Social networks and the paradox of voting*

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Abstract

People vote although their marginal gain from voting is zero. We contribute to the resolution of this paradox by presenting a model for equilibrium configuration of voting attitudes. Each individual is seen as an element of a social network, within which pairs of individuals express ideas and attitudes, exerting mutual influence. We model the role of such networks in propagating the mutual influence across pairs of individuals. We show that it suffices that a small set of individuals have a strong feeling about voting to generate a significant turnout in elections.

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1 Introduction

It has long been recognized that high turnout in elections presents a basic puzzle: the marginal impact of each individual's vote is negligibl and yet people vote. The voter-participation paradox can be traced back to Downs (1957). Several explanations have been put forward to explain it. Among others, we can mention Tullock (1967), Frey (1971), Gooding and Roberts (1975), Palfrey and Rosenthal (1983, 1985), and Ledyard (1984).

In a brief account of these contributions, Tullock (1967) considers voters who obtain utility from voting and Gooding and Roberts (1975) allow for ethical voting. In both cases, an additional benefit is added to the action of voting. Frey (1971) discusses a different issue. He makes the argument the jobs of high-income voters endow them with superior information. This, in turn, motivates higher participation. More recently, the game theoretic approach of Palfrey and Rosenthal (1983, 1985) and Ledyard (1984) offers an explanation to the voting paradox. In a world of rational voters, the crucial element is the expected benefit from voting.[1] Since this expected benefit hinges upon the probability of a voter casting the decisive vote, expectations about other people's vote are relevant. Thus, it is not surprising that most contributions have focused on the benefit definition, and in particular, identifying benefits that do not depend on the outcome of the election (or decision). Sieg and Schulz (1995) take a different route. They question the full rationality and strong information requirements. In an attempt to cope with bounded rationality and incomplete information of voters, they use evolutionary game theory to address the issue. Shachar eand Nalebuff (1999) put forward the idea that political leaders will exert more effort in bringing people to vote in close elections. Yet, no social mechanism of influence interchange is made explicit. More recently, Castanheira (1999) uses a generalized model of Poisson games (Myerson 1994) to explain turnout rates and why they might be decreasing in population size. In his model, voters are rational and costs of voting, the information set of voters and the institutional framework are presented in a more general way than in previous works. Finally, Coate and Conlin (2002) present a distinct framework, in which voting turnout results from a contest between two opposing groups. The starting point of their theory is Harsanyi (1980)'s work. Harsanyi (1980) suggests that voting turnout can be understood by people acting according to rule-utilitarianism. That is, each individual takes the action tghat, if adopted by all society members, maximizes social welfare, defined by the sum of individual utilities.[2] Coate and Conlin (2003) provide an empirical analysis, based on liquor laws referenda. To our purposes, the significant point to note is that the fraction of supportrs of a platform, which will be a driving force for voting turnout in their model, is exogenously given and characterized by a statistical distribution. Thus, explicit social interaction, the focus of our paper, is also absent.

Overall, these are "warm glow" explanations of voting. This type of explanation, implicitly or explicitly, assumes that voting is a consumption good, although Downs (1957) is probably closer to seeing voting as an investment.

This takes us to the empirical evidence. In a recent paper, Guttman, Hilger and Schachmurove (1994) found evidence that voters see the act of voting as a consumption good.[3] Voters obtain utility from casting a vote, independently of who wins. Of course, utility is higher if the preferred candidate (or platform) wins. The implication of this empirical finding is that there is less to the voting paradox than one may have thought at first. However, the foundations of this utility are not well known. That is, current economics literature does not address the reasons why voting may be a consumption good.[4]

In a new and different direction, Ianni and Corrodi (2000) present a model with interaction between agents – each voter has an incentive to conform with the perceived winning side. This information is conveyed by an electoral poll, freely available. In this case, there are no social contacts at the individual level, which is the mechanism we focus on.

We contribute to the resolution of the paradox by presenting, up to our knowledge, a new explanation.[5] We see each individual as being part of a social network. Each member of society has a set a social relation, more or less extensive. Within this network, people express ideas and attitudes, exerting mutual influence across each pair of individuals. Society is just the set of all social-relations networks of each person. Thus, if a set of members has a positive attitude towards voting, they influence their social links, which in turn exert influence upon others, and so on, in a domino effect of social relations. We show that it suffices that a small set of individuals have a strong feeling about voting to create a process that leads to massive turnout in elections.

The model does not explain each individual's vote, just the decision to vote rather than abstain. On this respect, it is quite distinct from previous explanations. The benefit from voting is independent of whom or what the individual voted for.

The conditions for our explanation to work require that, at the start, the majority of people are neutral about voting (that is, they are indifferent between voting or not) and remaining population has a strong interest in voting.

The neutral voters can be identified with all rational voters who understand how negligible their vote's impact really is (in the absence of coordination or coalition formation among voters). The second group can be identified with those people who have something to gain from the vote.

Thus, we do not need a society-wide "warm-glow", or a detailed computation of costs and benefits of voting (seen as either consumption or investment or both)

2 Individual voting attitudes

Consider a system of N individuals, each one willing, or not, to vote. This willingness defines as a voting attitude. We assume that attitudes evolve in time due to the influence that agents exert over each other, until an equilibrium set of attitudes is reached.[6] After that point in time attitudes are assumed not to change anymore until there is some action based upon the attitudes.

Fix some point in time and let the voting attitude of individual i be expressed by a statement "yes" or "no" which is modelled as a binary variable $s_i = \pm 1, i = 1, \ldots, N$. These statements are interpreted to have the following meaning. If $s_i = +1$, individual i is willing to vote, if $s_i = -1$, individual i does not wish to vote.

2.1 The Effect of Social Norms

Under no interaction between individuals, we assume that an isolated individual will not discriminate between a positive and a negative statement, and will decide with equal probability in favor of $s_i = +1$ or $s_i = -1$. Given no social interaction these choices are independent of other individuals' choices. Let the mean choice in the set of the N individuals be denoted by

$$m = \frac{\sum s_i}{N}. \tag{1}$$

Notice that the fraction of people saying that they are willing to vote is given by

$$f = \frac{m+1}{2}. \tag{2}$$

Clearly, the average value of m is zero.

In the setting with no direct interaction between individuals, let us consider that the individuals are in a coercive situation, where a social pressure to conform exists. This coercion may be exerted by a majority, the existence of leaders, communication media and/or other factors, like ethical values or social norms, that may influence the direction of voter's voting attitude. Let h_c denote the intensity of this coercion. Its sign defines whether this coercion is in the direction of inducing voting, in which case $h_c > 0$, or in the direction of inducing people not to vote, in which case $h_c < 0$. We assume that isolated individuals conform with these social forces and each of them will choose the attitude that maximizes

$$u_{1i} = h_c s_i. \tag{3}$$

No matter how small h_c is, a small degree of coercion induces massive voting or massive abstention.

2.2 The Effect of Social Interaction

We now turn to the effect of interactions, exchanges and contacts between individuals, abstracting from the effect of external coercion. Considering a pair of individuals i and j, they can either agree with respect to the voting attitude, in which case $s_i s_j = +1$, or disagree, coming into conflict, in which case $s_i s_j = -1$. We introduce $J > 0$ as a measure of the degree of interaction or exchange. The level of agreement for a given pair (i, j) is thus measured by

$$J s_i s_j, \tag{4}$$

being $+J$ in case of agreement and $-J$ in case of disagreement. A given individual i interacts with, say, n other individuals, labeled $i_1, i_2, \ldots, i_n$, with a set of given attitudes $\{s_j\}_{j \in \mathcal{I}}$, where $\mathcal{I} = \{i_1, i_2, \ldots, i_n\}$. We assume that, in the absence of external coercion, individual i chooses his/her attitude such as to maximize the degree of agreement (the attitude is kept constant over all social contacts)

$$u_{2i} = J \sum_{j \in \mathcal{I}} s_i s_j. \tag{5}$$

Let us now determine what happens when both effects, social external coercion and interactions between individuals, occur simultaneously. In that case, it is obvious that every agent will choose the attitude that is aligned with h_c, that is to say, choose the attitude with the same signal as h_c. In that way, each individual i maximizes the sum

$$G_i = u_{1i} + u_{2i} = J \sum_{j \in \mathcal{I}} s_i s_j + h_c s_i \tag{6}$$

and, at the same time, maximizes each of its components.

We now introduce a function $\mathcal{H}$ to measure the degree of collective convergence, agreement or divergence for any given configuration of attitudes $\{s_i\}_{i=1,\dots,N}$, as given by

$$\mathcal{H}\left(\{s_i\}, J, h_c\right) = \sum_{i=1}^{N} G_i = J \sum_{i=1}^{N} \sum_{j \in \mathcal{I}} s_i s_j + h_c \sum_{i=1}^{N} s_i \tag{7}$$

In this case, under the optimal individual choice of attitudes, $\mathcal{H}$ is also being maximized and attains either the value

$$\mathcal{H}\left(\{s_i\}, J, h_c > 0\right) = \frac{JN}{2} n + N h_c \tag{8}$$

if $h_c > 0$ or

$$\mathcal{H}\left(\{s_i\}, J, h_c < 0\right) = \frac{JN}{2} n - N h_c \tag{9}$$

if $h_c < 0$.

2.3 The Effect of Personal Values

Until now we have discussed two effects: the tendency to conform with social external norms and the interaction with other individuals. We now turn to a third relevant factor, namely the fact that each person, in her or his capacity as a group member, is *a priori* bound to a certain attitude by common representations and norms. An additional factor is then required in order to convey all that is incultated in each person by the culture in which he or she lives, leading the person to be 'personally' inclined to opt, for example, for a positive rather than a negative attitude. This factor should act on each individual like the external coercion factor, except that it is different for each individual. If, for individual i, the intensity of this factor is h_i, the isolated influence of this additional factor leads him or her to maximize

$$u_{3i} = h_i s_i. \tag{10}$$

Here, h_i may vary in sign and intensity from individual to the other. Depending on the nature of the model to be implemented, one may use either a configuration of known $\{h_i\}$ or else, assume a probability distribution $p\{h_i\}$. Together with the other factors, we assume that individuals choose their attitudes so as to maximize

$$H_i = u_{1i} + u_{2i} + u_{3i} = J \sum_{j \in \mathcal{I}} s_i s_j + h_c s_i + h_i s_i. \tag{11}$$

3 The voting outcome

Our model assumes that each individual chooses the attitude in order to maximize his/her utility function (11). However, the value attained depends on the others' attitudes. In other words, this maximization is conditional on other individuals' optimal choices.

To generate the equilibrium values of utility, we assume a dynamic process of voting attitudes, in which we take as primitives the behavioral specifications previously introduced. Equilibrium is attained when the aggregated utility is maximized given all these constraints.

3.1 Aggregating utilities

We follow Galam and Moscovici (1991) to account for the emergence of a group as such, and assume that, in equilibrium, the interaction of individual i with each of his/her neighbors with attitude s_j can be replaced with the interaction with an average attitude taking

$$s_j = \frac{1}{N-1} \sum_{k=1, k \neq j}^{N-1} s_k \tag{12}$$

If this is the case, the n neighbors become identical and

$$\sum_{j\in\mathcal{I}} s_j = \frac{n}{N-1}\sum_{k=1,k\neq j}^{N-1} s_k.$$

Notice, that as N increases without bound, the sum above tends to nm. Defining h_J as $J\sum_{j\in\mathcal{I}} s_j$ and substituting the sum above in the expression for H_i we get

$$\begin{aligned} H_i &= h_J s_i + h_c s_i + h_i s_i \\ &= \tilde{h}_i s_i, \end{aligned}$$

where

$$\tilde{h}_i = h_J + h_c + h_i.$$

In other words, the result of the assumption underlying (12) is that the attitudes are formally not coupled any more.[7] Each agent will maximize H_i above by choosing s_i =sign $\tilde{h}_i$. Hence, the aggregate utility

$$\begin{aligned} \mathcal{H}\left(\{s_i\}, J, h_c, \{h_i\}\right) &= \sum_{i=1}^{N} H_i \\ &= Nm\left(h_J + h_c\right) + \sum_{i=1}^{N} h_i s_i \end{aligned}$$

is being maximized in the process of individual maximization of each agent. For large enough N, the aggregate utility may be rewritten as

$$\mathcal{H}\left(\{s_i\}, J, h_c, \{h_i\}\right) = Nm\left(h_J + h_c + \langle h_i\rangle\right), \tag{13}$$

where $\langle h_i\rangle$ denotes the expected value of the random variable h_i.

Thus, given $\{s_i\}, J, h_c$ and $\{h_i\}$, the value of aggregated utility $\mathcal{H}$ is a function only of the mean attitude m. Clearly, the optimal value of $\mathcal{H}$ must be associated with a unique value of m. A value of m, however, is not associated with a unique configuration of attitudes $\{s_i\}$. Several different configurations may lead to the same value of m. In that sense, equilibrium is not unique.

3.2 The Probability Distribution of m

In this section we derive the probability distribution of m in equilibrium. In order to do that, let N_+ be the number of individuals with a positive attitude and let N_- be the number of individuals with a negative attitude for a given mean attitude m. It then follows that

$$\begin{aligned} N &= N_+ + N_- \\ Nm &= N_+ - N_- \end{aligned}$$

and the possible number of configurations of attitudes associated with m is

$$W(m) = \begin{pmatrix} N \\ N_+ \end{pmatrix} = \frac{N!}{N_+!N_-!} = \frac{N!}{\frac{N(1+m)}{2}!\frac{N(1-m)}{2}!}. \tag{14}$$

As N is very large, we use Stirling's formula to write

$$\ln W(m) \approx N\left[\frac{m}{2}\ln\left(\frac{1-m}{1+m}\right) - \ln\left(\frac{\sqrt{1-m^2}}{2}\right)\right]$$

Let us denote by $\phi(m)dm$ the probability that the level of aggregate utility $\mathcal{H}(m)$ is between E and $E + dE$. We assume that the logarithm of this probability density $\phi(m)$ is proportional to $-E$. In other words, there exists a positive D such that

$$\phi(m) \propto \exp\left[-\mathcal{H}(m)/D\right]. \tag{15}$$

Given that m can be attained with $W(m)$ possibly different configurations, we furthemore assume that

$$\phi(m) \propto W(m)\exp\left[-\mathcal{H}(m)/D\right].$$

Normalizing this expression, we get

$$\phi(m) = \frac{W(m)\exp\left[-\mathcal{H}(m)/D\right]}{\int_{-1}^{+1} W(m)\exp\left[-\mathcal{H}(m)/D\right]}.$$

When N is sufficiently large, the probability distribution of m will be centered in the maximum of the numerator. In other words, recalling that for large N the field h_J behaves as Jnm,

$$\lim_{N\to\infty}\frac{1}{N}\ln\phi(m) = \inf_m \lim_{N\to\infty}\left\{\frac{1}{N}\ln W(m) - \frac{1}{D}\left[\frac{Jnm^2}{2} + mh_c + m\left(\frac{\sum h_i}{N}\right)\right]\right\}$$

First-order conditions after taking that limit[8] imply that the minimum is attained at

$$m = \int p(h_i)\tanh\left[(nJm + h_c + h_i)/D\right] dh_i. \tag{16}$$

This equation gives the implicit equilibrium value of m.

4 Voting pattern

In the absence of h_c and $\{h_i\}$, a geometrical interpretation can be made for the equilibrium value of m. Equation (16) becomes

$$m = \tanh(am) \tag{17}$$

where

$$a = \frac{nJ}{D}. \tag{18}$$

Graphical analysis of the intersection of the two functions, $y = m$ and $y = \tanh(am)$, show that if $a \leq 1$, there is only one solution, namely $m = 0$. If $a > 1$, however, a positive solution $m^*(a)$ and a negative (symmetric) solution $-m^*(a)$ exist. Notice that $m^*(a)$ is increasing with a and that $m = 0$ is still a solution of the first order condition. However, if $a > 1$, the solution $m = 0$ no longer yields to a maximum.

From this interpretation, it follows that consensus is more easily reached with small D. It is interesting to notice that consensus is not the typical outcome of these equilibria. Rather than pushing to assimilate individuals to one another, the process leading to equilibrium emphasizes those features which differentiate them from each other. The mean attitude m that is attained will depend therefore on the degree of allowance for divergence. Since consensus is attained for small D, we associate a high D to people that may diverge more freely with their attitudes. In this sense, let D denote to a certain extent the degree of 'democracy'. On the other hand, a higher degree of voting attitudes can be obtained as J increases, reflecting the intensity of the influence due to exchange among the individuals.

An interesting effect is the one associated with D. Suppose that the ability to accommodate divergence within society increases (that is, D increases), then the equilibrium turnout at elections is smaller. It becomes less important to comply with the social norm, which leads to a lower voting level.

It is also clear that an increase in the number of social contacts, n, has the same qualitative impact of an increase in J. This prediction finds empirical support in the work of Coate and Conlin (2002). They find there is a higher probability of voting in more densely populated areas. Thus, a higher turnout is associated with a larger network of social contacts, under the assumption that people in more densely populated areas are influenced by the attitudes of a greater number of their fellow citizens.

Most of these effects do carry on to the (analytically) more complex cases of positive h_c and $\{h_i\}$. While taking h_c and h_i to be zero is useful to provide some insight into the basic working of the model, it is hardly satisfying. We now extend our analysis by way of numerical simulations. We take h_i to have a uniform distribution on $[-1, 1]$.[9]

Taking first the case of $h_i \in [-1, 1]$ and $h_c = 0$, the equilibrium condition entails three possible equilibria: one at 50% voting, one with a high turnout (close to 1) and one with a low turnout (close to 0).

Take now $h_c > 0$. A positive social attitude towards voting increases, naturally, the equilibrium number of votes (in both types of equilibrium). Moreover, if h_c is sufficiently large, the abstention-dominant equilibrium disappears. For a sufficiently high value of h_i multiple equilibria cease to exist and only the high-voting equilibrium survives.

The model can also generate some testable implications. For example, if the ability to allow for divergence without affecting social cohesion is greater among young people, then we should see a higher abstention rate for the younger than for the older groups of the population.

Another interesting implication can be derived. If the new information and communication technologies broaden each individual's network of influences, then equilibrium turnouts become closer to the all vote or no one votes. It also leads to a multiple equilibria environment, all other factors constant.

A final issue to explore is whether a stronger positive attitude by a fraction of the population can act as a substitute for the number of people with such a positive attitude. It turns out that a minimum of population favoring voting must exist, irrespective of the intensity of preference for voting, in order to generate the high turnout equilibrium. This minimum fraction of the population with a positive attitude can be understood as a requirement for a minimum number of social interactions needed to set into move a general preference towards voting.

Formally, assume that a fraction $(1-p)$ of the population is neutral with regard to voting, while fraction p has positive attitude $h_i = h^* > 0$ towards voting. The value p compatible with m being an equilibrium value is given by

$$p = \frac{m - \tanh(am)}{\tanh\left(am + \frac{h_i}{D}\right) - \tanh(am)} \tag{19}$$

From this,

$$\lim_{h_i \longrightarrow \infty} p = \underline{p} > 0 \tag{20}$$

Thus, as claimed, a high turnout equilibrium requires that a minimum fraction of the population has a positive attitude towards voting (in the absence of a general social norm in that direction).

5 Final remarks

In this work, we present a novel explanation for high turnouts at elections, despite the apparent negative cost-benefit assessment of such decision. Other motives have been proposed in the literature. They require a high level of rationality or elements outside economics to characterize all voters.

Our explanation is complementary to the previous work in the following sense. It is sufficient that a small sub-set of the population decides to vote, based on any of the motivations that appeared previously in the literature, coupled with social networks and a mass of 'neutral' population to induce high turnouts at elections.

To a certain extent, this means that benefits from voting are not independent from each person's social links.

We can link our model to previous work. In Palfrey and Rosenthal (1983, 1985) and Sieg and Schulz (1995), voters indifferent between two alternatives do not vote whenever they face strictly positive costs of voting. We suggest that even individuals ex-ante indifferent between two platforms (or two candidates) may end up voting, due to social network influence. Relating to the empirical evidence of voting as a consumption good (Guttman *et al.*, 1995), our framework provides a foundation for their findings: participation has some value due to each voter's social network. As to the Frey (1971) argument of better information of high-income people, we provide an alternative route. High-income voters may have a larger social network, exposing them to a stronger pressure to vote. In equilibrium, they will be more likely to vote. In regression analysis, income may just be a proxy for network size. At the least, this raises an empirically interesting issue, calling for further testing.

In a sense, our explanation is a formalization of the notion of a "warm glow" reason for voting.

An interesting question is why we do not see as an outcome of the model a huge volume of abstention. If initial strong feelings against voting exist, we can obtain such result. Moreover, such outcomes are not uncommon in real elections. Of course, our model is also able to generate low turnout elections. We did not emphasize this side of our results, as it is quite difficult to design an empirical test to distinguish our explanation, built on the working of social networks, from a more standard high cost – low benefit explanation.

Endnotes

1. Biker and Ordeshook (1968) pointed out that instrumental voting, that

is, voting to influence decisions and policy, is hardly rational.

2. This idea is further developed in Feddersen and Sandroni (2001).

3. Earlier work of Ashenfelter and Kelley (1975) also offers empirical support for the consumption good theory of voting. Another early study is due to Silver (1973). Reviews of evidence on voting participation can be found in Matsusaka and Palda (1993) and Struthers and Young (1989). See also Matsusaka and Palda (1999).

4. Other related works are due to Aldrich (1993,1997) and Feddersen and Pesendorfer (1996).

5. Harbaugh (1996) justifies turnout at elections by peer pressure, which is close in spirit to our analysis.

6. Equilibrium is characterized as a configuration of attitudes that attains a fixed point of the dynamics that describes their evolution in time.

7. Equilibrium is characterized as a configuration of attitudes that attains a fixed point of the dynamics that describes their evolution in time.

8. To see this, use Stirling's formula to rewrite $W(m)$. Since $\mathcal{H}$ is proportional to N, the Laplace asymptotic method applies to evaluate

$$\lim_{N\to\infty} \frac{1}{N} \ln \phi(m)$$

as described.

9. Similar qualitative results are obtained under a normal distribution for h_i. Details available from the authors upon request.

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