a given date. We shall see that bankers have no incentives for sharing the records or the function f with non bankers.

In each period, people are twice randomly matched in pairs. First each non banker is matched with a banker of the same type. In these meetings there is no scope for consumption and production, although announcements can be recorded, and capital and monetary assets, to be defined below, can change hands. Then, in a second meeting, each type e banker is matched with a type d banker, and each type e non banker is matched with a type d non banker. In the second meeting, the realization of productivity shocks occur and production takes place.

We assume that people cannot precommit to future actions, so that those who produce or give up assets have to get a future reward for doing so. As in Cavalcanti and Wallace (1999), bankers can be induced to produce and transfer assets without receiving something tangible in exchange, because they can be rewarded and punished in the future for actions they take currently. Unlike their environment however, non bankers here can in principle transfer assets to bankers without receiving something tangible, although that cannot happen in meetings between two non bankers, where they must receive something tangible in order to produce.

We assume that each person has a technology that permits the person to create indivisible and perfectly durable objects called notes. The notes issued by bankers are distinguishable from those issued by non bankers. We also assume that any person can always destroy or freely dispose of notes that the person has acquired. As in Cavalcanti and Wallace (1999), desirable allocations do not require note issue by non bankers, and might require note issue and redemption (destruction) by those whose histories are fully known, the bankers. In allocations where bankers are not creating notes, the monetary assets are interpreted as outside money in constant supply. We assume that bankers are not able to see the assets of a non banker in a meeting. If capital or money is observable, a larger set of inside-money allocations could in principle be studied, but the essence of our findings would also apply.

We also assume that in a meeting with production, capital units can be transferred only after production takes place. This assumption prevents capital sharing by having consumers transferring it to producers, who use it for production, and receiving it back in the same meeting. To keep the model simple, we assume that each person can carry from one meeting to the next a pair $(a, m) \in \{0, 1\}^2$, that is, at most one unit of capital and at most one unit of money. We let $k \in [0, 2]$ be the total measure of the existing supply of capital. When $k = 2$, each person starts at date 0 with one unit of capital and the model more closely resembles typical random-matching specifications.

Although formal definitions of allocations are given at different contexts in the sections that follow, it is useful to anticipate here the criteria we use to compare them. We are interested in steady states where people of types e and d are treated symmetrically within the same sector. For instance, regarding the non-bank sector, we associate allocations to a list (p, q, y), where p is a measure of potential producers, q is a measure of potential consumers, and y is the level of consumption and production going on in non-bank meetings. The qualification "potential" will be used in connection to specific money and capital holdings that enable individuals to engage in trade. From society's point of view, non-bank welfare U_{n} will be defined as a multiple of

$$\frac{1}{1-\beta}pq[u(y) - y],$$

that is, the present discounted value of the product of two terms: a measure of trade frequency, pq, and a measure of social gains per meeting, $u(y) - y$. It can be shown that this value, after normalizing constants are taken into account, correspond to the expected discounted utility, faced by each non banker at the very first date of the economy. This interpretation of U_{n} requires that types, asset holdings, and credit records be allocated randomly to non bankers at the first date, according to the stationary distributions chosen as steady states.[1] The type notations, d and e, are thus not necessary in the description of allocations. It is also not important to associate π to productivity risk. It could represent preference risk as in Diamond and Dybvig (1983) as well.

3 Benchmark allocations

In our benchmark allocations, non bankers are never handed out passwords, and capital does not change hands. For ease of exposition, we assume in this section that each person starts with a unit of capital, so that $k = 2$. Our goal is to describe an allocation in which the bank and non-bank sectors live in isolation from each other, with non bankers using outside money, and to show that this allocation is optimal when $\pi = 1$.

We assume a symmetric distribution of outside money among non bankers, and ignore inside money. We let q denote the measure of non-bank consumers starting with a unit of outside money, multiplied by 2π, and let p denote the

[1]This is equivalent to assuming the the first date of the model economy is even with probability one half, and odd with probability one half.

measure of non-bank producers starting without a unit of outside money, also multiplied by 2π. We make use below of the fact that p is the probability of meeting with a productive producer, who is willing to trade for money because he or she does not have a unit of money. We begin by specifying the set of symmetric and stationary allocations satisfying sequential, individual rationality constraints that, for short, we also call participation constraints.

Corresponding to these constraints, as Cavalcanti and Wallace (1999) describe, there is a trading mechanism in which people in meetings play a coordination game: two people in a meeting consider a trade proposed by the social planner; if they say *agree*, then that trade occurs; if at least one say *disagree*, then each person leaves the meeting and proceeds to the next date with what the person owned prior to the meeting. An allocation is implementable if it is a subgame perfect equilibrium for this coordination game or, more conveniently, if and only if it satisfies the participation constraints dictated by individual rationality.

It is important to take into account the consequences of not having non-bank announcements recorded by bankers, together with symmetry and stationary restrictions. Let us recall first that all notes issued by non bankers, being uniform, are treated the same. This implies that a non banker will never produce or transfer capital in order to acquire a note issued by another non banker because the non banker could always hand out his or her own note instead. Moreover, without capital transfers, since in meetings between bankers and non bankers there is no production, and because the non banker remains anonymous in the absence of passwords, note issue by bankers to non bankers is not consistent with a steady state. Indeed, bankers have nothing to offer to non bankers when it comes the time to retire or destroy such notes. We use that fact and ignore note issue in what follows.

We can thus ignore meetings between bankers and non bankers. Let us first describe the part of the symmetric and stationary allocations that are relevant to non bankers.

When two non bankers meet, trade can occur only if the producer does not have money and the consumer does, since trade among non bankers must be *quid pro quo* trade. Let us consider that the mechanism in play recommends a level of output y in such a meeting, to be produced by the producer in the meeting and consumed by the consumer in the meeting. If y is acceptable to both parties, the consumer transfers his or her note to the producer; otherwise, nothing happens in the meeting and they go on to the next date.

In order to express the participation constraints, we first describe in terms of y the expected discounted utilities of non bankers as of the beginning of a period before meetings. We let v be that of a non banker consumer with one unit of money, and w be that of a non banker producer without money. Given that the probability of finding a productive producer without money is p, and the probability of finding a consumer with money is q, the stationary values v and w satisfy

$$v = p[u(y) + \beta w] + (1 - p)\beta w^{\emptyset} \tag{1}$$

and

$$w = q[-y + \beta v] + (1 - q)\beta v^{\emptyset}, \tag{2}$$

where $w^{\emptyset}$ is the value of a producer with money, and $v^{\emptyset}$ is that of a consumer without money. Moreover, current-period consumers without money have to wait for the next period, when they become producers, to engage in trade. As a result of the unit upper bound on money holdings, the same applies to current-period producers with money. Hence, we also have

$$v^{\emptyset} = \beta w \tag{3}$$

and

$$w^{\emptyset} = \beta v. \tag{4}$$

These expressions are written under the presumption that when y is produced and consumed, a unit of outside money changes hands. In this section, the participation constraints for non bankers assume that defection on the part of non bankers goes unpunished because such defection does not become part of a public record. Thus, the participation constraints are simply that trade is weakly preferred to leaving the meeting with what was brought into the meeting. There are two such constraints, one for the consumer and one for the producer:

$$u(y) + \beta(w - w^{\emptyset}) \geq 0 \tag{5}$$

and

$$-y + \beta(v - v^{\emptyset}) \geq 0. \tag{6}$$

In addition to the conditions of non negative gains from trade, the measures p and q have to be consistent with stationarity. If there are $1/2 - p/2\pi$ potential producers with money in the current period, while q times $p/2\pi$ producers (without money) engage in trade in the current period, then next's period measure of consumers with money, multiplied by 2π, is given by $\pi - p + pq$. The stationarity requirement for q is thus

$$q = \pi - p + pq, \text{ with } p,\ q \in [0, \pi], \tag{7}$$

which also implies that for p, namely $p = \pi - q + pq$. We arrive at the following definition of implementable allocations, when the banking sector is ignored.

Definition 1 *A benchmark suballocation (y, p, q) for non bankers is implementable if there exists (v, w) such that (1-7) hold.*

The expected utility for non bankers satisfies

$$2\pi U_n = qv + (\pi - q)v^{\emptyset} + pw + (\pi - p)w^{\emptyset},$$

provided that the expectation is taken with respect to the stationary (and initial) distribution of money holdings. Then one benchmark planning problem is the following.

Subproblem 1 Maximize U_n by choice of an implementable benchmark suballocation (y, p, q).

The welfare criterion leading to the definition of U_n gives the same weight to all individuals at date 0. Due to the bilateral trade structure, whenever there is a flow of trade, there is a corresponding flow of utility for consumers $u(y)$ and disutility y for producers. Since the stationary volume of trade is $\pi(p/2\pi)(q/2\pi)$, it follows that U_n must be equal to the present discounted value of the constant flow $pq[u(y) - y]/4\pi$, that is,

$$U_n = \frac{1}{1-\beta}\frac{pq}{4\pi}[u(y) - y]. \tag{8}$$

Equation (8) can also be derived by combining the definition of U_n with equations (1-4) and (7). For solving the optimum problem, we use (3-4) to rewrite (1) and (2) in the matrix notation

$$M \begin{pmatrix} v \\ w \end{pmatrix} = \begin{pmatrix} pu(y) \\ -qy \end{pmatrix}, \tag{9}$$

where

$$M = \begin{pmatrix} 1-(1-p)\delta & -\beta p \\ -\beta q & 1-(1-q)\delta \end{pmatrix}, \tag{10}$$

and $\delta = \beta^2$. We also use (1-2) and (3-4) to write the participation conditions (5) and (6) as

$$u(y) + \beta w - \delta v \geq 0, \text{ or } \quad v \geq 0, \tag{11}$$

and

$$-y + \beta v - \delta w \geq 0, \text{ or } \quad w \geq 0. \tag{12}$$

The participation constraint for the producer, (12), is equivalent to the inequality $\beta v \geq y$. Since $y \geq 0$, then (12) implies (11). Solving now for w and v in (9), for a given y, yields, after some simple algebra, the condition that $w \geq 0$ if and only if

$$u(y) \geq \frac{y}{\beta}[(1-\delta)\frac{1}{p} + \delta], \tag{13}$$

when p is positive.

Therefore, the solution to subproblem 1 can be found by maximizing $pq[u(y) - y]$ subject to the producer's participation constraint, (13), and the stationarity requirement that $q = \pi - p + pq$. For β sufficiently high, the participation constraint does not bind and the solution is given by $u'(y) = 1$ and $p = q$ satisfying $p^2 - 2p + \pi = 0$. This choice of (p, q) corresponds to the quantity of outside money that maximizes the flow of trade pq. When (13) is violated for such a p and the first-best level of output, satisfying $u'(y) = 1$, then the social planner has to trade-off a reduction in the social surplus, $u(y) - y$, and in the trade volume pq, for an increase in p which weakens the participation constraint. When the constraint binds, the solution has $w = 0$.

We now describe the part of the symmetric and stationary allocations that are relevant to bankers. Here we ignore meetings between bankers and non bankers. Regarding meetings between bankers, we first note that money is not needed to induce bankers to produce, because the trade histories of bankers are known. We assume that society keeps a binary record r for each banker. At date 0, all bankers are assigned the record $r = 1$. Let us consider that the mechanism in play recommends a level of output y in a meeting where two bankers have the good record $r = 1$, and the producer draws the high productivity shock. If y is acceptable to both parties, y is produced and the record of each banker remains $r = 1$. Otherwise, nothing happens in the meeting, but each banker in disagreement with the planner recommendation has his or her record updated for $r = 0$. At the next date, the mechanism recommends zero production in meetings where consumers have the bad record $r = 0$.

It follows that the expected discounted utility for a banker with record $r = 0$ is zero, because that banker lives in autarky. In our notation, we anticipate the result that there is no defection with implementable allocations, and describe in terms of y the expected discounted utilities of bankers with $r = 1$ as of the beginning of a period before meetings. We let $\overline{v}$ be that of a banker consumer, and $\overline{w}$ be that of a banker producer. The stationary values $\overline{v}$ and $\overline{w}$ satisfy

$$\overline{v} = \pi[u(y) + \beta\overline{w}] + (1 - \pi)\beta\overline{w} \tag{14}$$

and

$$\overline{w} = \pi[-y + \beta\overline{v}] + (1 - \pi)\beta\overline{v}. \tag{15}$$

The participation constraints are now

$$u(y) + \beta\overline{w} \geq 0 \tag{16}$$

and

$$-y + \beta\overline{v} \geq 0, \tag{17}$$

since the payoff from defection is zero. Ignoring now the non-bank sector, we consider the following suballocations:

Definition 2 *A benchmark suballocation y for bankers is implementable if there exists $(\overline{v}, \overline{w})$ such that (14-17) hold.*

It follows now that $U_b = (\overline{v} + \overline{w})/2$—the expected utility of bankers, an expectation taken after assigning equal probability that the initial date is even or odd. Then our benchmark bank optimization problem is the following.

Subproblem 2 Maximize U_b by choice of an implementable benchmark suballocation y.

The solution to problem 2 is trivial. The producer's participation constraint is easily found to be

$$u(y) \geq \frac{y}{\beta}[(1-\delta)\frac{1}{\pi} + \delta], \tag{18}$$

which corresponds to the non-bank participation constraint (13) when $p = \pi$. The optimum production level corresponds to the minimum between the y that satisfies this constraint with equality, and the first-best level of production, the y such that $u'(y) = 1$. A comparison with the optimum for non bankers leads to the following conclusion about outside-money allocations.

Proposition 1 *Assume that $k = 2$. If $\pi = 1$, then optimization of benchmark allocations yields $U_n = U_b$. Hence outside money is essential (and inside money is not) for these parameters. However, when $\pi < 1$, that optimization yields $U_n < U_b$, with welfare increasing in π.*

The proof in the appendix is straightforward. The non-bank constraint set is increasing in p and, as just noted, coincides with that of bankers if and only if $p = \pi$. It follows from the stationarity restrictions on p that, if $\pi < 1$, then $p = \pi$ only if $q = 0$. Moreover, when participation constraints do not bind, although production levels are the same in both sectors, the volume of trade is higher in the banking sector, since the stationary values of p, such that $p = q$, are lower than π by a difference that is decreasing in π.

4 Capital allocations

We now assume that $k < 2$, so that there is a potential scarcity of capital. Our goal in this section is to revisit the properties of the benchmark allocations of the previous section. We start by looking at the bank sector in isolation.

With the deterministic pattern of consumption and production dates for each individual, there is room for reallocating capital from producers to consumers after production takes place. A simple allocation for bankers can do just that. The planner's recommendation is that every producer transfers his or her capital holdings to the consumer at the end of the meeting if the consumer does not have a unit of capital already. This allocation maximizes the chances of production at the next date, and does not deal with any new participation constraint. For the producer is content with transferring capital, since capital is of no use after defection for a banker who has to live in autarky.

Let us suppose that the measure of capital in the bank sector is $k_b \leq 1$. Then according to the recommended allocation, the measure of bank producers with capital in the beginning of a period, multiplied by 2, is $\tau \equiv 2\min\{k_b, 1/2\}$. We describe in terms of y the expected discounted utilities of bankers holding a units of capital, at the beginning of a period before meetings. We let v_a be that of a bank consumer, and w_a be that of a bank producer. Given that the probability of finding a producer with capital is τ, and the probability of finding a consumer with capital is $\theta \equiv 2k_b - \tau$, the stationary values satisfy

$$v_0 = \tau[\pi u(y) + \beta w_1] + (1 - \tau)\beta w_0, \tag{19}$$

$$v_1 = \tau[\pi u(y) + \beta w_1] + (1 - \tau)\beta w_1, \tag{20}$$

$$w_0 = \beta v_0 \tag{21}$$

and

$$w_1 = \theta[-\pi y + \beta v_1] + (1 - \theta)[-\pi y + \beta v_0]. \tag{22}$$

When $k_b = 1/2$, the bank sector can still attain benchmark consumption levels by having capital changing hands every period. In a somewhat trivial way, capital transfers trace the distribution of marginal utility of consumption across individuals. Producers in the current period, having a high marginal utility of consumption in the next period, transfer capital. Even when $k_b < 1/2$, the study of desirable allocations is similar in spirit to the analysis of the previous section because the participation constraints remain essentially the same:

$$u(y) + \beta\min\{w_0, w_1\} \geq 0 \tag{23}$$

and

$$-y + \beta\min\{v_0, v_1\} \geq 0. \tag{24}$$

We consider now the following class of capital allocations for the bank sector.

Definition 3 *A suballocation y for bankers is implementable with capital k_b if there exists (v_a, w_a) such that (19-24) hold for $\tau = 2\min\{k_b, 1/2\}$ and $\theta \equiv 2k_b - \tau$.*

It follows now that $U_b(k_b) = (\bar{v} + \bar{w})/2$, where $\bar{v} = \theta v_1 + (1 - \theta)v_0$ and $\bar{w} = \tau w_1 + (1-\tau)w_0$, with the understanding that the expectation underlying the definition of U_b takes into account the stationary (and initial) distribution of capital holdings for bankers. Then our next subproblem is the following.

Subproblem 3 Maximize $U_b(k_b)$ by choice of an implementable suballocation y with capital k_b.

The solution to subproblem 3 can be described as follows.

Lemma 1 *The maximum in subproblem 3, when the productivity risk is π, coincides with the optimum benchmark welfare for bankers when the productivity risk is $\tau\pi$.*

We next show that the non-bank sector cannot be as efficient as the bank sector in allocating scarce capital. For ease of exposition, when describing implementable suballocations, we anticipate some of the consequences of capital being scarce in the non-bank sector, say with measure $k_n \leq 1$. It is desirable that producers without money hold as much capital as possible, and that consumers without capital hold as much money as possible, being able to buy capital and to become producers in the future. A consumer without money and without capital is destined to autarky when credit is ignored, as we are assuming in this section.

Let us anticipate, therefore, that non bankers without money are given the priority for receiving capital at date 0, and that the initial measure of non bankers without capital receives a unit of money. Let us consider the planner recommendation that one unit of money be exchanged, in productive meetings, for a level of output y together with a unit of capital as well. Then the initial distribution of money and capital can be made stationary as long as consumers with money do not start with capital. If capital is not scarce, the planner can either dispose of capital at date 0, or suggest that money buy goods only, not capital, when the consumer has capital already.

Given the above considerations, when describing desirable allocations, it suffices then to distinguish four values for non bankers: v is the value of a consumer with money (with or without capital); $v^\emptyset$ is that of a consumer without money and with capital; w is that of a producer without money and with capital; and $w^\emptyset$ is that of a producer with money (with or without capital). Since $v' = \beta w$ and $w' = \beta v$, the relevant expected discounted utilities satisfy

$$M \begin{pmatrix} v \\ w \end{pmatrix} = \begin{pmatrix} pu(y) \\ -qy \end{pmatrix}, \tag{25}$$

as before, except that p is now the measure of producers with capital and without money, multiplied by 2π, and q is the measure of consumers with money, also multiplied by 2π. Regarding the feasibility of p and q, in addition to the stationarity requirement,

$$q = \pi - p + pq, \text{ with } p,\ q \in [0, \pi], \tag{26}$$

there is now a capital constraint: the measure of producers without money and with capital, plus the measure of consumers without money and with capital, cannot exceed k_n. Since these measures correspond, respectively, to p and $\pi - q$, after the scale factor 2π is taken into account, the capital constraint is

$$p + \pi - q \leq 2\pi k_n. \tag{27}$$

The participation constraint for consumers and producers are, as before,

$$v \geq 0 \text{ and } w \geq 0. \tag{28}$$

We have hence chosen to examine the following class of capital suballocations, when the banking sector is again ignored.

Definition 4 *A suballocation (y, p, q) for non bankers is implementable with capital k_n if there exists (v, w) such that (25-28) hold.*

The welfare measure for non bankers is $U_n(k_n)$ satisfying

$$2\pi U_n(k_n) = qv + (\pi - q)\beta w + pw + (\pi - p)\beta v.$$

The modified non-bank subproblem is the following.

Subproblem 4 Maximize $U_n(k_n)$ by choice of an implementable suballocation (y, p, q) with capital k_n.

The following lemma shows that capital scarcity plays the same role as the condition $\pi < 1$ regarding the essentiality of outside money, as stated in proposition 1.

Lemma 2 *The maximum in subproblem 4 is bounded above by the benchmark welfare of non bankers, and is strictly lower than the optimum* $U_b(k_n)$ *for* $k_n < 1/2$*, even when* $\pi = 1$.

Let us suppose that the benchmark non-bank solution is (y, p, q). We have thus reached the conclusion that the supply of capital affects welfare if $k_b < 1/2$ or, using $k = k_b + k_n$ and the capital constraint (27) to take into account the non-bank sector, if $k < 1+(p-q)/2\pi$. If the non-bank producer's constraint is not binding, then the benchmark solution has $p = q$, and capital is scarce if $k < 1$. Otherwise, that solution has $p > q$ and capital can be considered scarce for levels of k higher than one. For low levels of capital, the social planner has to make a choice of how much capital to allocate to each sector in the context of an outside-money allocation. In face of the results in proposition 1, and lemmas 1 and 2, maximization of the economy-wide welfare criteria, $\min\{U_b(k_b), U_n(k_n)\}$, would require a greater allocation of capital to the non-bank sector, namely, $k_n > k_b$ such that $U_b(k_b) = U_n(k_n)$.

Such considerations are made so far in the context of a bank sector that does not trade with non bankers. When $\pi < 1$, the use of outside money in the non-bank sector is such that a measure of capital remains idle in the hands of consumers, who were not able to sell capital in the previous period. On the same token, there is in every period a measure of producers with money and without capital. There would exist thus scope for the bank sector to trade capital with non bankers, and in the context of full anonymity of non bankers, such trades require the use of money. Intermediation of capital could in principle be implemented with outside money, by having the bank sector refluxing outside money with sales and purchases of capital from the non-bank sector, and by using banker histories to implement trades within the bank sector. The extent of outside-money intermediation would nonetheless be limited by the ability of the bank sector in refluxing outside money among bankers. The absence of such limitations is a defining feature of inside money.

There is however a reason why intermediation without credit cannot work in this framework. With stationary allocations, any inflow of capital into a sector has to be matched by an equal outflow. The only non bankers who could potentially buy capital from bankers are producers with money. But since they can buy both goods and capital from non-bank producers without money, they do not have any incentive to trade with bankers. We shall show that inside money can get around this problem when we allow non bankers to build a credit record with the bank sector. Before turning to credit allocations, we leave to the reader a general definition of implementable outside-money allocations, when trade across sectors is allowed, as well as the straightforward proof of the following proposition.

Proposition 2 *Without passwords, optimum allocations have no trade between sectors, and have at least as much capital allocated to the non-bank sector as to the bank sector.*

We have thus a motivation for studying capital intermediation with credit in the next section. There is however a reason to study inside-money allocations, even when capital is not scarce, but $\pi < 1$. That reason comes from the possibility of using inside money in connection with personalized credit, in such a way designed to insure non bankers against the risk of unsuccessful trade attempts. We shall proceed by assuming first that $k = 2$ in the next section, and by returning to the discussion of capital intermediation as a restriction to inside-money allocations.

5 Credit allocations

We assume first that $k = 2$ and ignore issues related to the allocation of capital. We now assume that each non banker is initially given a pair of numbers (i, s) that are private information to each individual. We assume however that all bankers know the function f mapping identities to passwords, $s = f(i)$, but do not share this information with non bankers. Since bankers' histories are fully known, they can be punished for sharing f with non bankers, and therefore never do so in the equilibrium context of implementable allocations. As a result, in a meeting with a banker, a non banker can announce a pair $(i, f(i))$ and have the history of *that* meeting recorded.

This information structure opens up the possibility that bankers condition trade to the recorded histories of non bankers. Optimum allocations would make as much use as possible of recorded histories. For the purposes of this paper, it suffices to study a simple scheme that we call *binary credit records.*

The planner assigns to each non banker a *balance* $z \in \{0, 1\}$. The planner recommends bankers to issue inside money, upon request, to non bankers with $z = 1$. When money is issued, the record is updated to $z = 0$. The planner also recommends bankers to destroy money, upon request, offered by a non banker with $z = 0$. When that happens, the record is updated to $z = 1$. Since only inside money is issued, in stationary allocations only inside money is held. Our goal is to show that if the discount factor β is sufficiently high, such a credit allocation attains a higher welfare than the benchmark allocations.

We now turn to describe participation constraints. We first recall that in meetings between bankers and non bankers there is no *single coincidence of wants* by assumption. So the only thing that can happen in these meetings in the updating of credit records, with creation or destruction of money. These activities do not affect the payoff of banks. Hence, the part of an inside-money allocation that is relevant to bankers can be analyzed as in the previous sections.

Regarding non bankers, we distinguish the following values after meetings with bankers take place, but before they are matched with non bankers: v_z is the value of a consumer with money and credit record z, and w_z is that of a producer without money and credit record z. It is also useful to think of z as money holdings *deposited* in the bank sector. We could also assign a value function to consumers without money, say $v_z^\emptyset$. It just happens, however, that if $z = 0$ then the consumer cannot buy goods in the current period, or make a deposit into his or her account in the next period, and thus $v_0^\emptyset = \beta w_0$. If $z = 1$, the consumer has no incentives to make a withdraw from a banker in the next period, when he or she becomes a producer. Hence $v_1^\emptyset = \beta w_1$. Likewise, we could also have assigned a value to producers with money, $w_z^\emptyset$. If $z = 0$, the producer will need the money anyway, so he or she makes no deposit next period and $w_0^\emptyset = \beta v_0$ (stationarity is of course important here). If $z = 1$, the producer cannot improve his or her record further, and essentially for the same reason, $w_1^\emptyset = \beta v_1$.

The values v_z and w_z should hence satisfy the following system of equations for a given y:

$$w_1 = q[-y + \beta v_1] + (1 - q)\beta v_0, \tag{29}$$

$$w_0 = q[-y + \beta v_0] + (1 - q)\delta w_0, \tag{30}$$

$$v_1 = p[u(y) + \beta w_1] + (1 - p)\delta v_1 \tag{31}$$

and

$$v_0 = p[u(y) + \beta w_0] + (1 - p)\beta w_1, \tag{32}$$

where p and q are the respective measures of producers without money and of consumers with money, integrated over the distribution of credit records, and multiplied by 2π.

The terms multiplying $1 - q$ and $1 - p$ in the first and fourth equations are written assuming that consumers with good credit, who need money, agree to withdraw from the banker and have the record updated to $z = 0$; and that producers with bad credit, who were not able to spend money in the previous period, agree to deposit with the banker and have their record updated to $z = 1$. The participation constraints regarding these transactions are, respectively,

$$v_0 \geq \beta w_1 \tag{33}$$

and

$$w_1 \geq \beta v_0. \tag{34}$$

The first inequality assures that a non banker is willing to borrow when there is an opportunity, a constraint that is easily satisfied by a stationary allocation with discounting, provided that value functions are nonnegative. We call the second inequality a *deposit* constraint. It assures that a non banker is willing to deposit money with a banker, and to become a producer currently without money, just for the sake of improving his or her credit record. It can be easily verified that this constraint is equivalent to the requirement that a producer with good credit is willing to produce in exchange for money, namely,

$$y \leq \beta(v_1 - v_0).$$

The participation constraint for producers with bad credit is

$$w_0 \geq 0, \tag{35}$$

which is the same as the inequality $\beta v_0 \geq y$. Finally, there are the participation constraints for consumers, requiring that v_1 and v_0 be nonnegative, and which are again implied by the producer's constraints.

Next, we discuss feasible measures of producers without money and consumers with money, p and q. It is intuitive that a credit allocation in this framework can increase *both* measures, as result of the intermediation implemented by bankers. We leave the details of the derivation of stationarity restrictions on p and q to the proof of the following lemma.

Lemma 3 *The set of stationary measures* (p, q)*, associated to the system (29-32), and implied by stationary distributions of credit records, is fully characterized by*

$$q = \pi - p + pq + \frac{1}{2}(\pi - p)^{\frac{1}{2}}(\pi - q)^{\frac{1}{2}}, \text{ with } p, q \in [0, \pi]. \tag{36}$$

The lemma shows that credit allows for an increase, associated to the term $\frac{1}{2}(\pi - p)^{\frac{1}{2}}(\pi - q)^{\frac{1}{2}}$, in the set of feasible measures of potential producers and consumers. In the proof, it is used the fact that a measure of producers, in proportion to $p(1 - q)$, fail to acquire money, but is able to make withdraws at the next date because they have good records. Likewise, a measure of consumers, in proportion to $q(1-p)$, fail to spend their money holdings, and is able to make deposits at the next date in order to leave the bad-record state. When $p = q$, half of the non-bank public holds a bad record, and the other half holds a good record. We now define the allocations of interest.

Definition 5 *A credit allocation* $(y_{\mathrm{b}}, y, p, q)$ *is implementable with* $k = 2$*, if* y_{b} *is a benchmark suballocation for bankers, and there exists* $(v_{\mathrm{z}}, w_{\mathrm{z}})$*, such that (29-36) hold.*

We are interested in maximizing $U = \min\{U_{\mathrm{b}}, U_{\mathrm{n}}\}$, where U_{b} is the benchmark welfare for bankers, and U_{n} satisfies

$$U_{\mathrm{n}} = \frac{1}{1-\beta}\frac{pq}{4\pi}[u(y) - y].$$

The optimization problem when $k = 2$ is as follows.

Problem 5 Maximize U by choice of an implementable credit allocation $(y_{\mathrm{b}}, y, p, q)$ with $k = 2$.

The following lemma and proposition shows that the potentially binding constraint for the non-bank subproblem is the deposit constraint (33), which is satisfied for any y, with $u(y) > y$, if β is sufficiently high. Their proofs are omitted because they are particular cases of more general propositions proved below for $k \leq 2$.

Lemma 4 *There exists a unique solution* (w_z, v_z) *to the system (29-32) for any* $y \geq 0$*. The solution is a linear transformation of* $u(y)$ *and* y*, and such that, as a function of* y*, each value is concave and eventually becomes negative. For* $y > 0$*,* $w_0 = 0$ *implies* $w_1 < \beta v_0$*. Moreover, if* $u(y) > y$*, then for* $\beta \in (0, 1)$ *sufficiently high, all participation constraints are satisfied.*

It thus follows that for β high enough, the deposit constraint does not bind for y such that $u'(y) = 1$, the first-best level of production. For this level of output, the volume of trade can be maximized with $p = q$. In summary, we have the following welfare result.

Proposition 3 *Assume* $k = 2$*. If* $\pi < 1$ *but* β *is sufficiently high, then credit is essential, in the sense that the maximum in problem 5 is greater than the benchmark welfare.*

We now turn to the intermediation of capital. Inside-money is destroyed in a credit allocation when a non banker producer with $z = 0$ makes a deposit. The only reason for this non banker to actually make a deposit, instead of holding on to money and waiting to become a consumer next period, is the potential gain of producing again currently to acquire more money. Without capital, the non banker will choose not to deposit. A necessary condition for a credit allocation to be implementable, therefore, is that depositors have access to capital.

With the benchmark allocation there is a need to allocate capital to a mass of non bankers without money. It is necessary to allocate a measure of capital to p producers and $\pi - q$ consumers, divided by 2π. If constraints do not bind and $p = q$, the capital requirement is precisely one half. With inside money, when $p = q$, the minimum capital requirement is still the same, as long as capital is transferred in meetings whenever money is. Hence, credit allocations with $k_n = 1/2$ and $p = q$ would require that bankers transfer capital to depositors, and that consumers, making withdraws, transfer capital to bankers. Our goal is to show that there is a social gain when the bank sector also reallocates capital. If the bank sector is not reallocating capital, then it can only expect deposits from consumers with capital. This requires giving consumers with money some capital at date 0, so that $k_n > 1/2$ even when $p = q$.

There are basically three ways to get deposits going on when capital is scarce. The first is to ask bankers to give capital to all depositors. The second is to allocate capital to some consumers with money. When the consumer meets with a producer with capital and there is production, then that producer keeps his or her capital holdings, and thus becomes a consumer with capital and money next date. If he or she has $z = 0$, and is not able to consume next date, then he or she can make a deposit afterwards in a meeting with a banker. The third way, is to do a combination of the first two. A banker would then transfer capital to some depositors without capital, but not transfer capital to depositors with capital.

The best combination depends on the level of capital scarcity. To fix ideas, let consider again the case $p = q$. The number of potential depositors is proportional to $\frac{1}{2}p(1-p)$, since $\frac{1}{2}$ of the p consumers with money would like to improve their credit record, and do not get to spend money holdings with probability $1-p$. Let us suppose that the planner wants to have a fraction x of the potential depositors being able to deposit. Implementing x might require some randomization from bankers. If the bank sector does not intermediate capital, implementing x requires, in addition to $k_n = \frac{1}{2}$, allocating at least $xq = xp$ (divided by 2π) extra capital to consumers with money at date 0. If all capital allocated to depositors come solely from intermediation, then the bank sector can implement x by disposing of $x\frac{1}{2}p(1-p)$ (divided by 2π) units of capital per period, a much lower capital requirement.

Now, there might be a cost of having bankers transferring capital. As assumed in section 2, bankers meeting with depositors are themselves producers. Taking capital away from bank producers reduces bank welfare. However, if capital is sufficiently scarce and $p = q$, allocating the extra mass xp to bankers, and have them transfer a fraction $x\frac{1}{2}p(1-p)$ to non bankers should provide a higher welfare than by shutting down capital intermediation. If capital is not as scarce, so that the cost of forgone bank production in proportion to $x\frac{1}{2}p(1-p)$ becomes suboptimal, then a combination of capital intermediation and date-0 transfers to non bankers is desirable.

We now revise the Bellman equations to allow for deposits with frequency x. The only required change is to write the equation for potential depositors as

$$v_0 = p[u(y) + \beta w_0] + (1-p)[x\beta w_1 + (1-x)\delta v_0], \qquad (37)$$

where the probability of deposit, $x \in [0, 1]$, corresponds to the probability of the depositor having access to capital as a result of a previous trade, or as a result of a current bank transfer. Hence x corresponds to the probability of the union of the event that a producer with $z = 0$ trades with a consumer with capital, and the event that he or she trades with a consumer without capital but receives a capital transfer from a banker at the time of depositing. It is straightforward to show that, with equation (37), the new system of Bellman equations corresponds the previous one when $x = 1$, and to the benchmark system when $x = 0$, since $v_0 = v_1$ and $w_0 = w_1$ must follow for $x = 0$. After some simple algebra, the new system can be written in matrix notation as

$$M \begin{pmatrix} v_0 \\ w_0 \end{pmatrix} = \begin{pmatrix} pu(y) \\ -qy \end{pmatrix} + x \begin{pmatrix} (1-p)\beta(w_1 - \beta v_0) \\ 0 \end{pmatrix} \qquad (38)$$

and

$$M \begin{pmatrix} v_1 \\ w_1 \end{pmatrix} = \begin{pmatrix} pu(y) \\ -qy \end{pmatrix} + \begin{pmatrix} 0 \\ (1-q)\beta(v_0 - \beta w_1) \end{pmatrix}, \qquad (39)$$

where M is as defined in the benchmark case.

There are no changes in the participation constraints. The frequency x is feasible if it is consistent with stationarity restrictions on p and q, and with capital stocks. We spell out these requirements in a lemma.

Lemma 5 *The set of stationary measures (p, q, x), associated to the system (38-39), and implied by stationary distributions of credit records, is fully characterized by*

$$q = \pi - p + pq + A_x(p, q), \text{ with } p, q \in [0, \pi], \qquad (40)$$

where $A_0 \equiv 0$ and, for each $x \in (0, 1]$, $A_x(\cdot)$ is a nonnegative and concave function. Moreover, (p, q, x) is consistent with k_n if the following capital constraint is satisfied:

$$p + \pi - q + A_x(p, q) \leq 2\pi k_n. \qquad (41)$$

The function A_x, defined in the appendix, coincides with $\frac{1}{2}(\pi - p)^{\frac{1}{2}}(\pi - q)^{\frac{1}{2}}$ when $x = 1$.

Since participation constraints do not depend on x directly, we can now define implementable credit allocations for $k \leq 2$.

Definition 6 *A credit allocation (y_b, y_n, p, q, x) is implementable with $k = k_b + k_n$, if y_b is an implementable suballocation for bankers with capital k_b, and there exists (v_z, w_z) such that (33-35) and (38-41) hold for (y_n, p, q, x, k_n).*

The optimum problem is stated as follows.

Problem 6 Maximize $\min\{U_b, U_n\}$ by choice of an implementable credit allocation (y_b, y_n, p, q, x) with capital $k_b + k_n \leq k$.

The following lemma presents a result, simplifying the study of participation constraints for credit allocations to a simple inequality, as in the benchmark case.

Lemma 6 *A credit allocation* (y_b, y_n, p, q, x) *satisfies the participation constraints (33-35) and the system (38-39) if and only if*

$$u(y) \geq \frac{y}{\beta}[(1-\delta)\frac{1}{p} + \delta + \frac{\det(M)}{(1-q)\delta p^2}]. \tag{42}$$

Inequality (42), the credit counterpart of (13), is a necessary and sufficient condition for the deposit constraint $w_1 \geq \beta v_0$ to be satisfied. It is shown in the proof that the determinant of the matrix M, $\det(M)$, is positive and converges to zero as β approaches one. On the other hand, for a fixed p, increases in q due to capital scarcity tightens the deposit constraint. We have now the following welfare result.

Proposition 4 *Assume* $\pi < 1$. *If* β *is sufficiently high, the maximum in problem 6 is greater than the benchmark welfare. If, in addition, capital is scarce* $(k < 1)$ *and* π *sufficiently high, then capital intermediation is essential, in the sense that bankers trade capital with non bankers in an optimum.*

6 Concluding remarks

We have used mechanism design, to study deposits and capital intermediation, in a monetary model of banking in the spirit of Cavalcanti, Erosa and Temzelides (1999). With our formulation, concepts in Diamond and Dybvig (1983), Green (1987), and Cavalcanti and Wallace (1999), can be used to study banking regulation such as Friedman's proposal.

We have shown that deposit contracts may include capital intermediation if capital is sufficiently scarce. For simplicity, we have restricted allocations to a simple scheme in which the non-bank public is monitored with two levels of credit ratings. A more sophisticated monitoring scheme, allowing the non-bank public to make more deposits than what we have allowed, can in principle produce higher welfare, provided that deposit constraints are satisfied. In our model, bankers distribute capital more efficiently than non bankers, weakening capital constraints, as a result of information asymmetries. It is thus reasonable to conjecture that intermediation would survive to more sophisticated monitoring arrangements, as long as non bankers remain partially anonymous, a necessary condition for fiat money to be essential in trade.

Our results require a notion of capital scarcity. But caution should be used with a literal interpretation. If capital were divisible, and its marginal product always positive in our model, then scarcity would always exist in a sense, because some people would be more inclined to use capital than others. We have not pursued such an avenue for obvious reasons of tractability. However, we believe that a connection of our model with extensions of the environment in Green (1987), would be a natural step in this research.

As a by-product, our model confirms the essentiality of uncertain time profiles of consumption, like in Diamond and Dybvig (1983), for a role of inside money. As shown, when there is no trade risk, outside money is essential in our model. There are thus important implications from removing consumption randomness in models of money.

Regarding banking, we can place Friedman's proposal against the background of information requirements. Suppose that the banking system is divided into two groups, one group that works as a pure depositary institution, and a second group that attempts to intermediate capital without issuing deposits. If capital is scarce, then the pure intermediaries cannot promise to have capital available to non bankers upon request, as a result of the matching frictions in the model. As a result, no allocation in which non bankers transfer capital to intermediaries without the benefits of money deposits can be implemented. In our model, Friedman's proposal therefore eliminates capital intermediation. Moreover, the extent of inside money creation is reduced because the quantity of capital that can be allocated to potential depositors is reduced by the lack of intermediation.

In our model, preventing deposit information to be linked to capital intermediation, would leave all capital trading to the non-bank public. This is of course an extreme case, and a more general model would produce a smaller but positive role for intermediaries under the Friedman's rule. Our model however has the advantage of showing that fiat money and capital do not have to be viewed necessarily as competing assets. We have shown that with incomplete insurance, individuals move randomly across wealth positions. Instead of emphasizing a choice between short- and long-run returns, we have emphasized choices of capital control and spending positions.

We have shown a negative side of Friedman's proposal, with respect to the optimal use of information. However, when capital intermediation takes place in an optimum, it does so by refluxing capital from consumers to producers, and in this sense, the banking system can be considered illiquid. We have only discussed equilibria where intermediation is never disrupted. If consumers were instead to stop transferring capital to bankers at some date, the bank system would have difficulties in honoring intermediation in the future. Discussing disruptive outcomes is problematic with mechanism design, given our assumptions of no aggregate uncertainty and perfect bank monitoring, and therefore lies beyond the scope of this paper. The model can provide, however, for new insights in comparison to Diamond and Dybvig (1983), since our bankers are never illiquid in the sense of fiat assets, a possibility that cannot be discussed in a capital-theoretic model.

Finally, our paper can be better situated in the literature with the help of the following rough division of approaches for evaluating proposals like Friedman's. On one hand, there are capital-theoretic models in the spirit of Diamond and Dybvig (1983), which derives a welfare gain from the financial system illiquidity, due to the random consumption of a monitorable non-bank public. On the other hand, there are monetary models, where money is required to overcome transaction difficulties, which focus on the monitoring of the supply of money by a monetary authority. There seems to be much to be learned from models in which the private supply of money is widespread, but the intermediation of real resources is yet nontrivial. In this paper, we have presented such a model.

In summary, we have chosen to study allocations in which bank difficulties are ruled out. Allocations however display a degree of illiquidity when there is capital intermediation. This could make bank instability a potential concern under different assumptions about aggregate uncertainty, and bank monitoring. It is not clear however whether our model can eventually support laissez-faire banking, or instead complex forms of regulation. In this sense, Friedman's proposal is still open for debate and future research in monetary theory. We feel however that a better understanding of the private supply of liquidity requires disaggregated models of banks, and that our model is a step in the right direction.

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Appendix

[to be included]