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Life Expectancy, Educational Attainment, and Fertility Choice*

Abstract

This paper explores the role of mortality as a determinant of educational attainment and fertility, both during the demographic transition and after its completion. Two main points distinguish our analysis from the previous ones. Together with the investments of parents in the human capital of children, traditional in the fertility literature, we introduce investments of adult individuals (parents) in their own education, which ultimately determines productivity in both the goods and household sectors. Second, we let adult longevity affect the way parents value each individual child. Increases in adult longevity or reductions in child mortality eventually raise the investments in adult education. Together with the higher utility derived from each child, this tilts the quality-quantity trade off towards less and better educated children, and increases the growth rate of the economy. This setup can explain both the demographic transition and the recent behavior of fertility in "post-transition" countries. Evidence from historical experiences of demographic transition, and from the recent behavior of fertility, education, and growth generally supports the predictions of the model.

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1 Introduction

Major demographic changes swept the world in the course of the last century. Today, mortality reductions have reached virtually every corner of the globe – with some unfortunate exceptions – and the vast majority of the world population lives in countries where fertility has already shown a significant decline, and where population is expected to stabilize within the next 50 years (Robinson and Srinivasan, 1997). These are the main features of the so called demographic transition.

Generally, the transition has been characterized by significant reductions in mortality followed by reductions in fertility, implying a period of intense population growth, which progressively diminishes as fertility starts to decline (see Heer and Smith, 1968; Cassen, 1978; Kirk, 1996; Mason, 1997; and Macunovich, 2000). In the demographers' view, "if there is a single or principal cause of fertility decline, it is reasonable to ascribe it to falls in mortality, which was the major cause of destabilization" (Kirk, 1996, p.379).[1]

Also, developed countries have recently experienced increasingly low fertility levels. In 1999, countries like Belgium, Germany, and Spain had fertility rates around 1.5, significantly below the replacement rate ($\simeq 2.1$). Furthermore, several developed countries continue to experience decreasing fertility rates, despite very low fertility levels and the fact that the demographic transition is already completed. This phenomenon has been largely overlooked both empirically and theoretically by the demographic and economic literature. It points to the necessity of understanding the recent behavior of fertility from a more general perspective, not restricted to the process of demographic transition.

The goal of this paper is to explore the role of life expectancy as a determinant of educational attainment and fertility, both during the demographic transition and after its completion. In the last decades, economists have become increasingly interested in the question of fertility choice and investments in human capital. Although the more recent phenomenon of low and decreasing fertility in developed countries has not received much attention, the process of demographic transition has been the subject of numerous studies (see, for example, Becker, 2000; Becker, Murphy, and Tamura, 1990; Ehrlich and Lui, 1991; Sah, 1991; Meltzer, 1992; Tamura, 1996; Blackburn and Cipriani, 1998; Galor and Weil, 2000; and Kalemli-Ozcan et al, 2000). We believe that our approach improves upon this literature in many respects. We deal with adult longevity, child mortality, human capital, and fertility in a straightforward way, which allows us to understand

[1] Initial economic conditions varied a lot in the different experiences of demographic transition. Also, there is debate regarding the chronology of events in some of the first European countries to experience the demographic transition, but this sequence is generally regarded as an accurate description.

the recent behavior of fertility in the more developed countries, ignored by the above-cited studies and incompatible with most of their results. Additionally, we are able to explain the fertility transition in a more complete fashion.

In the model, mortality is assumed to be exogenous at the individual level, and fertility is assumed to be always an object of individual choice.[2] Although there are various actions that agents can take to improve their life prospects, our interest here is focused on the gains in life expectancy observed in the last two centuries, which were largely due to scientific revolutions and technical improvements. At the individual level, these were partly exogenous. And even though these gains were endogenous to the economic system as a whole, they were, to some extent, still exogenous to most of the less developed countries. These countries experienced longevity gains independent of improvements in economic conditions, partly as a consequence of the absorption of knowledge generated elsewhere and of the help provided by international aid programs (see Preston, 1975 and 1980; and Kirk, 1996).

Theoretically, two points distinguish our analysis from the previous ones. First, we let adult longevity affect the way in which parents value each individual child, in much the same way that the number of children does in the traditional literature. This assumption is simply an extension of the widely accepted effect of child mortality on fertility to later ages. Intuitively, it can also be understood in these terms, once one considers that individuals are not only concerned with the survival of their children, but also with the continuing survival of their whole descent. Acknowledging the importance of adult longevity to the way in which parents value each individual child has important consequences in terms of fertility choices. This hypothesis alone helps explain the behavior of fertility after the demographic transition.

Second, together with the traditional emphasis on investments of parents in the human capital of children, we introduce investments of adult individuals (parents) in their own education. This distinction between "basic" and "adult" human capital helps separate the effects of child mortality from those of adult longevity on investments in human capital and growth.[3]

These two features of our theory play central roles in the mechanics of the model. Briefly,

[2] The issue of fertility choice in underdeveloped economies is controversial in the demographic literature. Nevertheless, evidence indicates that there is always some margin of choice. Several kinds of actions taken in 'pre-modern' societies, directly or indirectly, affect fertility outcomes, including marriage patterns, breast feeding habits, abortion, and sexual practices (see Demeney, 1979; Caldwell, 1981; Kirk, 1996; and Mason, 1997). We abstract completely from this issue here, and assume that fertility was always, at no cost, an object of individual choice. Even though we acknowledge that technical developments made the control over the number and timing of births more precise, we believe that this is an issue more related to the uncertainty of the outcome than to the ideal choice itself, and our interest concerns mainly the latter.

[3] Furthermore, this approach is more realistic and brings the theory closer to the empirical accounts that justify the impacts of life expectancy on educational investments (see, for example, the discussion on rates of return in Meltzer, 1992).

increases in adult longevity or reductions in child mortality eventually raise the investments in education, which increase the productivity of individuals both in the labor market and in the household sector. Also, higher life expectancy tilts the quantity-quality trade-off towards less and better educated children and tends to move the economy out of a "Malthusian" equilibrium. Once the economy abandons the "Malthusian" regime, increases in life expectancy cause reductions in fertility, increases in educational attainment, and increases in the growth rate of the economy.

In the empirical discussion, we present different sets of evidence to support the predictions of the model. We justify the exogenous role played by life expectancy by showing that recent reductions in mortality were largely independent of improvements in economic conditions, so that they indeed appear to be a driving force behind changes in other demographic variables. We discuss historical experiences of demographic transition and argue that, generally, they agree with the patterns generated by the model. Finally, we test the theoretical predictions in relation to fertility, educational attainment, and growth in 'modern' economies, using a cross-country panel with data between 1960 and 1990. In brief, the estimated model implies that a 10 year gain in adult longevity reduces fertility by 1.7 points, increases the average schooling in the population by 1 year, and increases the growth rate by 5%; a reduction of 100 per one thousand in child mortality implies a decrease of 1.5 points in the total fertility rate.

The structure of the paper may be outlined as follows. Section 2 motivates the discussion by showing that while the cross-sectional relation between income and some key demographic variables (life expectancy, fertility, and schooling) has been monotonically shifting in the last 35 years, the relation between life expectancy and the same demographic variables has remained considerably stable. Section 3 discusses the structure of the model, and the effects of adult longevity and child mortality on both the steady-state with growth and the Malthusian equilibrium. Section 4 summarizes the theoretical results and describes histories of demographic transition that would be expected to arise from our model. Section 5 discusses how the model fits the empirical evidence. We discuss how some recorded demographic transition histories fit the patterns described in section 4, and analyze whether the more recent cross-country behavior of fertility, education, and growth supports the predictions of the model. The final section summarizes the main results of the paper.

2 The Behavior of Life Expectancy, Fertility, and Educational Attainment after the Demographic Transition

The traditional growth literature looks at income as the single variable either driving or summarizing the changes in all relevant development outcomes. In this perspective, gains in per capita income improve nutrition and health consumption, which reduces mortality rates; income gains also change the quantity-quality trade off in terms of number and education of children, which reduces fertility and increases human capital investment. Statements like these are common places in the economics profession, and it seems fair to say that they give an accurate description of the consensus regarding the main changes taking place during the process of economic development.

Even though there is obviously a lot of truth to this view, it is far from giving a complete picture of reality. Recently, the relationship between income and crucial demographic variables, such as life expectancy or fertility, has been clearly unstable. Figures 1 to 3 illustrate the changing relationships between income, life expectancy, fertility, and educational attainment.[4] To concentrate on economies that share the same demographic regime, the figures refer only to countries that had already started the demographic transition in 1960.[5]

Figure 1 shows that, for constant levels of income, life expectancy has been rising.[6] Logarithm curves are fitted to the 1960 and 1995 cross sectional relation between per capita GNP and life expectancy. For lower levels of income, life expectancy at birth has increased by more than five years in the period between 1960 and 1995. This means, for example, that a country with per capita GNP of US$5,000 in 1995 had a life expectancy roughly 10% higher than a country with per capita GNP of US$5,000 in 1960.

Figure 2 tells an analogous story for the relationship between income and fertility. Again, curves are fitted to the 1960 and 1995 cross sectional relationship between income and fertility. For constant levels of income, fertility has been falling. These reductions have been as large as 2 points for countries with per capita income around US$3,000, and even larger for poorer countries.

Finally, as Figure 3 shows, the story is not different for the relationship between education and income. Logarithm curves are fitted to the cross sectional relationship between income and average schooling in 1960 and 1990. Gains in average schooling in the period were usually over 1 year, for constant levels of income.

One immediately wonders whether these changes in life expectancy, education, and fertility are interrelated, and what the specific mechanisms connecting them are. An insight in this direction

[4] The general results illustrated in Figures 1 to 5 do not depend in any way on the specific statistics used, or on the presence of any particular country in the sample. Detailed description of the variables is saved until the empirical section. The logarithm curves used are of the general form $y = \alpha + \beta \ln(x)$, and the power curves used are of the general form $y = \alpha x^{\beta}$.

[5] A more precise reason for the restricted sample is given in the theoretical section. Empirically, some objective criterion defining whether a country already started the demographic transition has inevitably to be chosen. Our choice is the cutoff point "countries that had life expectancy at birth above 50 years in 1960", also to be justified later on. The results do not depend on the specific criterion chosen, and we believe that there should not be much discussion regarding the countries actually included in the sample (generally, OECD, Latin American, East Asian, and some Arab and North African countries).

[6] This phenomenon was first noticed by Preston (1975), who analyzed data between 1930 and 1960.

is gained by looking at the relation between life expectancy and the other two variables.

In Figure 4, we plot the cross sectional relation between life expectancy and fertility in 1960 and 1995. The lines are polynomials (3^{rd} order) fitted to the different years.[7] At first sight, the shift in the position of the curve suggests that a change in the relationship is being portrayed. But if we look closely, there is not much overlapping of the two curves, and when the overlapping does actually occur, the points relative to the different years are more or less evenly distributed over the same area. The two segments look more like an approximation to a single stable nonlinear function than a description of a changing relationship. This point is further explored by fitting a single nonlinear function (3^{rd} order polynomial) to the whole data set, assuming that a stable relation is present throughout the period. Visually, the curve seems to have a good fit, and the functions estimated separately for each sub-period seem to merge into it. The single fitted line actually explains more of the overall variation in the data than the two polynomials fitted independently to each year (R^2 of 0.78, against 0.76 for 1960 and 0.21 for 1995). The interesting part is that this curve does not separate points from 1960 and 1995 as being below or above it, as a curve fitted to all the data in Figures 1, 2, or 3 would do. Points from the different years are distinguished as being more on its left portion or on its right portion, as if countries were sliding on this curve through time, via increases in life expectancy and reductions in fertility.

The results regarding life expectancy and educational attainment are even stronger. Figure 5 plots the cross sectional relationship between these two variables in 1960 and 1990, and fits a power curve to each year. The stability of the relationship through time is clear. Indeed, the two curves almost merge into each other for the region over which there are observations for both years. Again, countries seem to be sliding on this curve through time, as life expectancy and educational attainment rise simultaneously.

So, for constant income levels, life expectancy is rising, fertility is declining, and educational attainment is increasing. At the same time, changes in fertility and schooling are following very closely the changes in life expectancy, such that, for constant life expectancy at birth, these variables seem to be constant. While fertility and education are direct objects of individual choice, life expectancy has a large exogenous component, related to scientific knowledge and technological development. This reasoning suggests that exogenous reductions in mortality together with a stable behavioral relationship between life expectancy and the other demographic variables are the driving forces behind these changes. In what follows, we develop a theory along these lines. Our goal is to explain the facts discussed above, together with the triggering of the demographic transition, as being determined by exogenous increases in life expectancy.

[7] Democratic Republic of Korea and Guyana are outliers excluded from Figure 4 for illustrational purposes.

3 Theory

3.1 The Structure of the Model

Assume an economy inhabited by adult individuals, who live for a deterministic amount of time, when they work, consume, invest in their own education, have children, and invest in the education of each child. The model is the usual 'one sex model,' common to the fertility literature. We abstract from uncertainty considerations, to concentrate on the impact of adult longevity and child mortality on the direct economic incentives at the individual level. To make the model treatable, we also abstract from the presence of physical capital. Individuals, or households, have an endowed level of what we call 'basic' human capital (determined from previous generation's decisions), based on which they decide on how much to invest in their own 'adult' education. Adult education determines productivity both in the labor market and in the household sector. Households possess backyard technologies for producing goods, adult human capital, and basic human capital, and they decide on how to allocate their time across these different activities in order to maximize utility. As we will see later on, changes in adult life expectancy and child mortality will change the incentives to engage in these different activities.

In the model, adults live for T periods, and at age τ they have children. A fraction β of the born children dies before reaching adulthood. Parents derive utility from their own consumption in each period of life ($\frac{c(t)^\sigma}{\sigma}$), and from the children they have. Childhood can be thought of as an instantaneous phase: as soon as individuals are born they become adults, and there is no decision to be taken as a child.

We assume that adults are concerned directly with the level of human capital of their children, via a constant elasticity function $\frac{h_c^\alpha}{\alpha}$, what is sometimes called a paternalistic approach. The traditional literature on economics of fertility usually assumes that the value that parents place on the human capital (or utility function) of each child is an increasing and concave function of the number of children. Since we are incorporating longevity and child mortality into the analysis, we also take into account the effect of these variables. We assume that, together with the number of children, parents also care about how long each child will live, in such a way that the relevant variable is the total lifetime of the surviving children ($(1-\beta)nT$), or what we call the total of 'child-years.' How much adults value the human capital of each child is an increasing and concave function of the total lifetime of the children, where this function is given by $\rho(.)$. But as a fraction β of the children will not reach adulthood, not all of them will enjoy these T years of life. As we treat n here as a continuous variable, we simply assume that $(1-\beta)n$ out of n born children will reach adulthood, avoiding thus the problems related to the uncertainty regarding the survival of each individual child. Therefore, $(1-\beta)nT$ assumes the role usually played by n alone in the traditional economic analysis of fertility. Intuitively, this set up extends the logic usually applied to the child mortality rate to later ages. It is a natural extension, once one considers that individuals are not only concerned with the survival of their children, but also with the continuing survival of their whole descent.[8] Additionally, we also assume that there is a tendency towards satiation in terms of the total of child-years, in the sense that for sufficiently high values, its marginal utility is zero. This seems to be a sensible hypothesis once, holding T constant, we think about the biological constraints that nature imposes on the bearing and timing of births. It will also be important to assure that increases in life expectancy will eventually move the economy out of a Malthusian equilibrium.

Individuals face goods and time constraints: they have to allocate their total lifetime between working (l), raising kids (b), and investing in their own education (e); and they have to allocate goods between their own consumption and children's fixed costs (f). Parents' income is determined by how much human capital they have (H_p) and by how much they work, and borrowing from future generations or monetary bequests are not allowed. Finally, we assume that the production functions of adult human capital, basic human capital, and goods are multiplicative on human capital and time. This means that adult human capital increases the individual's productivity both in the labor market and in the household production of basic human capital, and that basic human capital increases the productivity of education in generating adult human capital.

Given these assumptions, individuals solve the following problem (IP):[9]

$$\max_{\{c(t),n,l,b,e\}_{t\in[0,T]}} \left\{ \int_0^T \exp(-\theta t)\frac{c(t)^\sigma}{\sigma}dt + \rho[(1-\beta)nT]\frac{h_c^\alpha}{\alpha} \right\} \quad \text{(IP)}$$

subject to

[8] In this case, individuals take into account that their children will need enough time to have their own children and raise them.

[9] Additionally, if we assume that parents enjoy having children only to the extent that they share part of their lifetime, the second term in the expression has to be integrated over time from τ to T, and discounted at the rate θ. As we discuss later on, the different specifications have exactly the same qualitative predictions.

$$
\begin{aligned}
y &\geqslant \int_0^T \exp(-rt)c(t)dt + \exp(-r\tau)nf, \\
T &\geqslant l + bn + e, \\
H_p &= Aeh_p + H_o, \\
h_c &= DbH_p + h_o, \\
y &= lH_p, \\
& \quad h_p \text{ given.}
\end{aligned}
$$

where $D, A > 0$; $0 < \sigma, \alpha < 1$; $u'(.), \rho'(.) > 0$; $u''(.), \rho''(.) < 0$; $\rho'(x) = 0$ for some $x = \overline{x} > 0$.

f is the goods fixed cost of having a child, b is the time investment in human capital per child, and e is the time devoted to adult education. $c(t)$ is adult consumption at instant t, l is lifetime labor supply, n is the number of children, and $\rho(.)$ is an increasing and concave function. D, A, H_o, and h_o are technological parameters, r is the interest rate, and θ is the subjective discount rate.

We distinguish between basic human capital and adult human capital: h denotes the kind of human capital formed during childhood, in which parents can invest, related to basic education and skills, and emotional development; H denotes the kind of human capital that can be obtained during young adulthood, related, for example, to college or graduate education, or to professional training. This set up assumes that individuals enter adulthood with a given level of basic education (h_p), and then, by deciding on how much to invest in their own education, they choose a level of adult human capital (H_p). h_c is the level of basic human capital that parents gives to each of their children. H_o and h_o denote the levels of adult and basic human capital that individuals have, even in the absence of investments of any sort in education, maybe determined from innate skills or natural learning throughout life. As will be clear in the following sections, these factors play an important role in allowing for the existence of a so called Malthusian steady-state, with no investment in human capital and zero growth.

To concentrate on the issues of interest, we depart from this formulation and introduce some simplifying assumptions. Since our central interest is the long run behavior of the economy, mainly the inter-generational fertility and human capital decisions, we abstract from life cycle considerations by assuming that subjective discount rates and interest rates equal zero. Given the separability of the utility function over time, this implies constant consumption throughout life.

Incorporating these hypotheses, the objective function and the goods constraint can be rewritten as:

$$\max_{\{c,n,l,b,e\}} \left\{ T\frac{c^{\sigma}}{\sigma} + \rho[(1-\beta)nT]\frac{h_c^{\alpha}}{\alpha} \right\},$$

and

$$lH_p \geqslant Tc + fn.$$

This is the benchmark model that guides our theoretical discussion. In the next subsection, we analyze the effects of adult longevity on educational attainment, fertility, and economic growth.

3.2 The Role of Adult Longevity

3.2.1 Static Implications of Longevity Gains

In this subsection, we look at the individual decision taking the initial level of basic human capital as given (h_p). In the following subsections, we discuss the implications of this decision process to the growth rate and dynamic behavior of the economy, and look at the properties of an equilibrium with zero growth and no investments in human capital.

As we hold child mortality constant, we save in notation by omitting the parameter β. Also, given that we look at an equilibrium with growth, the parameters f, h_o, and H_o become irrelevant as time goes by, so we ignore them. Defining $A_p = Ah_p$, $D_p = DA_p = DAh_p$, substituting for l in the time constraint, and for h_c in the utility function, the first order conditions (foc's) for, respectively, c, n, b, and e can be written as:

$$Tc^{\sigma-1} = \frac{T}{A_p e}\lambda, \quad (1)$$

$$T\rho'(nT)\frac{(D_p be)^{\alpha}}{\alpha} = b\lambda, \quad (2)$$

$$\rho(nT)(D_p be)^{\alpha-1}D_p e = n\lambda, \quad (3)$$

$$\rho(nT)(D_p be)^{\alpha-1}D_p b = \left(1 - \frac{Tc}{A_p e^2}\right)\lambda; \quad (4)$$

where λ is the multiplier on the constraint above.

Using equations 2 and 3 from the foc's, we get:

$$nT\frac{\rho'(nT)}{\rho(nT)} = \alpha. \quad (5)$$

Define $\varepsilon(nT) = nT\frac{\rho'(nT)}{\rho(nT)}$, the elasticity of the altruism function ($\rho(.)$) in relation to its argument. The expression above states that the agent will equate the elasticity of the altruism function to the constant elasticity of the h_c sub-utility: $\varepsilon(nT) = \alpha$.

If $\varepsilon(.)$ is monotonic, this implies that nT will always be constant, and that exogenous changes in T will have the following effect on n:

$$\frac{dn}{dT} = -\frac{\varepsilon'(nT)n}{\varepsilon'(nT)T} = -\frac{n}{T} < 0. \tag{6}$$

The equalization of elasticities expressed in equation 5 comes from the fact that n and b enter in a multiplicative way both in the objective function (via the sub-utility functions) and in the constraint. But the simple expression obtained above hinges on the additional assumption of constant elasticity for the h_c sub-utility function. What this buys us is the independency of n in relation to all other exogenous variables apart from T. With a more general specification, h_c would show up in the right hand side of 5, and it would allow the other exogenous variables to affect the optimal choice of n. But also in this case, the force working towards a negative relationship between n and T would still be present, even though it could possibly be weakened by the adjustment on h_c. The important factor here is the presence of T in the discount function $\rho(.)$, and the way in which T and n enter inside this function. As long as we have a specification where n and T have similar effects on $\varepsilon(.)$, there will be a tendency for n and T to move in opposite directions.[10] This is the role played here by the assumption that parents see number of children and adult lifetime of each child in similar ways, such that the relevant variable in determining how much parents care for each individual child is the total lifetime of the children, or the total of 'child-years'.

Using equations 1, 3, and 4 from the foc's, we get:

$$\begin{aligned} A_p e^2 &= Tc + A_p ebn, && (7) \\ \rho(nT)(D_p be)^{\alpha-1} D &= nc^{\sigma-1}. && (8) \end{aligned}$$

The constraint gives us $Tc + A_p ebn = TA_p e - A_p e^2$. Together with equation 7, this implies

$$e = \frac{T}{2}, \text{ and } \frac{de}{dT} = \frac{1}{2}. \tag{9}$$

Educational attainment increases with longevity. This should be expected, since increases in longevity increase the period over which the returns from investments in education can be

[10] More precisely, if the altruism function assumes the general form $\rho(n,T)$, and $\varepsilon(n,T) = \frac{\rho_n(n,T)n}{\rho(n,T)}$ denotes its elasticity in relation to n, the condition for $\frac{dn}{dT}$ to be negative is that $sign\{\varepsilon_n(n,T)\} = sign\{\varepsilon_T(n,T)\}$, where the subscripts denote partial derivatives.

enjoyed. Technological parameters, such as A and D, do not appear in expression 9 because they affect the costs and benefits of investments in education in the same way.[11] Although we see e here as a measure of educational attainment, it can also be regarded in more general terms as the specialization of individuals in the social division of labor. In this sense, this result is the one observed by Becker (1985) and Becker and Murphy (1992), where increases in the total time available for labor market activities tend to increase the amount of specialization.

With expressions 6 and 9 in hand, we can use equations 7 and 8 to determine the effects of exogenous changes in T on c and b (see Appendix A.1). This gives us

$$\frac{db}{dT} = \frac{-\left\{nc^{\sigma-1}\left[\frac{1}{T}+(1-\sigma)\frac{A_p}{2c}(\frac{bn}{T}+\frac{1}{2})\right]+\rho(nT)(\alpha-1)(D_pbe)^{\alpha-2}DD_p\frac{b}{2}\right\}}{\rho(nT)(\alpha-1)(D_pbe)^{\alpha-2}DD_p\frac{T}{2}-(1-\sigma)n^2c^{\sigma-2}\frac{A_p}{2}} \lesseqgtr 0,$$

and

$$\frac{dc}{dT} = \frac{A_p\left\{n^2c^{\sigma-1}\left[\frac{1}{T}+(1-\sigma)\frac{A_p}{2c}(\frac{bn}{T}+\frac{1}{2})\right]+\rho(nT)(\alpha-1)(D_pbe)^{\alpha-2}DD_p(nb+\frac{T}{4})\right\}}{\rho(nT)(\alpha-1)(D_pbe)^{\alpha-2}DD_pT-(1-\sigma)n^2c^{\sigma-2}A_p} \lesseqgtr 0.$$

Both $\frac{dc}{dT}$ and $\frac{db}{dT}$ can be either positive or negative, but, as shown in Appendix A.1, they cannot be both negative at the same time. c or b must necessarily increase as T increases, and both can increase at the same time. This is an obvious result once we realize that an increase in T also means an expansion in the constraint set. Since n goes down as T increases, and e increases only proportionally to T, the additional resources have to be 'consumed' either via a raise in b or via a raise in c, and possibly via both.

The specific signs of $\frac{dc}{dT}$ and $\frac{db}{dT}$ depend on the values of the parameters, but the forces at work can be understood by looking at the individual problem. We know that, as T increases, the shadow price of the time b invested in h_c (n) goes down, and the productivity of this investment goes up (e), so that h_c must increase in the new optimum, even though b itself may decrease. Depending on the magnitude of the decrease in this shadow price, and on the concavity of the sub-utility functions (σ and α), it will be worthwhile for the individual also to increase c together with h_c, or to let c decrease as h_c increases.

It is easy to show that h_c unequivocally increases as T increases. Since $h_c = D_pbe$, we have that $\frac{dh_c}{dT} = D_p(b\frac{de}{dT} + e\frac{db}{dT})$, which gives:

$$\frac{dh_c}{dT} = \frac{-D_p\left\{2c^{\sigma-1}n\left[1+(1-\sigma)\frac{bn}{c}A_p\right]+(1-\sigma)nc^{\sigma-2}A_p\frac{T}{2}\right\}}{\rho(nT)(\alpha-1)(D_pbe)^{\alpha-2}DD_pT+(\sigma-1)n^2c^{\sigma-2}A_p} > 0.$$

[11] This result is analogous to the one originally obtained by Ben-Porath (1967), regarding the effect of the price of services of human capital.

It may seem counterintuitive that c may actually go down as T increases, but it is important to keep in mind exactly what this theoretical experiment corresponds to. Here, we are analyzing an increase in T holding constant the level of basic human capital of parents (h_p). So, the result means that individuals entering adulthood that face an increase in their life expectancy will increase their own education and the basic education that they give to their children. And it may even be the case that they reduce their own consumption in each period in order to be able to invest more in the children's human capital. This is different from analyzing what will be the effect of T on the consumption pattern across generations. As we will see now, the model predicts that increases in T increase the growth rate of consumption across generations.

3.2.2 Dynamic Implications of Longevity Gains

The Possibility of a Steady-State

The possibility of a steady-state in this economy rests on the values of the parameters α and σ. Technological factors summarized by the goods constraint in problem IP imply that, in any steady-state, c and h_p must necessarily grow at the same constant rate from one generation to the next. But the individual maximization problem tells us, through equation 8, that c and h_p growing at the same rate will not be consistent with the optimal choices of the different generations, unless $\alpha = \sigma$. Therefore, for a steady-state to exist in this economy, it must be the case that $\alpha = \sigma$, so that individuals from different generations will make optimal choices such that c and h_p will grow at the same constant rate, and b, n, e, and l will be constant.

This can be formally seen once we realize that, in terms of the individual problem stated in IP, for a steady-state to exist it must be the case that the agent will not change his decisions regarding n, b, l, and e as h_p increases. This means that the different generations, who differ only in terms of their endowed h_p and see it as a given parameter, will translate the higher levels of basic human capital in increased consumption, leaving b, n, e, and l unchanged.

From the results obtained before, we already know that $\frac{dn}{dh_p} = \frac{de}{dh_p} = 0$. We can use equations 7 and 8 to show how b and c respond to changes in h_p. This gives the following expressions:

$$\begin{aligned} \frac{db}{dh_p} &= \frac{(\sigma-1)be}{h_p[(\sigma-\alpha)bn+(\alpha-1)e]} - \frac{b}{h_p} \gtrless 0, \text{ and} \\ \frac{dc}{dh_p} &= \frac{Ae^2(1-\alpha)(bn-e)}{T[(\sigma-\alpha)bn+(\alpha-1)e]} > 0, \end{aligned}$$

where the sign of $\frac{dc}{dh_p}$ comes from the fact that $\sigma < 1$.

As mentioned before, a steady-state requires a constant b with an increasing h_p. This will only happen here if $\sigma = \alpha$, in which case we have $\frac{db}{dh_p} = 0$ and $\frac{dc}{dh_p} = \frac{A}{T}e(e - bn)$. It is immediate to see that, in this case, c and h_p will grow at the same constant rate, given by $(1+\gamma) = \frac{h_c}{h_p} = DAbe$.

If $\sigma \neq \alpha$, there is no steady-state, and b will increase or decrease over time (with the increase in h_p) until a corner solution is reached. Rewrite $\frac{db}{dh_p}$ in the following way:

$$\frac{db}{dh_p} = \frac{b}{h_p} \frac{(\sigma-\alpha)(e-bn)}{[(\sigma-\alpha)bn + (\alpha-1)e]}.$$

So, if $\alpha > \sigma$, we have $\frac{db}{dh_p} > 0$; and if $\alpha < \sigma$, we have $\frac{db}{dh_p} < 0$, since $\sigma < 1$.

The intuition for this result is clear. If $\alpha > \sigma$, the sub-utility function related to h_c is less concave than the one related to c, such that when h_p grows from one generation to the next, younger generations tend to increase h_c more than proportionately to c, and this is achieved through increases in b. The same sort of argument works for the case where $\alpha < \sigma$, implying that h_c is increased less than proportionately to c, and that this is achieved through reductions in b. When $\alpha = \sigma$, every generation is just happy to increase c and h_p in the same proportion in relation to the previous generation, in which case b remains unchanged and we have a steady-state. If this actually happens, we will necessarily have h_p and c growing at the same constant rate through time. From now on, when discussing an equilibrium with growth, we will implicitly assume that $\alpha = \sigma$, so that a steady-state exists.

The Effect of T on the Steady-State Growth Rate

The production function of h_c implies that the growth rate of basic human capital is given by[12] $(1+\gamma) = \frac{h_c}{h_p} = DAbe$. From the goods constraint on problem IP we have that $Ah_ple = Tc$, which implies that, in steady-state, c will grow at the same rate of h_p, namely, $(1+\gamma)$. The same will also be true for the level of adult human capital (H_p), as can be seen from the production function $H_p = Aeh_p$.

The effect of longevity gains on the growth rate of this economy is given by

$$\frac{d(1+\gamma)}{dT} = DA\left(b\frac{de}{dT} + e\frac{db}{dT}\right) > 0,$$

where the sign comes from the fact that, as proved in subsection 3.2.1, $\left(b\frac{de}{dT} + e\frac{db}{dT}\right) > 0$. Longevity gains increase the steady-state growth rates of consumption and all forms of human capital across generations.

We see the intuition for these results as follows. As longevity increases, incentives to invest in adult human capital increase, so that e – the amount of time devoted to parent's own education,

[12] If $DAbe < 1$, there is no growth in steady-state. In this case, H_o and h_o will be important in determining the human capital and consumption levels in equilibrium.

or the educational attainment – increases. Once educational attainment and adult human capital (H_p) are higher, the individual becomes more productive in investing in children's human capital. The higher life span of each child also tilts the quantity-quality trade off towards less and better educated children, what reduces fertility. Together with the higher adult productivity in the household sector, this increases the level of basic human capital given to each child. Higher basic human capital, and more investments in adult education (higher educational attainment), end up increasing the growth rate of the economy.

The goal of this section is to stress the role played by adult longevity, through changes in the return to education and the way parents value each child, in the fertility and educational choices. Even though the unambiguity of some of the effects depends on the functional forms adopted, these forces will always be at work, no matter how the model is specified. Our approach shows that, under reasonable assumptions, the role played by longevity gains is important enough to reduce fertility, increase educational attainment, and raise the growth rate of the economy.

3.2.3 The Malthusian Equilibrium

The model developed in the previous subsections can, with little modifications, accommodate a so called Malthusian equilibrium, where investment in all forms of human capital are at corner solutions and fertility varies positively with consumption and production. Besides, the model allows the characterization of the fertility transition as a natural consequence of the escape from such a steady-state, caused by successive increases in adult longevity.

We reincorporate the goods fixed cost of children (f) and the lower bound levels of basic and adult human capital (h_o and H_o) into the model. As mentioned before, in an equilibrium with consumption and all forms of human capital growing, these constant terms become irrelevant, and all conclusions discussed in the previous subsections hold. But in an equilibrium with zero growth and no investment in human capital these elements play a key role.

A Malthusian equilibrium in this set up is a situation where $h_p = h_o$, and the optimal choice of the individual implies $b = e = 0$. Collapsing all the constraints into only one and writing the problem in terms of $\{c, n, b, e\}$, this equilibrium is characterized by the following foc's, where λ is still the multiplier on the constraint:

$$\begin{aligned} c^{\sigma-1} &= \frac{\lambda}{H_o}, \\ T\rho'(nT)\frac{h_o^{\alpha}}{\alpha} &= \frac{f}{H_o}\lambda, \\ \rho(nT)h_o^{\alpha-1}DH_o &< n\lambda, \\ 0 &< \left[1 - \frac{Ah_o(Tc + fn)}{H_o^2}\right]\lambda. \end{aligned}$$

We call this corner solution a Malthusian equilibrium because, in a situation like this, changes in productivity – brought about, for example, by exogenous changes in H_o – will be positively correlated with changes in both consumption and fertility (for proof and further discussion, see Appendix A.2).

While this corner solution holds, changes in T will only be associated with changes in c and n. Working with the first two foc's and the constraint, we get the effects of T on c and n:

$$\frac{dn}{dT} = \frac{\frac{f^2 n}{T^2}(\sigma-1)c^{\sigma-2} - \frac{h_o^{\alpha}}{\alpha}[nT\rho''(nT)+\rho'(nT)]}{T^2\rho''(nT)\frac{h_o^{\alpha}}{\alpha} + \frac{f^2}{T}(\sigma-1)c^{\sigma-2}} \gtrless 0, \text{ and}$$

$$\frac{dc}{dT} = \frac{f\frac{h_o^{\alpha}}{\alpha}[2nT\rho''(nT)+\rho'(nT)]}{T^3\rho''(nT)\frac{h_o^{\alpha}}{\alpha} + f^2(\sigma-1)c^{\sigma-2}} \gtrless 0.$$

Appendix A.2 shows that $\frac{dc}{dT}$ and $\frac{dn}{dT}$ may be positive or negative, but both cannot be negative at the same time. Either c or n must increase as T increases, since an increase in T corresponds to an outward shift in the constraint. Besides, $-nT\rho''(nT) < \rho'(nT) < -2nT\rho''(nT)$ is a sufficient condition for both $\frac{dc}{dT}$ and $\frac{dn}{dT}$ to be positive. The specific signs of $\frac{dc}{dT}$ and $\frac{dn}{dT}$ depend on the properties of the $\rho(.)$ function. This is expected, since the only way by which T changes the equality between marginal rate of substitution and price ratios of n and c is via the marginal utility of n (see foc's above).

While stuck in this Malthusian equilibrium, an economy can behave in many distinct ways as longevity increases: c and n may increase, c may increase and n decrease, or n may increase and c decrease. But as T keeps growing, no matter what happens to n and c, the inequalities characterizing the Malthusian equilibrium (last two foc's above) are eventually broken, and the economy enters in the dynamic process described in the previous subsections, where consumption and human capital grow from one generation to the next, and fertility declines with increases in longevity. Appendix A.2 proves this claim.

The intuition for the escape from the Malthusian regime is the following. As adult longevity increases, returns from investment in adult education also increase, because of the longer period over which education is productive. So, if gains in adult longevity are big enough, parents will start investing in their own education, and we will have $e > 0$. In relation to investments in basic human capital, the story is not so simple. As adult longevity gains take place, the total number of 'child-years' (nT) certainly increases, from the expansion of the constraint set and the concavity of the sub-utility functions. Generally, depending on the properties of $\rho(.)$, it could be the case that fertility would also keep growing and the corner solution on b would never be broken. The role played by the assumption "$\rho'(x) = 0$ for some $\overline{x} > 0$" is exactly to guarantee that, for nT

sufficiently high, fertility will stop increasing and investments in children's human capital will be eventually undertaken (making $b > 0$). If this assumption holds, sufficiently large adult longevity can always guarantee positive investments in adult and basic human capital (b and $e > 0$). After this threshold point is reached, further increases in longevity trigger the demographic transition, and the economy moves into a sustained growth path.

In this setup, the only engine behind the demographic transition and the escape from the Malthusian steady-state is the exogenous change in longevity. In the next subsection, we show that reductions in child mortality can play a similar role, both in terms of the steady-state with growth and the escape from the Malthusian equilibrium.

3.3 The Role of Child Mortality

3.3.1 Child Mortality in the Equilibrium with Growth

We now reintroduce child mortality into the analysis, under the assumption that costs related to having and educating children depend on the total number of born children. Under this assumption, the individual problem is exactly the same stated in the beginning of section 3. We start by analyzing the static implications of child mortality reductions, and then go on to discuss its effects on the growth rate of the economy and on the possibility of escape from the Malthusian steady-state. First order conditions for the equilibrium with growth are identical to the ones from section 3.2.1, apart from equation 2, which becomes

$$(1-\beta)T\rho'[(1-\beta)nT]\frac{(D_p be)^{\alpha}}{\alpha} = b\lambda, \tag{2'}$$

and from the fact that $(1-\beta)$ should be introduced multiplying nT inside $\rho(.)$, whenever $\rho(.)$ appears.

To explore the properties of this equilibrium as β changes, we follow the same steps from subsection 3.2.1. Using equations 2' and 3, we get

$$\varepsilon[(1-\beta)nT] = \alpha,$$

so that $\frac{dn}{d\beta} = \frac{n}{1-\beta} > 0$. The model implies a constant total lifetime of surviving children. In a sense, parents have a kind of target of 'child-years', and they increase fertility when child mortality increases, to guarantee the achievement of this 'goal.'

Using foc's 3, 4, and the constraint, we get the same expression for e that we had before ($e = T/2$), which implies that $\frac{de}{d\beta} = 0$. Together with equation 1 and the constraint, this yields:

$$\frac{db}{d\beta} = \frac{(\sigma-1)bnc^{\sigma-2} - \frac{2c^{\sigma-1}}{A_p}}{(1-\beta)[(1-\sigma)nc^{\sigma-2} + \frac{\rho[(1-\beta)nT](1-\alpha)h_c^{\alpha-2}D^2T}{n}]} < 0, \text{ and}$$
$$\frac{dc}{d\beta} = \frac{\rho[(1-\beta)nT](\alpha-1)h_c^{\alpha-2}D_pDeb + nc^{\sigma-1}}{(1-\beta)[(1-\sigma)nc^{\sigma-2} + \frac{\rho[(1-\beta)nT](1-\alpha)h_c^{\alpha-2}D^2T}{n}]} \lesseqgtr 0.$$

Also, since $h_c = DAeh_pb$, we have $\frac{dh_c}{d\beta} < 0$.

In an equilibrium with growth, reductions in child mortality will reduce fertility, increase investments in basic human capital, and leave adult educational attainment unchanged (so that h_c will increase). Parents' consumption may go either up or down, depending on the value of the parameters.

The growth rate of this economy is given by $(1+\gamma) = DAeb$, so it is easy to see that when there is a change in β, this rate changes by $\frac{d(1+\gamma)}{d\beta} = DAe\frac{db}{d\beta} < 0$. Increases in child mortality reduce the steady-state growth rate of the economy, via reductions in the investment in basic human capital.

Here, the main engine is the reduction in fertility. As child mortality decreases and fertility is reduced, resources are freed up to be used either in producing c or h_c. But the reduction in n also represents a reduction in the shadow price of h_c in relation to c, such that h_c will certainly increase (via an increase in b), and c may go either up or down, depending on how strong the income effect is.

3.3.2 Child Mortality and the Malthusian Equilibrium

We use the same strategy adopted in subsection 3.2.3 to characterize the Malthusian equilibrium in this economy. In this case, the corner solution yields:

$$\varepsilon[(1-\beta)nT] > \frac{\alpha Df}{h_o}, \text{ and}$$
$$TAh_o < H_o,$$

which are analogous to the inequalities obtained before. The behavior of n and c in this equilibrium can be analyzed using the foc's and the constraint:

$$\frac{dn}{d\beta} = \frac{T\frac{h_o^\alpha}{\alpha}\{\rho'[(1-\beta)nT] + (1-\beta)nT\rho''[(1-\beta)nT]\}}{(1-\beta)^2T^2\rho''(nT)\frac{h_o^\alpha}{\alpha} + \frac{f^2}{T}(\sigma-1)c^{\sigma-2}} \lesseqgtr 0, \text{ and}$$
$$\frac{dc}{d\beta} = \frac{-f\frac{h_o^\alpha}{\alpha}\{\rho'[(1-\beta)nT] + (1-\beta)nT\rho''[(1-\beta)nT]\}}{(1-\beta)^2T^2\rho''(nT)\frac{h_o^\alpha}{\alpha} + \frac{f^2}{T}(\sigma-1)c^{\sigma-2}} \gtreqless 0.$$

Note that these two expressions will never have the same sign: if one is positive, the other must be negative. This had to be the case, since changes in β do not change the individual constraint, so that if the agent wants to increase the 'consumption' of some 'good', he has to decrease the 'consumption' of the other.

Anyhow, no matter what happens to n and c, reductions in child mortality will increase the total number of surviving 'child-years' ($(1-\beta)nT$), and will push the economy away from the Malthusian steady-state, into a steady-state with growth and positive investments in human capital. The difference here is that, at first, when β changes, nothing happens to the incentives to invest in adult human capital (second inequality), and only investments in basic human capital are undertaken. Only after basic human capital (h_c) is accumulated from one generation to the next, the incentives to invest in adult education increase. And if child mortality reduction is large enough, the economy enters a sustained growth path. These claims are proved in Appendix A.3.[13]

3.3.3 Costs of Children Depending on Number of Surviving Children

In our analysis of the effects of child mortality, we assumed that costs of children depend on the number of born children. Our results would change sensibly if costs of having children depended on the number of surviving children. Which one of the two specifications is the most accurate description of reality is an empirical matter. It probably depends crucially on which phase of childhood concentrates most of the reductions in mortality. We come back to this discussion in the empirical section, but now we briefly explore the theoretical consequences of changing the assumptions related to the costs of having children.

Once we assume that costs of children depend on the number of surviving children, the time and goods constraint in problem IP have to be substituted by the following:

$$\begin{aligned} lH_p &\geqslant Tc + f(1-\beta)n, \text{ and} \\ T &\geqslant l + b(1-\beta)n + e, \end{aligned}$$

and the rest of the problem remains unchanged.

The only role of a change in child mortality in this set up will be to change the fertility rate, in such a way as to maintain exactly the same number of surviving children ($\overline{n}$, the net fertility

[13] There are some appealing variations of the basic model that do not introduce any major change in terms of the qualitative results. For example, if instead of facing increases in their longevity together with their children's, parents face only increases in children's longevity, we arrive at similar conclusions. Generally, increases in future generations' adult longevity have the same short run effects of reductions in child mortality, and the same long run effects of overall increases in adult longevity. Also, if utility from children depends only on shared lifetime (between parents and children), there is no change at all in the qualitative results.

rate), given constant values of the other parameters. Since child mortality affects the costs and benefits of having children in the same way, parents have a target number of surviving children that is kept no matter what is the child mortality rate. Reductions in child mortality reduce the fertility rate, and leave the other variables unchanged. There is no effect on growth or human capital accumulation, and reductions in child mortality do not tend to move the economy out of a Malthusian regime.

4 Synthesis of the Predictions of the Model

This section takes a step back and looks at the predictions of the model from a broader perspective. We describe histories of demographic transition that would come out of a model as the one outlined in the previous section. We look descriptively at the possible paths that an economy starting from a Malthusian equilibrium would follow as improvements in adult longevity and child mortality took place. Predictions related to adult longevity are basically the same across all the different specifications discussed. In relation to child mortality, predictions may be quite different, depending on the assumptions related to the costs of raising children.

We concentrate the discussion on fertility (n), educational attainment (e), and consumption or growth (c or γ), as exogenous changes are assumed to take place in child mortality (β) and adult longevity (T). Table 1 summarizes the effects of T and β on these three endogenous variables.

For economies starting from the Malthusian equilibrium, the effects of T and β are very similar. At first, educational attainment does not change, while consumption and fertility may go either up or down. In any case, we know that increases in T and reductions in β will tend to move the economy out of the Malthusian equilibrium, and into a steady-state with growth and investments in human capital. Given our assumptions, in the moments preceding the threshold of this transition, individuals will generally be in a situation where $\varepsilon[(1-\beta)nT]$ has relatively small values, due to an almost constant $\rho[(1-\beta)nT]$, and a decreasing $\rho'[(1-\beta)nT]$ (equal to zero for big enough $(1-\beta)nT$). This implies that the terms on $\rho'(.)$ and $\rho''(.)$ in $\frac{dn}{dT}$ and $\frac{dn}{d\beta}$ will become quantitatively less important, the closer we are to the beginning of the transition. If this process is strong enough such that $\rho'(.)$ and $\rho''(.)$ become irrelevant in determining the signs of $\frac{dn}{dT}$ and $\frac{dn}{d\beta}$, we will have that, immediately before the transition, fertility will be positively affected by gains in adult longevity, and will not respond to changes in child mortality. In this moment, consumption will be relatively insensitive to changes in both adult longevity and child mortality.

Once this threshold is reached, the economy enters a sustained growth path. In the new equilibrium, increases in adult longevity and reductions in child mortality increase the economy's growth rate and reduce fertility. Educational attainment also rises as adult longevity increases, but will not change with reductions in child mortality. In our set up, there is nothing that ties down the equilibrium fertility to some specific value. As long as life expectancy keeps growing, fertility will be reduced, possibly to values below replacement rate.

Another variable that deserves special attention from the perspective of the demographic transition, but that we did not mention yet, is population. At any point in time, population is an intricate function of the cumulative effect of past fertility and child mortality rates on initial population levels, and also a function of adult longevity. If we normalize our model in such a way that parents have children in the end of their first period of life ($\tau = 1$), and we call P_s the population at period s, we have that:

$$P_s = \sum_{j=s-T}^{s-1} \left[\prod_{i=s-T}^{j} (1-\beta_i) n_i \right] P_{s-T-1} = \sum_{j=s-T}^{s-1} \left[\prod_{i=0}^{j} (1-\beta_i) n_i \right] P_0,$$

where $s > T$, and P_0 is the initial population.

This expression helps understand the direct effect of longevity on population size. For illustrational purposes, assume an economy in a Malthusian long run equilibrium, where the net fertility rate ($\overline{n} = (1-\beta)n$) equals one. If this has been the case for a sufficiently long time, we have that $P_s = P_{s-T-1} \sum_{j=1}^{T-1} [(1-\beta)n]^j = P_{s-T-1}(T-1)$. Additionally, assume that the fertility rate does not respond much to changes in adult longevity in this equilibrium, such that $\frac{dn}{dT} \simeq 0$. In this case, gains on T are translated in an almost one to one basis into population increases: for T large enough, a 50% gain in adult longevity will represent a long run increase of almost 50% in population.

With child mortality reductions, the effects are even more dramatic, given the cumulative way in which it enters the determination of population size. Even if fertility falls, as long as its fall does not offset completely the child mortality reduction, the long run effects will be enormous. This is clear once we look at the net fertility rate $\overline{n}$ as a single variable. If we start from a Malthusian equilibrium where $\overline{n} = 1$, and there is a reduction in child mortality that is not completely offset by fertility declines, we will end up in a situation where $\overline{n} > 1$. Given constant values for the other parameters, $\overline{n}$ determines alone the rate of population growth and, thus, any $\overline{n} > 1$ represents a population growing exponentially at a constant rate through time. No matter how low this rate is, this means population explosion in the long run.[14]

What this means is that, as adult longevity increases and child mortality declines, and as the economy approaches the transition point, population should be growing considerably fast. This is even more so once we remember the point mentioned earlier, that, on the verge of the transition,

[14] For example, a population growing at 2% a year doubles in 36 years.

fertility is likely to rise with adult longevity and not to be very responsive to child mortality changes. And, from a simple accounting perspective, the growth will be faster, the faster are the longevity gains and child mortality reductions.

Thus, our model predicts that Malthusian economies experiencing increases in life expectancy ($(1-\beta)T$) would go through an initial phase with consumption and fertility changing in different ways, and with population increasing rapidly. This population increase would be driven mainly by the gains in life expectancy itself. If these gains were significant enough, economies would move to a new equilibrium with growth. From this point on, educational attainment would start rising and fertility would be reduced as life expectancy raised. Further increases in life expectancy in this new equilibrium would be associated with further reductions in fertility, and increases in growth and educational attainment. These are the basic facts that we will be looking for in our empirical investigation of the demographic transition, and of the recent behavior of fertility, educational attainment, and growth in post-transition countries.

5 Empirical Evidence

5.1 The Nature and Timing of Mortality Changes

The engine behind the demographic transition in the theory discussed above is the exogenous rise in life expectancy. For the model to be empirically relevant, it must be the case that longevity gains actually preceded fertility reductions in the real experiences of demographic transition, and that the mortality reductions were to some extent exogenous to economic development. Economic growth, maybe determined by technical development, could be the driving force behind the whole process, reducing mortality via improved nutrition and living conditions, and reducing fertility via the usual income induced quantity-quality trade off. To show that mortality was, per se, a driving force in the process, we have to show that some of the changes in life expectancy were not due to material improvements, nor to changes in behavior induced by increased income.

We do not claim that improvements in living conditions do not affect life prospects. This relation is, indeed, an important part of the mechanism of checks and balances behind the Malthusian model. Our claim is just that changes in life expectancy at birth from 40 to more than 70 years, like the ones experienced during the demographic transition, are not entirely due to material improvements.

The role of material conditions in determining mortality, mainly through nutrition, was extensively discussed by Fogel (see, for example, Fogel, 1994). His analysis shows that nutrition explains 90% of the mortality reductions in France up to 1870, but only 50% after that. Survival rates to the age of 15 in France rose from 67 in 1880, to 89 in 1935. Half of this change cannot be accounted for by improved nutrition, so that a gain of 10% in the probability of survival seems to come from sources unrelated to improved material conditions. It is also interesting to note that exactly from 1870 on, the French crude birth rate started to show a consistent declining trend.

The evidence for the less developed countries also indicates that a considerable part of the gains in life expectancy was not related to economic development. Preston (1980) discusses in detail the causes of mortality declines in less developed countries during the twentieth century, and argues that approximately 50% of the decline was not directly related to improved material conditions.

The evidence comes both from the changing relation between economic development and life expectancy, and from the diseases responsible for mortality declines. The changing cross-sectional relation between income and life expectancy was already mentioned (see Figure 1). Similar evidence is available regarding the relation between life expectancy and nutrition. Table 2, reproduced from Preston (1980, p.305), contains information on life expectancy at birth in 1940 and 1970, for countries at different income and nutrition levels. Life expectancy gains took place in all the nutrition brackets shown in the Table. For the lowest nutrition group (less than 2,100 calories daily), there was an increase of 10 years in life expectancy at birth. Preston (1980) also relates life expectancy changes to income and calories consumption, and concludes that approximately 50% of the changes in life expectancy were due to 'structural factors,' unrelated to economic development.

A look at the diseases responsible for mortality declines in developing countries leads to similar conclusions. Table 3, reproduced from Preston (1980, p.300), presents the approximate percentage of the mortality decline in less developed countries accounted for by different diseases. Preston argues that the role of economic development in reducing mortality probably operated mostly through influenza/pneumonia/bronchitis, for which there was no effective deployment of preventive measures, and diarrheal diseases, for which the improvements came mainly through improvement in water supply and sewerage (Preston, 1980, p. 313). Apart from these diseases, preventive measures were probably the most effective ones. Simple changes in public practices and personal health behavior, brought about by knowledge previously inexistent, allowed for significant reductions in mortality at very low costs (Preston, 1996, p.532-4).[15] Table 3 shows that this

[15] Most dramatically, the acceptance of the germ theory – developed on the turn of the nineteenth to the twentieth century – allowed for inexpensive gains in life expectancy via simple preventive measures (Vacher, 1979; Ram and Schultz, 1979; Preston, 1980 and 1996; Ruzicka and Hansluwka, 1982). Also, throughout the twentieth century, health programs became increasingly dissociate of the countries' economic conditions, and more dependent on the concerns of the developed world. Even though the monetary value of the help was relatively small, the larger contributions came in the form of development of low cost health measures, training of personnel, initiation of programs, and more effective and specific interventions (see Preston, 1980, p.313-5; and Ruzicka and Hansluwka, 1982). This, to some extent, helped to dissociate gains in life expectancy from improvements in economic conditions.

view generates numbers similar to the ones obtained in the income–nutrition–mortality analysis, with a little more than 50% of the life expectancy gains being unrelated to economic development per se.

The chronology of events in the historical experiences of demographic transition also supports the exogenous role played by mortality. Even though there is considerable consensus in the demographic literature regarding this fact,[16] we present here further evidence in this direction. Since the econometric modelling of the transition itself is a difficult task, the historical evidence constitutes an informal test of some of the predictions of the model.

Figure 6 presents data for life expectancy at birth, total fertility rate, and real wages in England, between 1541 and 1921. The life expectancy and fertility statistics are the usual ones, and they were constructed by Wrigley et al (1997, Appendix 9), based on reconstitution of families from parish records; data for the period posterior to 1871 was obtained from Keyfitz and Flieger (1968). The real wage data refers to the daily wage rate of building craftsmen, expressed in units of a composite good; these numbers were calculated by Wrigley and Schofield (1981, Appendix 9), based on the original work of Brown and Hopkins (1956).

Figure 7 presents similar statistics for the case of Sweden, between 1736 and 1936. Instead of life expectancy at birth, the mortality statistic is survival rate to the age of fifteen. These series were obtained from Eckstein et al (1999) and Macunovich (2000). The real wage, for the period up to 1914, is the daily rate for male agricultural workers deflated by the price of rye and, for 1915 on, the summer wage for male casual labor deflated by a cost of living index. Real wage and fertility data come from Eckstein et al (1999).

The figure for England shows that consistent gains in life expectancy started happening around 1780, although life expectancy really took off only after 1840. Fertility, on the other hand, had a clear upward trend between 1660 and 1820. After that, it suffered a large reduction, until it started increasing again between 1840 and 1860. From 1860 on, fertility declined consistently, reaching its current levels. Real wages also showed a slight upward trend until 1800, when a structural break took place, and they started increasing rapidly.

For Sweden, events were a little smoother. Survival rates started increasing consistently around 1800, while fertility and real wages remained considerably stable during the first half of the nineteenth century. Around 1850, fertility rates started dropping and wages started rising.

Generally, both figures are consistent with our model, and suggest two different demographic regimes for each country. In both cases, the transition seems to have happened at some point in the nineteenth century. England experienced a Malthusian regime in the period between 1650

[16] See, for example, Heer and Smith (1968), Cassen (1978), Kirk (1996), Mason (1997), and Macunovich (2000).

and 1800, during which gradual long term increases in real wages were associated with gradual increases in fertility. After that, with consistent life expectancy gains, fertility started declining and real wages started rising. In Sweden, both real wages and fertility remained roughly constant until 1850, when, following increases in the survival rate, fertility dropped continuously and real wages started rising.

Even though the timing of changes in the cases of England and Sweden fit the general patterns generated by the model, the precise moment of the increase in life expectancy is not so clear, and this may bring some suspicion in relation to the sequence of events described above. The problem is that, in these cases, the process took place at a very slow pace, so it is difficult to identify exactly when it started. In this sense, a look at the more recent experiences of demographic transition may make things clearer. Developing countries that went or are going through the transition, did so in the second half of the twentieth century, and at a pace much faster than the one experienced by the industrialized nations. Data from the World Bank's *World Development Indicators* allow us to look at these cases, for the period between 1960 and 1995.

Figures 8, 9, and 10 illustrate cases of, respectively, African, Asian, and Latin American countries that have already started the demographic transition. Each figure contains nine graphs, with data related to life expectancy at birth and total fertility rate. Data are averages for each five year periods between 1960 and 1995. For all cases shown, life expectancy rises consistently through time, with the exception of the last couple of observations for the Democratic Republic of Korea, Kenya, and Zimbabwe. For several cases, it is also true that fertility falls constantly since the beginning of the period, what suggests that these countries had started their transitions before 1960. This would be the case of, among others, South Africa, Hong Kong, India, Republic of Korea, Brazil, and Chile. There are also several cases for which, during the initial decline in mortality, fertility remained roughly constant, or even increased, and after some later point it started to decrease at a very fast pace (for example, Algeria, Morocco, China, Indonesia, Bolivia, and Mexico). What we do not see are countries that, after experiencing large declines in fertility, experience increases that bring it back to its initial level. Once fertility decreases, it does not rise back again. Japan and Argentina, which may seem to contradict this claim, start the period already with considerably low fertility levels, and the magnitude of the changes they experience are minor in comparison to the other countries.

Finally, a couple of African countries have not, up to now, started their demographic transition. Figure 11 illustrates the experience of nine of these countries. In most cases, mortality reductions have been significant and consistent throughout the last 40 years. Nevertheless, no significant reduction in fertility can be noticed. In some countries, fertility has risen up to 1 point, while in others it has remained roughly constant. There are also cases for which reductions have been taking place, although the levels are very high and it is still too soon to identify any trend. What generally distinguishes these countries from the ones in the previous figures are the considerably higher mortality rates. In some cases, life expectancy at birth is, today, below 45 years (Ethiopia, Guinea-Bissau, and Sierra Leone). Presently, mortality levels as these ones are not observed in any other part of the world.

Summarizing, we see cases of modest longevity gains without fertility reductions, but we do not see cases of fertility reductions without longevity gains. The features of the data are consistent with the theory. Initial life expectancy gains, while the economy is still in the Malthusian equilibrium, may have distinct effects on fertility. But, inevitably, further mortality reductions end up moving the economy out of this equilibrium. Once this threshold is reached, fertility decreases with rises in life expectancy, and once this process starts, the trend is never reversed.

Additionally, the data supports the idea that there may be a cut off level of life expectancy that determines the escape from the Malthusian equilibrium. Strictly, this cut off level could be country specific, depending on cultural and natural aspects. But the evidence discussed above is consistent with a common threshold around 50 years of life expectancy at birth. If this is the case, reaching this level of longevity would mark the transition of a country from a Malthusian regime to an equilibrium with investments in human capital and the possibility of sustained growth.

In Figures 12 to 15 we explore this point further, by analyzing the behavior of fertility and educational attainment before and after the year when life expectancy at birth reaches 50. Obviously, we do not imply that this specific number is the precise point at which all the different countries start their demographic transition, but rather that it is a reasonable approximation to the moment of change in the demographic regime. All countries that reach this level of life expectancy within the period of time contained in the sample are included in the Figures. They are aligned in time according to the year when the threshold was reached, such that the year T is the "year when life expectancy at birth reached 50" for every country. Other years are measured as deviations from this reference point.

Figure 12 shows the behavior of fertility before and after year T. There seems to be some change in pattern, from stable to decreasing, as countries cross the threshold point, but since there is considerable heterogeneity in fertility levels at year T, the visualization of any trend is difficult. Therefore, Figure 13 looks at the same data, measured as the deviation of fertility in relation to its initial transitional level (year T). The pattern arises more clearly in this Figure. While fertility behaves very erratically before the year when life expectancy reaches 50, it shows a consistent downward trend for all countries after this cut off is reached.

Figures 14 and 15 do the same exercise for average schooling in the population above 25. The results show an analogous pattern. While educational attainment does not have any clear trend before life expectancy at birth reaches 50, it shows a consistent upward trend for all countries after this cut off point is reached.

The overall historical evidence is consistent with the predictions of the model regarding the behavior of the economy before and after the triggering of the demographic transition.[17] Besides, the data suggest that the transition usually starts at some moment around the time when life expectancy at birth reaches 50 years. This point will be important in our investigation of the recent behavior of fertility, educational attainment, and growth in "post-transition" countries.

5.2 The Behavior of the Economy in the Equilibrium with Growth – Evidence from a Panel of Countries

5.2.1 Estimation Strategy

In this section, we analyze the behavior of fertility, educational attainment, and growth in a panel of countries, between 1960 and 1990. Our goal is to test whether the relation between these variables and child mortality and adult longevity agrees with the predictions of the model. In the model, the behavior of the economy suffers a significant change as we move from the Malthusian equilibrium to the equilibrium with positive investments in human capital. For this reason, it does not make sense to compare pre to post-transition economies, since they respond differently to changes in the exogenous variables. We opt, thus, to look at economies that had already started the demographic transition in 1960 and, therefore, should behave according to the properties of the model in the equilibrium with growth.

Following section 4, we concentrate on the endogenous variables for which we have observable statistics: fertility, educational attainment, and growth. The data are averages for five year periods between 1960 and 1990. Variables corresponding to child mortality, adult longevity, fertility, and growth are taken from the World Bank's *World Development Indicators – 1999*. These are, respectively: child mortality rate before 1 year (*mort*), life expectancy conditional on survival to

[17] The behavior of population in the second half of the twentieth century is also consistent with the discussion in section 4. Heuveline (1999) uses counter factual projections of the behavior of mortality and fertility between 1950 and 2000 to disentangle the effect of these two variables on the world population. He extends the methodology applied by White and Preston (1996), by dividing the world population into regions, and projecting four counter factual scenarios for each of them separately. The projections are obtained by applying age and sex specific survival rates to the populations of the different regions, and applying age specific fertility rates to the female population by age. His analysis shows that mortality reductions of the second half of the twentieth century contributed to increase the world population by, at least, 33%, while fertility changes reduced it by 26%. Interestingly, had the fertility and mortality levels remained at their 1950 values, the world population today would be virtually the same as it actually is. Contrary to the common belief in the economics profession, the population explosion of the twentieth century was caused almost entirely by gains in life expectancy, with fertility changes working towards slowing down the process.

1 year ($adult1$), total fertility rate ($fert$), and growth rate of the real GNP per capita ($growth$). Educational attainment is measured by the average schooling in the population aged 25 and above ($schl$), from the Barro and Lee data set (see Barro and Lee, 1993). The sample is restricted only to countries for which data is available for all the variables and years. This leaves us 71 countries and 7 points in time, which gives a total of 497 observations. Summary statistics for the relevant variables are presented in Table 4. Appendix B describes the data in more detail, and enumerates the countries included in the sample.

The first order conditions for the individual problem implicitly give a set of reduced form equations that express fertility, education, and growth as functions of child mortality, adult longevity, and the other exogenous variables:

$$\begin{aligned} fert &= f(mort, adult1, X), \\ schl &= g(mort, adult1, X), \text{ and} \\ growth &= q(mort, adult1, X), \end{aligned}$$

where X denotes all other exogenous variables apart from child mortality and adult longevity.

Our strategy, in contrast with the traditional empirical growth literature, is to take the model seriously and estimate these reduced form equations. Apart from the life expectancy variables, the only exogenous factors in our model are related to tastes (parameters of the utility function) and technology (parameters of the production functions). To account for shifts in these exogenous variables across countries, we use country fixed effects, so that any systematic difference due to culture, religion, and technology is washed away. Also, to account for possible technological development or absorption through time, we include time dummies.

The issue of exogeneity of the life expectancy gains becomes extremely important here. To control for the changes in life expectancy that are simply a consequence of economic development, we include the natural logarithm of the real GNP per capita ($\ln gnp$) in the reduced forms above. This will isolate the changes due to life expectancy, from those directly attributable to development and to behavioral changes induced by increased in income.

With these considerations in mind, the basic specification of our empirical model is given by the following system of seemingly unrelated regressions:

$$\begin{aligned} fert_{it} &= \beta_0^f + \beta_1^f mort_{it} + \beta_2^f adult1_{it} + \beta_3^f \ln gnp_{it} + \alpha_i^f + \alpha_t^f + \varepsilon_{it}, \\ schl_{it} &= \beta_0^s + \beta_1^s mort_{it} + \beta_2^s adult1_{it} + \beta_3^s \ln gnp_{it} + \alpha_i^s + \alpha_t^s + \upsilon_{it}, \text{ and} \\ growth_{it} &= \beta_0^g + \beta_1^g mort_{it} + \beta_2^g adult1_{it} + \beta_3^g \ln gnp_{it} + \alpha_i^g + \alpha_t^g + \omega_{it}, \end{aligned}$$

where the β's are coefficients, the α's are country and time fixed effects, and ε_{it}, υ_{it}, and ω_{it} are random shocks.

Since these three equations are reduced forms from a system of simultaneous equations, ε_{it}, υ_{it}, and ω_{it} are likely to be correlated with each other. To address this problem, we estimate the system above using Zellner's GLS approach for seemingly unrelated regressions. To check the robustness of the results, we also include lagged dependent variables in the right hand side, and allow for panel specific AR1 auto-correlation in the residuals. As our fertility and schooling measures are population averages, they tend to present considerable persistence through time. Both the lagged dependent variable and the auto-correlated residuals help to check whether this persistence is in any way driving the results.

Finally, our goal is to test the model's predictions in relation to economies in the steady-state with growth, where there are positive investments in human capital. Not all the countries included in the sample had already left the Malthusian regime in 1960, and this may bias the estimates. For this reason, we estimate the model both for the whole sample and for a sample of selected countries, that supposedly had already started the demographic transition in 1960 (post-transition countries). Although it is always difficult to define a precise criterion that identifies a country as being or not in the Malthusian regime, this is a problem we cannot avoid. We assume that countries with life expectancy at birth above 50 years in 1960 had already escaped the Malthusian equilibrium, as the evidence from section 5.1 suggests.[18] The list of countries included in the selected sample is contained in Appendix B, and it shows that the criterion chosen seems to be a reasonable one.

5.2.2 Analysis of the Results

Tables 5 and 6 present the results of the estimation for, respectively, the whole sample and the selected sample. For illustrational purposes, Tables 5(a) and 6(a) contain results for regressions without country and time fixed effect. The first three columns in these tables are regressions of the dependent variables only on child mortality ($mort$) and adult longevity ($adult1$), and the last three columns also include income ($\ln gnp$) as an explanatory variable. Tables 5(b) and 6(b) contain the regressions with country and time fixed effects. The first three columns in these tables are regressions of the endogenous variables on child mortality, adult longevity, income, and country and time fixed effects. The three middle columns include lagged dependent variables in the right hand side, and the last three columns allow for panel specific auto-correlation in the residuals (AR1).

[18] We also estimate the model using another selection criterion: countries with fertility rate below 3.5 in 1960, or with reductions of more than 0.5 in the fertility rate between 1960 and 1965. The results are basically the same.

Concentrating on Table 6(b), we see that the model performs well empirically. All coefficients on adult longevity are significant and have the expected sign, except for the one in the fertility equation with lagged dependent variable. Apart from this case, coefficients are also very stable across the different specifications.

In relation to child mortality, all coefficients in the fertility equations are significant and have the expected effect. Two of the coefficients in the schooling equations, and one of the coefficients in the growth equations, are also significant, but with an effect opposite to the one predicted by the model. This strange result seems to be robust to different sample choices and specifications, as long as they include country and time dummies. It is interesting to note that in the regressions without country and time dummies (Table 6(a)), child mortality appears as significant and with the expected effect.

The idea that adult longevity reduces fertility and increases educational attainment and growth, while child mortality only has significant effects on fertility, supports the variation of the model explored in subsection 3.3.3. There, we argued that when costs of having children depend on the number of surviving children, changes in child mortality will only affect fertility, and leave educational attainment and growth unchanged. This could be a consequence of the fact that we are dealing here only with child mortality before 1 year of age, when probably not many investments in children have been undertaken. But, since reductions in child mortality have been largely concentrated on very early ages (mainly below 1 year), and mortality rates between 5 years and young adulthood are extremely low, we do believe that such a version of the model may be, empirically, the most relevant one.

The comparison of the results from the whole sample (Table 5(b)) with the ones from the selected sample (Table 6(b)) also agree with theory. Regarding adult longevity, none of the coefficients in the schooling equations in the whole sample are significant, and all the significant coefficients in the fertility and growth equations are considerably smaller than the ones obtained in the selected sample. In relation to child mortality, only one of the coefficients in fertility is significant in the whole sample. Since the predictions being tested apply strictly only to economies that already started the demographic transition, we should expect the whole sample to deliver results weaker than the ones found in the selected sample.

Finally, the quantitative implications of the estimated coefficients are also of interest. Given the numbers in Table 6(b), a 10 year gain in adult longevity implies a reduction of 1.7 points in the total fertility rate, an increase of 1 year in average schooling in the population above 25, and a growth rate higher by 5%. A reduction of 100 per one thousand in child mortality implies a decrease of 1.5 points in the total fertility rate.

These results are in line with a vast array of evidence from studies that try to estimate the economic impacts of life expectancy gains. Most notoriously, these include the positive effect of longevity on growth in the traditional growth regression literature, summarized and discussed in Barro and Sala-i-Martin (1995). Other examples are the case study for India of the effects of life expectancy gains on schooling and productivity (Ram and Schultz, 1979), and the simulation exercises performed by Bils and Klenow (2000), analyzing the role of life expectancy in explaining cross-country differences in schooling, productivity, and fertility.

5.2.3 Robustness of the Results: Different Age Mortality Rates

The theory developed here implies that adult longevity gains should increase investments in education, reduce fertility, and spur economic growth only if these gains took place during productive lifetime. Longevity gains concentrated at very old ages, when individuals do not or cannot work anymore, should not have the same effects, since they do not affect the horizon over which investments in human capital can be used. Given that significant reductions in mortality at old ages have been observed in the more developed countries, this issue should be of concern. If this is what is driving our empirical results, the evidence does not support the causal links predicted by the model.

To check whether the results discussed in the previous subsection reflect the causal relations stressed by the model, or some other spurious correlation between old age longevity and development, we repeat the exercise from Table 6(b) breaking down adult longevity into productive life expectancy and old age life expectancy. This is done using information related to the mortality rate between 15 and 60 years, together with child mortality and life expectancy at birth (for description of the variables, see Appendix B). The two constructed variables indicate expected years of life between 15 and 60 years (*adexp*), and life expectancy conditional on survival to 60 years (*oldexp*). We estimate regressions similar to the ones presented in Table 6(b), breaking down adult longevity into the two variables mentioned. The inclusion of the new variable reduces the sample considerably, so much so that we are left only with observations in every ten year periods in the interval between 1960 and 1990, and only 42 countries. The results are presented in Table 7(a).

As should be expected if the results were being driven by gains in longevity during productive life, old age life expectancy (*oldexp*) is not significant in any of the specifications. The effects of child mortality (*mort*) are extremely close to the ones observed in Table 6(b), both quantitatively and qualitatively. The results on productive adult life expectancy (*adexp*) are also similar to the ones obtained in Table 6(b). The effects on fertility and growth are always significant and with the expected signs, and quantitatively stronger than the ones obtained before. The effects on schooling, on the other hand, are borderline significant in only one of the cases.

Finally, to check whether taking child mortality before 1 year bias the results in any way, by leaving aside the significant mortality rates to which children are subject between the ages of 1 and 5, we repeat the exercise from Table 7(a) using child mortality rate before 5 years. This redefines the child mortality rate and old age life expectancy as, respectively, $mort5$ and $oldexp5$. The choice of this variable reduces the sample even further, and we are left with observations in every ten year periods in the interval between 1960 and 1990, and only 37 countries. Nevertheless, the results – presented in Table 7(b) – are still extremely similar to the ones from Table 7(a). The only noticeable difference is the fact that productive adult life expectancy ($adexp$) is not significant in the fertility regression in one of the specifications.

This evidence is supportive of the causal links established by the theory, since it shows that the effects captured on Table 6(b) come from longevity gains during productive years, and not from old age mortality reductions. This is the precise mechanism on which the logic of the model relies.

6 Concluding Remarks

This paper explores the link between life expectancy, educational attainment, and fertility choice. We show that, under reasonable conditions, mortality reductions can explain the movement of economies from a Malthusian equilibrium, with no investments in human capital, to a steady-state with growth. Further reductions in mortality in this steady-state with growth reduce fertility, increase educational attainment, and, thus, increase the growth rate of the economy. These features of the model help explain the demographic transition throughout the world and the recent behavior of fertility in "post-demographic transition" countries.

Two aspects of the model drive these effects, and distinguish our theoretical work from the previous literature. The utility that parents derive from each child is allowed to depend on the number of children and, additionally, on the lifetime that each child will enjoy as an adult. The way number of children and lifetime of each child interact in the parent's utility function is an important force behind the mechanics of the model.

Also, human capital investments are broken down in two pieces: basic investments, that take place during childhood and are done by parents; and adult investments, that take place during adulthood and are done by the individuals themselves. We interpret educational attainment as the time that an adult individuals spend on their own education. Besides being more realistic, this approach allows the model to distinguish between the effects of adult longevity and child mortality on investments in education and growth.

Empirically, we justify the exogenous role played by life expectancy by presenting evidence that roughly 50% of the changes in this variable during the last century were unrelated to simple material gains, or economic development. We show that the chronology of events in the experiences of demographic transition agree with the patterns generated by our model, and that the recent behavior of fertility, educational attainment, and growth in "post-demographic transition" countries also follows most of the theoretical predictions.

A Appendix: Analytical Results

A.1 The Effect of T on c and b in an Equilibrium with Growth

Using equations 5 to 9, one can show that:

$$\frac{db}{dT} = \frac{-\left\{nc^{\sigma-1}\left[\frac{1}{T} + (1-\sigma)\frac{A_p}{2c}(\frac{bn}{T} + \frac{1}{2})\right] + \rho(nT)(\alpha-1)(D_p be)^{\alpha-2} DD_p \frac{b}{2}\right\}}{\rho(nT)(\alpha-1)(D_p be)^{\alpha-2} DD_p \frac{T}{2} - (1-\sigma)n^2 c^{\sigma-2}\frac{A_p}{2}} \lesseqgtr 0,$$

and

$$\frac{dc}{dT} = \frac{A_p\left\{n^2 c^{\sigma-1}\left[\frac{1}{T} + (1-\sigma)\frac{A_p}{2c}(\frac{bn}{T} + \frac{1}{2})\right] + \rho(nT)(\alpha-1)(D_p be)^{\alpha-2} DD_p(nb + \frac{T}{4})\right\}}{\rho(nT)(\alpha-1)(D_p be)^{\alpha-2} DD_p T - (1-\sigma)n^2 c^{\sigma-2} A_p} \lesseqgtr 0.$$

But note that, if $\frac{dc}{dT} < 0$, we have

$$n^2 c^{\sigma-1}\left[\frac{1}{T} + (1-\sigma)\frac{A_p}{2c}(\frac{bn}{T} + \frac{1}{2})\right] + \rho(nT)(\alpha-1)(D_p be)^{\alpha-2} DD_p(nb + \frac{T}{4}) > 0.$$

Since $nb + \frac{T}{4} > \frac{nb}{2}$, this implies that

$$n^2 c^{\sigma-1}\left[\frac{1}{T} + (1-\sigma)\frac{A_p}{2c}(\frac{bn}{T} + \frac{1}{2})\right] + \rho(nT)(\alpha-1)(D_p be)^{\alpha-2} DD_p \frac{nb}{2} > 0,$$

which in turn implies that $\frac{db}{dT} > 0$.

Also, if $\frac{db}{dT} < 0$, we have

$$n^2 c^{\sigma-1}\left[\frac{1}{T} + (1-\sigma)\frac{A_p}{2c}(\frac{bn}{T} + \frac{1}{2})\right] + \rho(nT)(\alpha-1)(D_p be)^{\alpha-2} DD_p \frac{nb}{2} < 0.$$

Again, since $nb + \frac{T}{4} > \frac{nb}{2}$, this implies that

$$n^2 c^{\sigma-1}\left[\frac{1}{T} + (1-\sigma)\frac{A_p}{2c}(\frac{bn}{T} + \frac{1}{2})\right] + \rho(nT)(\alpha-1)(D_p be)^{\alpha-2} DD_p(nb + \frac{T}{4}) < 0,$$

which in turn implies that $\frac{dc}{dT} > 0$.

In words, $\frac{dc}{dT}$ or $\frac{db}{dT}$ may be negative, but both cannot be negative at the same time. If one of them is negative, the other must be positive.

A.2 The Effect of T on the Malthusian Equilibrium

A.2.1 The Malthusian Equilibrium

We call the corner solution in human capital investments a Malthusian equilibrium because, in a situation like this, changes in productivity – brought about, for example, by exogenous changes on H_o – will be positively correlated with changes in both consumption and fertility. This can be seen by analyzing the effect of an exogenous change in H_o. Using the first two foc's and the constraint, we arrive at:

$$\frac{dn}{dH_o} = \frac{f(\sigma-1)c^{\sigma-2}}{T^2\rho''(nT)\frac{h_o^\alpha}{\alpha}+\frac{f^2}{T}(\sigma-1)c^{\sigma-2}} > 0, \text{ and}$$

$$\frac{dc}{dH_o} = \frac{T\rho''(nT)\frac{h_o^\alpha}{\alpha}}{T^3\rho''(nT)\frac{h_o^\alpha}{\alpha}+f^2(\sigma-1)c^{\sigma-2}} > 0.$$

Fertility and consumption respond positively to exogenous increases in productivity. If we want this set up to display all the features of a so called Malthusian regime, including its checks and balances mechanisms and the zero long run growth in consumption, we can substitute H_o by some function $F(H_o, P)$. In this case, P denotes the aggregate population level, and $F_{H_o}(.,.) > 0$. Decreasing marginal product in the agriculture sector due to limited availability of land would be captured by $F_P(.,.) < 0$, such that, for a given level of H_o, individual productivity decreases with total population. This externality implies that exogenous increases in H_o generate short run increases on n and c, that are 'consumed' in the long run by the expanded population. With time, n returns to its long run equilibrium value – given by $(1-\beta)n = 1$, such that population is constant – which also pins down the long run value of consumption. In this equilibrium, there are no long run improvements on living standards, and population grows only to the extent allowed by exogenous technical improvements or positive natural shocks (increases on H_o). This additional feature of the model captures all the properties of what is known as a Malthusian regime, but to keep things simple we analyze the case where $F(H_o, P) = H_o$. As long as the effect of P takes some time to operate, our results will be quite general.

A.2.2 Changes on T while the Corner Solution Holds

In this case, only n and c will be affected by T. The effect of T on n and c in the Malthusian equilibrium can be obtained from the first two foc's and the constraint.

$$\frac{dn}{dT} = \frac{\frac{f^2 n}{T^2}(\sigma-1)c^{\sigma-2} - \frac{h_o^\alpha}{\alpha}[nT\rho''(nT)+\rho'(nT)]}{T^2\rho''(nT)\frac{h_o^\alpha}{\alpha} + \frac{f^2}{T}(\sigma-1)c^{\sigma-2}} \lesseqgtr 0, \text{ and}$$

$$\frac{dc}{dT} = \frac{f\frac{h_o^\alpha}{\alpha}[2nT\rho''(nT)+\rho'(nT)]}{T^3\rho''(nT)\frac{h_o^\alpha}{\alpha} + f^2(\sigma-1)c^{\sigma-2}} \gtreqless 0.$$

If $\frac{dn}{dT} < 0$, it means that $nT\rho''(nT)+\rho'(nT) < 0$, which implies that $2nT\rho''(nT)+\rho'(nT) < 0$, which in turn implies that $\frac{dc}{dT} > 0$.

Also, if $\frac{dc}{dT} < 0$, we have $2nT\rho''(nT)+\rho'(nT) > 0$, which implies that $nT\rho''(nT)+\rho'(nT) > 0$, which in turns implies that $\frac{dn}{dT} > 0$.

In other words, $\frac{dc}{dT}$ or $\frac{dn}{dT}$ may be negative, but both cannot be negative at the same time. Either c or n must increase as T increases. Besides, a sufficient condition for both of them to be positive is that $-nT\rho''(nT) < \rho'(nT) < -2nT\rho''(nT)$.

A.2.3 The Escape from the Malthusian Steady-State

We want to look at what happens to the two last first order conditions as T increases. We start by analyzing the steady-state where investment in both forms of human capital is zero, and show that as T increases, an interior solution tends to be achieved both on b and e. We then go on to show that, when an interior solution is actually achieved in one of these variables, further increases on T still tend to break the remaining inequality ($e > 0$ and $b = 0$, or $b > 0$ and $e = 0$).

i) $e = 0$ **and** $b = 0$

The last equation can be rewritten as $Ah_o(Tc + fn) < H_o^2$. From the constraint, we have $TH_o = Tc + fn$, so substituting back on the inequality we have $Ah_oT < H_o$. A, h_o, and H_o are given parameters, so as T increases this inequality clearly will eventually be broken.

Substituting for λ from the second foc, the first inequality can be written as $\varepsilon(nT) > \frac{\alpha Df}{h_o}$. From the expression that we had before for $\frac{dn}{dT}$, we can obtain $\frac{d(nT)}{dT}$:

$$\frac{d(nT)}{dT} = \frac{2\frac{f^2 n}{T^2}(\sigma-1)c^{\sigma-2} - \frac{h_o^\alpha}{\alpha}\rho'(nT)}{T\rho''(nT)\frac{h_o^\alpha}{\alpha} + \frac{f^2}{T^2}(\sigma-1)c^{\sigma-2}} > 0.$$

nT increases unequivocally as T increases. Since $\rho'(nT) = 0$ for big enough nT, $\varepsilon(nT)$ will eventually be arbitrarily close to zero, and the first inequality will be broken as T increases.

ii) $e > 0$ **and** $b = 0$

This solution is characterized by the following foc's:

$$\begin{aligned} Tc^{\sigma-1} &= \frac{T}{Aeh_o + H_o}\lambda, \\ T\rho'(nT)\frac{{h_o}^{\alpha}}{\alpha} &= \frac{f}{Aeh_o + H_o}\lambda, \\ \rho(nT){h_o}^{\alpha-1}D(Aeh_o + H_o) &< n\lambda, \\ (Tc + fn)Ah_o &= (Aeh_o + H_o)^2; \end{aligned}$$

and the constraint is $T = e + \frac{Tc+fn}{Aeh_o+H_o}$.

The constraint together with the last foc gives $e = \frac{T}{2} - \frac{H_o}{2Ah_o}$. Using this fact together with the first two foc's and the constraint, we get an expression for $\frac{d(nT)}{dT}$:

$$\frac{d(nT)}{dT} = \frac{[fn + \frac{(e+fn)}{2}]f(\sigma-1)c^{\sigma-2} - \frac{T^2}{2}\frac{h_o^{\alpha}}{\alpha}\rho'(nT)}{T\rho''(nT)\frac{h_o^{\alpha}}{\alpha} + \frac{f^2}{T^2}(\sigma-1)c^{\sigma-2}} > 0.$$

The second and third foc's characterize the corner solution with the same inequality that we had before: $\varepsilon(nT) > \frac{\alpha Df}{h_o}$. As we just showed that nT increases in this solution as T increases, and we know that $\varepsilon(nT)$ will tend to zero as nT increases, this inequality will tend to be broken and we will get an interior solution in b as T rises.

iii) $b > 0$ **and** $e = 0$

The third and fourth first order conditions in this case will be:

$$\begin{aligned} \rho(nT){h_c}^{\alpha-1}DH_o &= n\lambda, \\ \rho(nT){h_c}^{\alpha-1}DbAh_o &< \left[1 - \frac{(Tc+fn)Ah_o}{H_o^2}\right]\lambda; \end{aligned}$$

and the constraint will be $T = bn + \frac{Tc+fn}{H_o}$. Substituting for λ from one of the foc's into the other and using the constraint we can rewrite the inequality as $TAh_o < H_o$. So, as T increases, an internal solution on e tends to be achieved.

A.3 β and the Escape from the Malthusian Equilibrium

Starting from a position where the solution to the individual problem implies $e = b = 0$, changes on β will not affect the foc related to e. The effect on the foc for b will depend on the behavior of $(1-\beta)nT$. Given the expression for $\frac{dn}{d\beta}$, we have that

$$\frac{d[(1-\beta)nT]}{d\beta} = \frac{(1-\beta)T^2\rho'[(1-\beta)nT]\frac{h_o^\alpha}{\alpha} - f(\sigma-1)c^{\sigma-2}n}{(1-\beta)^2T^2\rho''(nT)\frac{h_o^\alpha}{\alpha} + \frac{f^2}{T}(\sigma-1)c^{\sigma-2}} < 0,$$

so that reductions on child mortality will increase the total number of 'child-years'. $(1-\beta)nT$ increases unequivocally as β decreases. So $\varepsilon[(1-\beta)nT]$ will also be arbitrarily close to zero for big enough $(1-\beta)nT$, and the first inequality will tend to be broken as β decreases.

So, differently, from increases in adult longevity, reductions in child mortality will tend unequivocally to take the economy to a transitional situation where $b > 0$ and $e = 0$. In this case, the corner solution in e will still be characterized by the same inequality above: $TAh_p < H_o$. As discussed before, for the first parents generation experiencing reductions in their children's mortality rate, $h_p = h_o$, and there is no tendency to break the corner solution on e. But as children who received positive investments in basic human capital become adults, this h_p in period t will assume some value $h_{c,t-1} > h_o$. If child mortality keeps being reduced from generation to generation, such that $h_{c,t+1} > h_{c,t} > h_{c,t-1}$ and so on, this inequality will also eventually be broken, and the economy will reach a steady-state with growth and positive investments in all forms of human capital.

B Appendix: Data

B.1 Definition of Variables

- child mortality rate (*mort*): Child mortality rate per 1,000, before 1 year – from the World Bank's *World Development Indicators*, 1999.

- adult longevity (*adult*1): Life expectancy conditional on survival to 1 year. Calculated from life expectancy at birth and child mortality rate – from the World Bank's *World Development Indicators*, 1999.

- young adult longevity (*adexp*): Expected years of life between 15 and 60 years. Calculated from adult mortality rate between 15 and 60 years – from the World Bank's *World Development Indicators*, 1999.

- old adult longevity using child mortality rate (*oldexp*): Life expectancy conditional on survival to 60 years. Calculated from life expectancy at birth, child mortality rate, and adult mortality rate between 15 and 60 years – from the World Bank's *World Development Indicators*, 1999.

- child mortality rate before 5 years (*mort*5): Child mortality rate per 1,000, before 5 years – from the World Bank's *World Development Indicators*, 1999.
- old adult longevity using child mortality rate before 5 years (*oldexp*5): Life expectancy conditional on survival to 60 years. Calculated from life expectancy at birth, child mortality rate before 5 years, and adult mortality rate between 15 and 60 years – from the World Bank's *World Development Indicators*, 1999.
- fertility (*fert*): Total fertility rate – from the World Bank's *World Development Indicators*, 1999.
- educational attainment (*schl*): Average schooling years in the population aged 25 and over – from the Barro and Lee (1993) updated data set.
- income growth (*growth*): growth rate of the GNP per capita (constant 1995 US$) – from the World Bank's *World Development Indicators*, 1999.
- income (*gnp*): GNP per capita (constant 1995 US$) – from the World Bank's *World Development Indicators*, 1999.

B.2 Countries Included in the Sample

- Whole sample (71 countries) – Table 5:

Algeria, Argentina, Australia, Austria, Bangladesh, Barbados, Belgium, Botswana, Brazil, Cameroon, Canada, Central African Republic, Chile, Costa Rica, Denmark, Dominican Republic, Ecuador, El Salvador, Fiji, Finland, France, Ghana, Greece, Guatemala, Guyana, Honduras, Hong Kong, Hungary, Iceland, Indonesia, Ireland, Israel, Italy, Jamaica, Japan, Kenya, Korea, Rep., Lesotho, Malawi, Malaysia, Malta, Mauritius, Mexico, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Norway, Pakistan, Panama, Papua New Guinea, Paraguay, Philippines, Portugal, Singapore, South Africa, Spain, Sri Lanka, Sudan, Swaziland, Sweden, Switzerland, Thailand, Togo, Trinidad and Tobago, Tunisia, United Kingdom, United States, Uruguay, Zambia.

- Selected Sample – Post-transition (life expectancy in 1960 greater than 50; 48 countries) – Table 6:

Argentina, Australia, Austria, Barbados, Belgium, Brazil, Canada, Chile, Costa Rica, Denmark, Dominican Republic, Ecuador, El Salvador, Fiji, Finland, France, Greece, Guyana, Hong Kong, Hungary, Iceland, Ireland, Israel, Italy, Jamaica, Japan, Korea, Rep., Malaysia, Malta, Mauritius, Mexico, Netherlands, New Zealand, Norway, Panama, Paraguay, Philippines, Portugal, Singapore, Spain, Sri Lanka, Sweden, Switzerland, Thailand, Trinidad and Tobago, United Kingdom, United States, Uruguay.

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Table 1: Effects of exogenous changes in *T* and *b* in the endogenous variables

Change	Equilibrium	*e*	*c/g* (Malthusian/growth eq.)	*n*
Gain in *T*	Malthusian	0	+/-	+/-
	'ss' with growth	+	+	-
Reduction in *b*	Malthusian	0	+/- or 0	+/- or -
	'ss' with growth	0	+ or 0	-

Table 2: Mean Life Expectancy at Birth of Countries in Various Ranges of National Income and Calorie Consumption, 1940 and 1970

Daily Calories Per Capita		**National Income per Capita in 1970 U.S. Dollars**				
		< 150	150 - 299	300 - 699	> 700	**All Income Levels**
< 2,100	1970	42.7	51.5	53.3	69.5	47.5
	1940	38.3	36.0			37.9
2,100 -2,399	1970	42.6	49.9	56.2	71.4	49.1
	1940	40.0	43.9	46.1		43.4
2,400 - 2,899	1970	45.4	57.9	61.3	68.0	61.4
	1940		44.1	50.4	59.6	51.1
> 2,900	1970				71.6	71.6
	1940			58.7	65.2	64.0
All Calories Levels	1970	42.7	52.4	57.8	70.8	55.9
	1940	38.6	42.4	52.2	64.1	52.2

Source: Preston (1980, Table 5.4, p.305).
Note: 1970 countries appear in the top rows, 1940 countries in the bottom rows.

Table 3: Diseases Responsible for LDC Mortality Declines and Methods That Have Been Used against Them

Dominant Mode of Transmission	Diseases	Approximate % of Mort. Decline in LDCs, 1900-70, Accounted for by Disease	Principal Methods of Prevention Deployed	Principal Methods of Treatment Deployed
Airborne	Influenza/ Pneumonia/ Bronchitis	30		Antibiotics
	Respiratory tuberculosis	10	Immunization; identification and isolation	Chemotherapy
	Smallpox	2	Immunization	Chemotherapy
	Measles	1	Immunization	Antibiotics
	Diphtheria/Whooping cough	2	Immunization	Antibiotics
		45		
Water-, food-, and feces-borne	Diarrhea, enteritis, gastroenteritis	7	Purification and increased supply of water; sewage disposal; personal sanitation	Rehydration
	Typhoid	1	Purification and increased supply of water; sewage disposal; personal sanitation; partially effective vaccine	Rehydration, antibiotics
	Cholera	1	Purification and increased supply of water; sewage disposal; personal sanitation; partially effective vaccine; quarantine	Rehydratation
		9		
Insect-borne	Malaria	13-33	Insecticides, drainage, larvicides	Quinine drugs
	Typhus	1	Insecticides, partially effective vaccine	Antibiotics
	Plague	1	Insecticides, rat control, quarantine	
		15-33		

Source: Preston (1980, Table 5.3, p.300).

Table 4: Summary Statistics

Variable		Mean	Std. Dev.	Min	Max
mort	Overall	58.35	46.65	4.58	204.00
	Between		42.80		
	Within		19.15		
adult1	Overall	66.87	8.57	43.95	79.18
	Between		8.11		
	Within		2.91		
fert	Overall	4.16	1.92	1.25	8.12
	Between		1.75		
	Within		0.83		
schl	Overall	4.67	2.82	0.04	12.00
	Between		2.71		
	Within		0.83		
growth	Overall	2.39	3.35	-20.68	16.74
	Between		1.91		
	Within		2.76		
gnp	Overall	6377.88	8325.06	103.51	46665.15
	Between		7933.15		
	Within		2670.76		

Notes: Sample is composed of 71 countries and 7 points in time (497 observations). Data are averages for five years periods, from 1960 to 1990. Variables are child mortality rate per 1,000 (before 1 year), adult life expectancy conditional on survival to 1 year, total fertility rate, average schooling in the population above 25, growth rate of the per capita GNP, and per capita GNP (in 1995 US$).

Table 5(a): Regression Results, Whole Sample, 71 countries, 1960-90

	fert	schl	growth	fert	schl	growth
mort	0.0210	-0.0296	0.0006	0.0158	-0.0180	0.0046
	0.0023	0.0041	0.0086	0.0023	0.0038	0.0089
	0.0000	0.0000	0.9450	0.0000	0.0000	0.6040
adult1	-0.0882	0.1173	0.0667	-0.0552	0.0429	0.0406
	0.0126	0.0221	0.0468	0.0126	0.0211	0.0495
	0.0000	0.0000	0.1540	0.0000	0.0420	0.4110
lngnp				-0.3885	0.8767	0.3066
				0.0490	0.0820	0.1925
				0.0000	0.0000	0.1110
constant	8.8325	-1.4500	-2.1033	9.9503	-3.9722	-2.9853
	0.9705	1.6987	3.6039	0.9251	1.5497	3.6371
"R-Sq"	0.79	0.69	0.03	0.81	0.75	0.03
N obs	497	497	497	497	497	497
N countries	71	71	71	71	71	71

Notes: Numbers below the coefficients are, respectively, standard errors and p-values. Dependent variables are total fertility rate, average schooling in the population above 25, and growth rate of the per capita GNP. Data are averages for five years periods, from 1960 to 1990. Independent variables are child mortality rate per 1,000 (before 1 year), adult life expectancy conditional on survival to 1 year, per capita GNP (in 1995 US$), and country and time fixed effects. Zellner's SUR approach used in all cases. R-Squared for the SUR estimations is based on the first stage OLS.

Table 5(b): Regression Results with Country and Time Fixed Effects, Whole Sample, 71 countries, 1960-90

	fert	schl	growth	fert	schl	growth	fert	schl	growth
mort	0.0040	0.0083	0.0066	0.0080	0.0031	0.0178	0.0025	0.0086	0.0144
	0.0025	0.0021	0.0131	0.0014	0.0021	0.0134	0.0022	0.0021	0.0117
	0.1030	0.0000	0.6140	0.0000	0.1370	0.1840	0.2530	0.0000	0.2180
adult1	-0.0758	0.0233	0.3451	-0.0287	0.0278	0.3143	-0.0637	0.0111	0.3884
	0.0179	0.0156	0.0952	0.0110	0.0158	0.1046	0.0150	0.0151	0.0847
	0.0000	0.1370	0.0000	0.0090	0.0790	0.0030	0.0000	0.4610	0.0000
lngnp	-0.3274	0.3790	1.1574	0.1137	0.2732	1.7556	-0.2758	0.3162	1.4118
	0.1052	0.0919	0.5598	0.0620	0.0894	0.6122	0.1059	0.0950	0.4993
	0.0020	0.0000	0.0390	0.0670	0.0020	0.0040	0.0090	0.0010	0.0050
lagged dep var				0.7107	0.4531	-0.0866			
				0.0252	0.0421	0.0452			
				0.0000	0.0000	0.0550			
constant	13.7540	-4.5716	-30.3154	1.8038	-3.7121	-31.0159	12.6186	-3.2858	-35.4969
	1.4086	1.2306	7.4962	0.9265	1.2351	8.3066	1.2911	1.2645	6.6535
country dummies	yes	yes	yes	yes	yes	yes	yes	yes	yes
time dummies	yes	yes	yes	yes	yes	yes	yes	yes	yes
ar1 disturbance	no	no	no	no	no	no	yes	yes	yes
"R-Sq"	0.94	0.98	0.44	0.98	0.99	0.50			
N obs	497	497	497	426	426	426	497	497	497
N countries	71	71	71	71	71	71	71	71	71

Notes: Numbers below the coefficients are, respectively, standard errors and p-values. Dependent variables are total fertility rate, average schooling in the population above 25, and growth rate of the per capita GNP. Data are averages for five years periods, from 1960 to 1990. Independent variables are child mortality rate per 1,000 (before 1 year), adult life expectancy conditional on survival to 1 year, per capita GNP (in 1995 US$), and country and time fixed effects. Zellner's SUR approach used in all cases, except the AR1 estimations. R-Squared for the SUR estimations is based on the first stage OLS.

Table 6(a): Regression Results, Selected Sample - Post-transition Countries (48 count.), 1960-90

	fert	schl	growth	fert	schl	growth
mort	0.0274	-0.0490	-0.0152	0.0247	-0.0370	-0.0102
	0.0030	0.0067	0.0117	0.0031	0.0064	0.0121
	0.0000	0.0000	0.1960	0.0000	0.0000	0.4000
adult1	-0.1333	0.0968	-0.1080	-0.1075	-0.0200	-0.1564
	0.0185	0.0408	0.0714	0.0197	0.0412	0.0773
	0.0000	0.0180	0.1300	0.0000	0.6270	0.0430
lngnp				-0.1827	0.8261	0.3422
				0.0546	0.1140	0.2140
				0.0010	0.0000	0.1100
constant	11.8391	0.7349	11.1149	11.6334	1.6651	11.5003
	1.4130	3.1202	5.4665	1.3914	2.9046	5.4511
"R-Sq"	0.74	0.51	0.01	0.75	0.58	0.01
N obs	336	336	336	336	336	336
N countries	48	48	48	48	48	48

Notes: Numbers below the coefficients are, respectively, standard errors and p-values. Only countries with life expectancy at birth above 50 years in 1960 included in the regressions. Dependent variables are total fertility rate, average schooling in the population above 25, and growth rate of the per capita GNP. Data are averages for five years periods, from 1960 to 1990. Independent variables are child mortality rate per 1,000 (before 1 year), adult life expectancy conditional on survival to 1 year, per capita GNP (in 1995 US$), and country and time fixed effects. Zellner's SUR approach used in all cases. R-Squared for the SUR estimations is based on the first stage OLS.

Table 6(b): Regression Results with Country and Time Fixed Effects, Selected Sample - Post-transition Countries (48 countries), 1960-90

	fert	schl	growth	fert	schl	growth	fert	schl	growth
mort	0.0158	0.0104	0.0278	0.0104	0.0073	0.0258	0.0175	0.0097	0.0438
	0.0031	0.0038	0.0184	0.0022	0.0038	0.0205	0.0030	0.0037	0.0160
	0.0000	0.0060	0.1300	0.0000	0.0560	0.2070	0.0000	0.0090	0.0060
adult1	-0.1845	0.0976	0.4954	-0.0171	0.0905	0.4894	-0.1634	0.0721	0.5755
	0.0210	0.0255	0.1244	0.0171	0.0273	0.1463	0.0196	0.0248	0.1045
	0.0000	0.0000	0.0000	0.3190	0.0010	0.0010	0.0000	0.0040	0.0000
lngnp	0.3442	0.1906	0.2513	0.2316	0.1375	0.8104	0.3471	0.1985	0.6024
	0.1215	0.1475	0.7211	0.0858	0.1500	0.8543	0.1183	0.1364	0.6260
	0.0050	0.1960	0.7270	0.0070	0.3590	0.3430	0.0030	0.1460	0.3360
lagged dep var				0.6386	0.3888	-0.0954			
				0.0349	0.0522	0.0592			
				0.0000	0.0000	0.1070			
constant	12.5436	-2.6264	-33.4922	0.6550	-2.0666	-34.9127	12.9110	-3.1230	-41.6277
	1.6895	2.0515	10.1200	1.4048	2.0340	11.4092	1.4055	1.7705	7.3997
country dummies	yes	yes	yes	yes	yes	yes	yes	yes	yes
time dummies	yes	yes	yes	yes	yes	yes	yes	yes	yes
ar1 disturbance	no	no	no	no	no	no	yes	yes	yes
"R-Sq"	0.94	0.96	0.44	0.97	0.97	0.48			
N obs	336	336	336	288	288	288	336	336	336
N countries	48	48	48	48	48	48	48	48	48

Notes: Numbers below the coefficients are, respectively, standard errors and p-values. Only countries with life expectancy at birth above 50 years in 1960 included in the regressions. Dependent variables are total fertility rate, average schooling in the population above 25, and growth rate of the per capita GNP. Data are averages for five years periods, from 1960 to 1990. Independent variables are child mortality rate per 1,000 (before 1 year), adult life expectancy conditional on survival to 1 year, per capita GNP (in 1995 US$), and country and time fixed effects. Zellner's SUR approach used in all cases, except the AR1 estimations. R-Squared for the SUR estimations is based on the first stage OLS.

Table 7(a): Robustness of the Results to Different Age Mortalities; Selected Sample – Post-transition Countries (42 countries), 1960-1990

	fert	schl	growth	fert	schl	growth	fert	schl	growth
mort	0.0146	0.0116	0.0283	0.0156	0.0150	0.0237	0.0162	0.0152	0.0217
	0.0045	0.0060	0.0247	0.0047	0.0077	0.0282	0.0043	0.0051	0.0179
	0.0010	0.0530	0.2510	0.0010	0.0510	0.4010	0.0000	0.0030	0.2240
oldexp	0.0028	0.1624	0.4501	-0.0109	0.1403	0.3623	-0.0013	0.1223	0.1729
	0.0674	0.0890	0.3671	0.0808	0.1356	0.5010	0.0638	0.0786	0.3050
	0.9670	0.0680	0.2200	0.8930	0.3010	0.4700	0.9840	0.1200	0.5710
adexp	-0.2579	0.0424	0.5158	-0.1325	0.1809	0.9764	-0.2530	0.0700	0.6493
	0.0464	0.0614	0.2531	0.0578	0.0916	0.3525	0.0437	0.0536	0.2026
	0.0000	0.4900	0.0420	0.0220	0.0480	0.0060	0.0000	0.1920	0.0010
lngnp	0.3687	0.2999	-0.1963	0.4562	0.2983	0.9878	0.3891	0.2078	0.3799
	0.1643	0.2171	0.8952	0.1687	0.2807	1.0536	0.1541	0.1892	0.6836
	0.0250	0.1670	0.8260	0.0070	0.2880	0.3480	0.0120	0.2720	0.5780
lagged dep var	no	no	no	yes	yes	yes	no	no	no
count, time dum.	yes	yes	yes	yes	yes	yes	yes	yes	yes
ar1 disturbance	no	no	no	no	no	no	yes	yes	yes
"R-Sq"	0.92	0.95	0.53	0.93	0.96	0.69			
N obs	168	168	168	126	126	126	168	168	168
N countries	42	42	42	42	42	42	42	42	42

Notes: Numbers below the coefficients are, respectively, standard errors and p-values. Only countries with life expectancy at birth in 1960 above 50 years included. Dependent variables are total fertility rate, average schooling in the population above 25, and growth rate of the per capita GNP. Data are averages for ten year periods, from 1960 to 1990. Independent variables are child mortality rate per 1,000 (before 1 year), expected years of adult life between 15 and 60, life expectancy conditional on survival to 60, per capita GNP (in 1995 US$), and country and time fixed effects. Zellner's SUR approach used in all cases, except the AR1 estimations. R-Squared for the SUR estimations is based on the first stage OLS.

Table 7(b): Robustness of the Results to Different Age Mortalities (Child Mortality up to 5 years); Selected Sample - Post-transition Countries (42 countries), 1960-1990

	fert	schl	growth	fert	schl	growth	fert	schl	growth
mort5	0.0104	0.0012	0.0090	0.0107	0.0034	-0.0053	0.0112	0.0053	0.0133
	0.0030	0.0039	0.0164	0.0034	0.0052	0.0195	0.0028	0.0032	0.0112
	0.0010	0.7570	0.5800	0.0020	0.5170	0.7860	0.0000	0.0990	0.2330
oldexp5	-0.0011	-0.0431	-0.1578	-0.0346	-0.0291	-0.4008	-0.0181	-0.0758	-0.2993
	0.0607	0.0778	0.3269	0.0737	0.1156	0.4339	0.0552	0.0647	0.2603
	0.9850	0.5800	0.6290	0.6390	0.8020	0.3560	0.7430	0.2410	0.2500
adexp	-0.2119	0.0421	0.8480	-0.0556	0.1180	1.4788	-0.2027	0.0765	0.9153
	0.0434	0.0556	0.2338	0.0605	0.0890	0.3449	0.0399	0.0469	0.1756
	0.0000	0.4490	0.0000	0.3590	0.1850	0.0000	0.0000	0.1030	0.0000
lngnp	0.6607	0.1251	-0.0555	0.6785	0.0657	1.9515	0.6885	-0.0266	0.4581
	0.1748	0.2239	0.9412	0.1787	0.2817	1.0605	0.1650	0.1901	0.6909
	0.0000	0.5760	0.9530	0.0000	0.8160	0.0660	0.0000	0.8890	0.5070
lagged dep var	no	no	no	yes	yes	yes	no	no	no
count, time dum.	yes	yes	yes	yes	yes	yes	yes	yes	yes
ar1 disturbance	no	no	no	no	no	no	yes	yes	yes
"R-Sq"	0.92	0.96	0.54	0.93	0.97	0.71			
N obs	148	148	148	111	111	111	148	148	148
N countries	37	37	37	37	37	37	37	37	37

Notes: Numbers below the coefficients are, respectively, standard errors and p-values. Only countries with life expectancy at birth in 1960 above 50 years included.. Dependent variables are total fertility rate, average schooling in the population above 25, and growth rate of the per capita GNP. Data are averages for ten years periods, from 1960 to 1990. Independent variables are child mortality rate per 1,000 (before 5 years), expected years of adult life between 15 and 60, life expectancy conditional on survival to 60, per capita GNP (in 1995 US$), and country and time fixed effects. Zellner's SUR approach used in all cases, except the AR1 estimations. R-Squared for the SUR estimations is based on the first stage OLS.

Figure 1: Relationship Between Per Capita Income and Life Expectancy at Birth - Transitional Countries (1960-95)

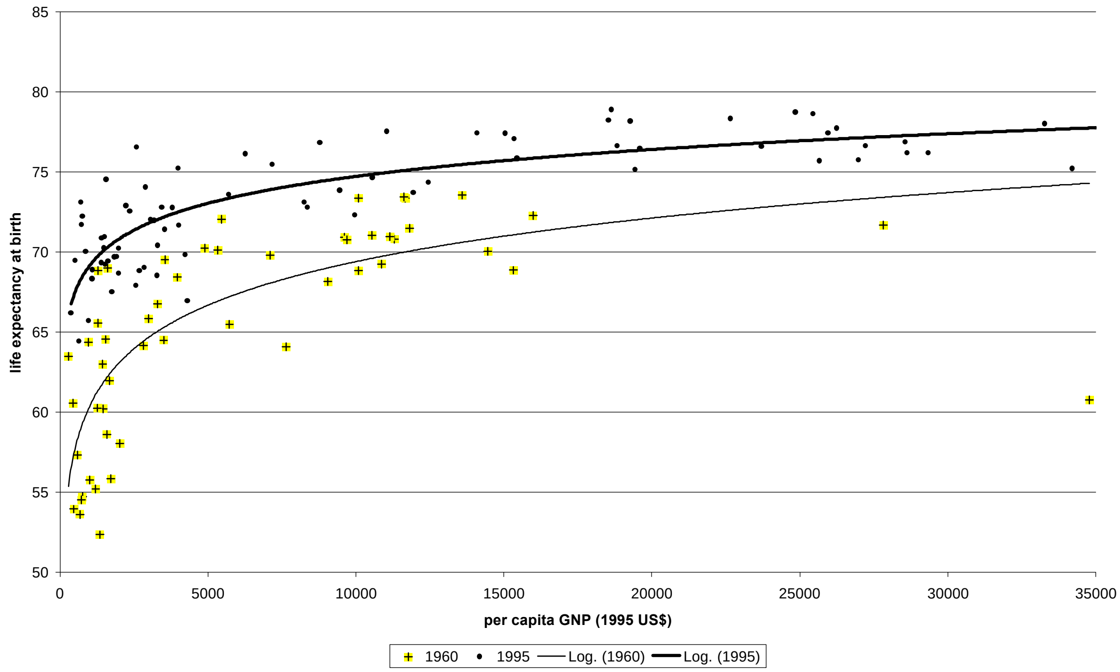

Figure 2: Relationship Between Per Capita Income and Fertility - Transitional Countries (1960-95)

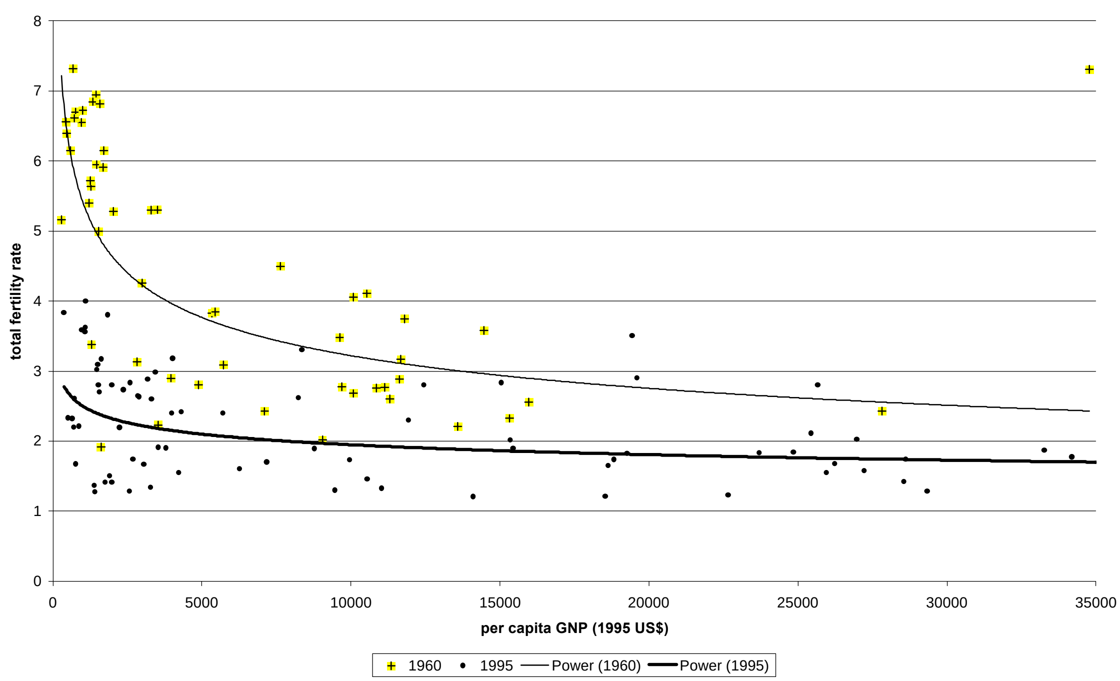

Figure 3: Relationship Between Per Capita Income and Schooling - Transitional Countries (1960-90)

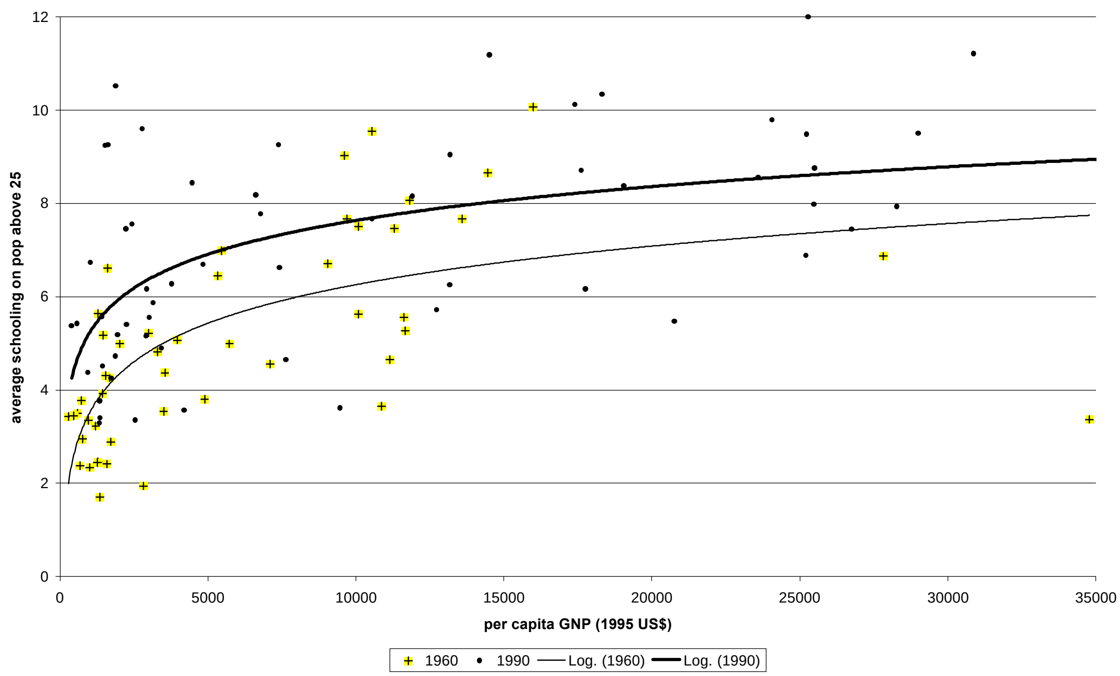

Figure 4: Relationship Between Life Expectancy and Fertility - Transitional Countries (1960-95)

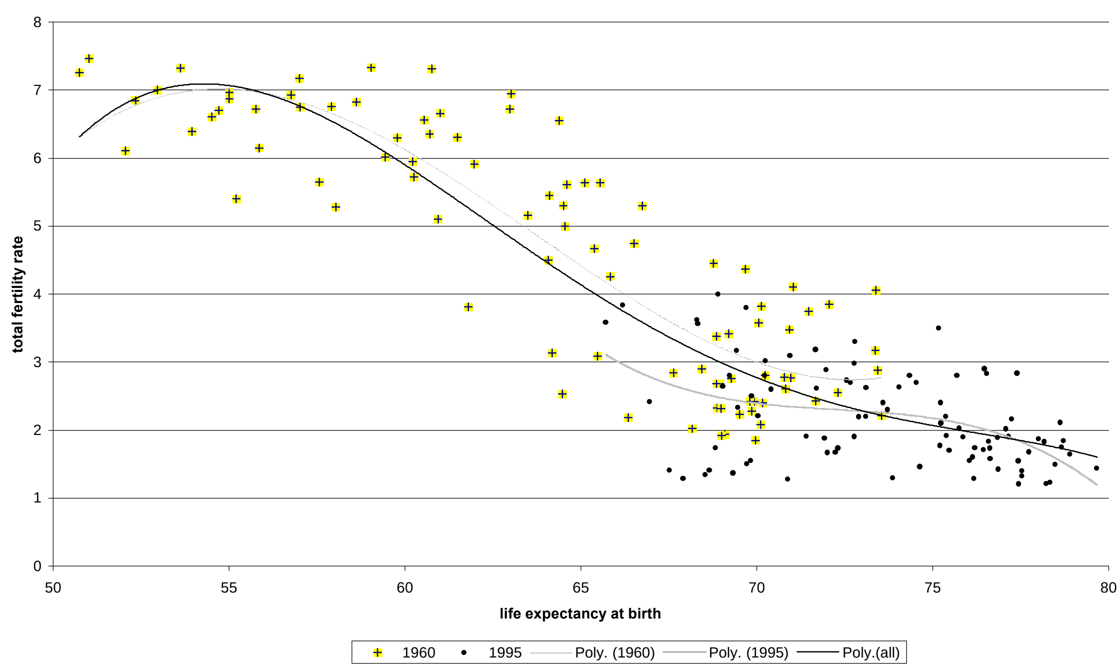

Figure 5: Relationship Between Life Expectancy and Schooling - Transitional Countries (1960-90)

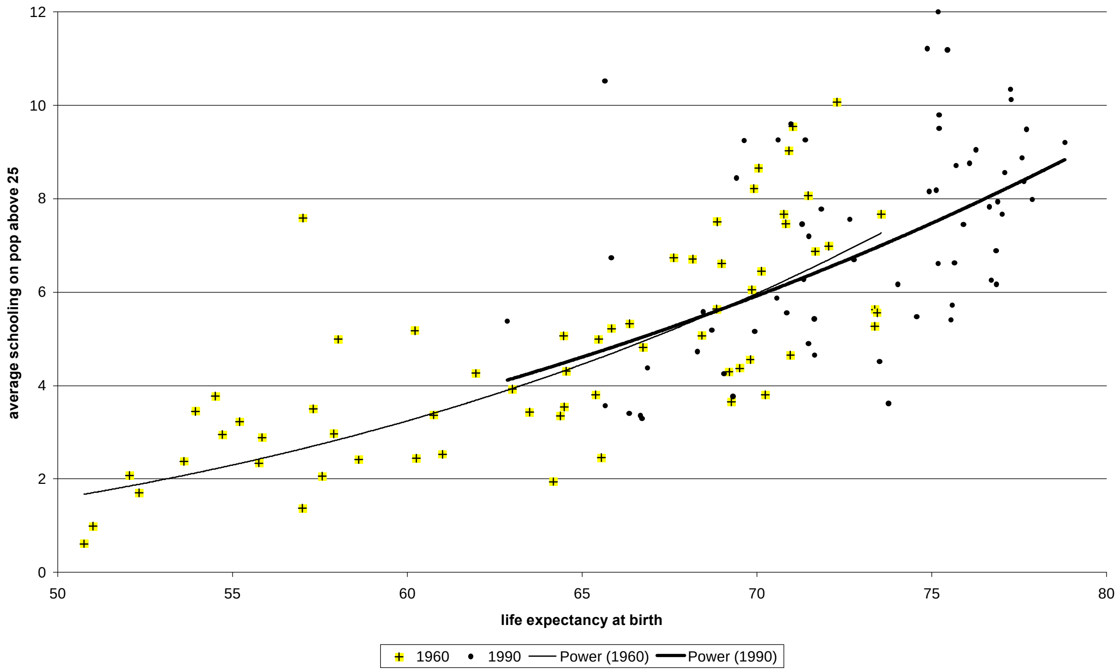

Figure 6: Demographic Transition in England

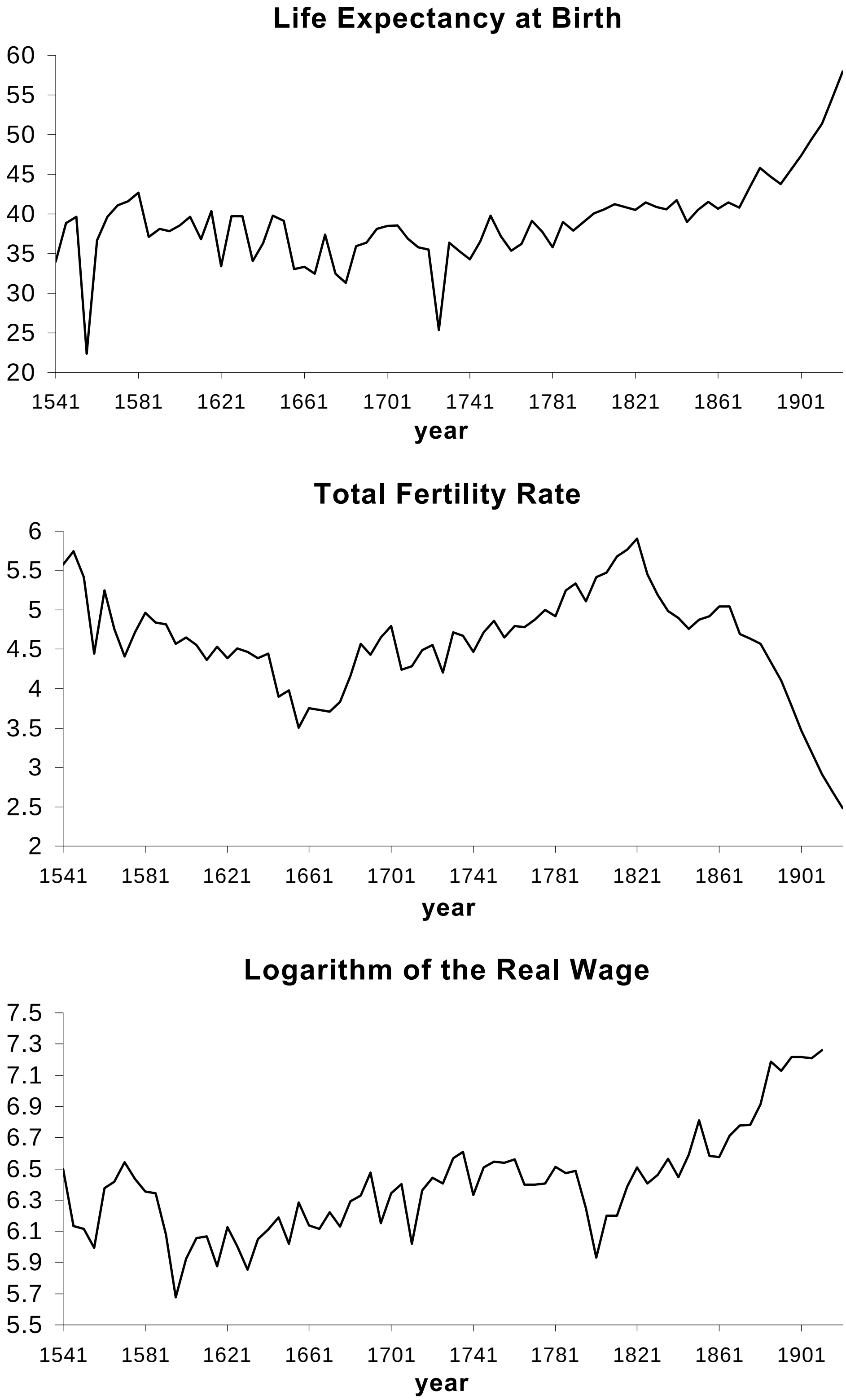

Figure 7: Demographic Transition in Sweden

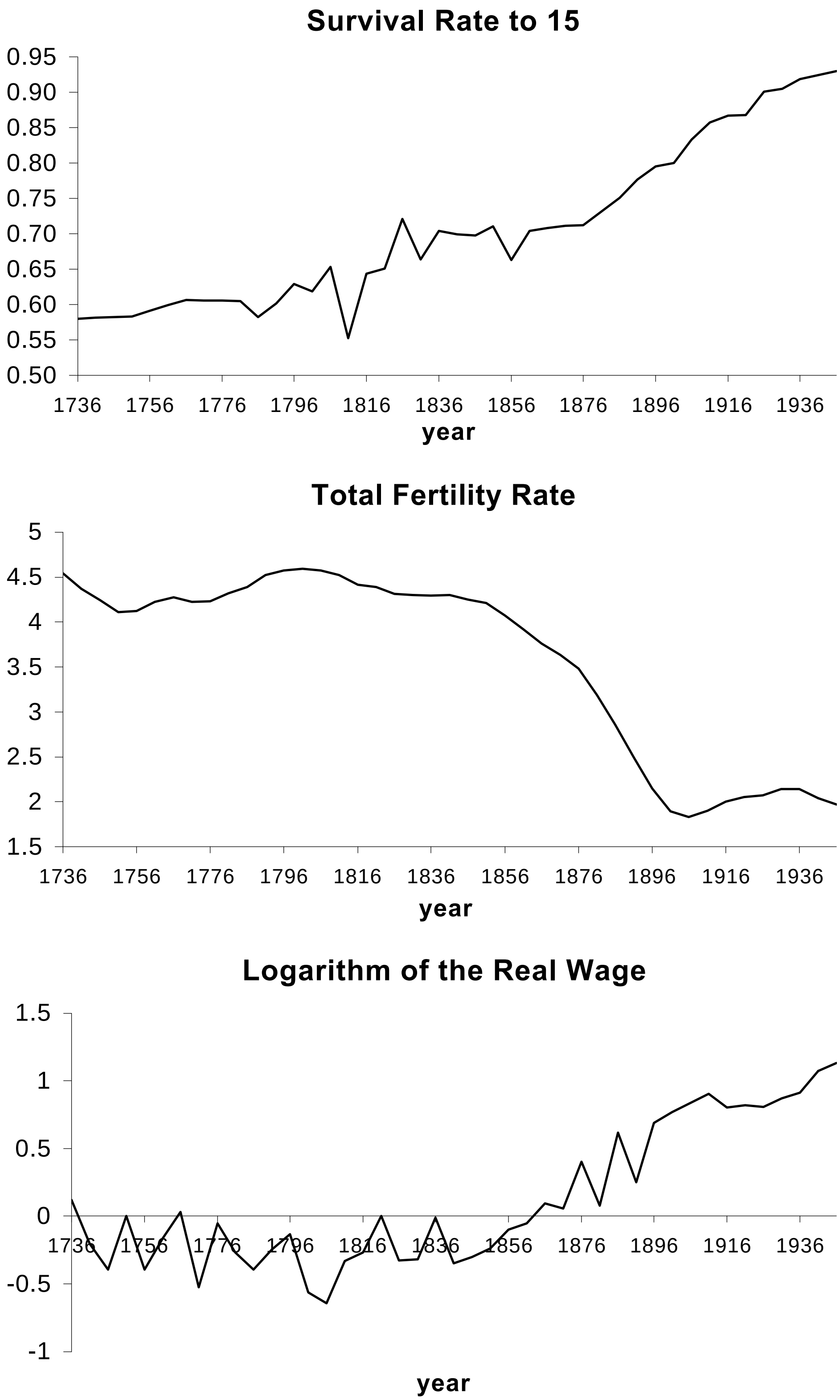

Figure 8: Total fertility rate and life expectancy at birth, transitional African countries, 1960-1995

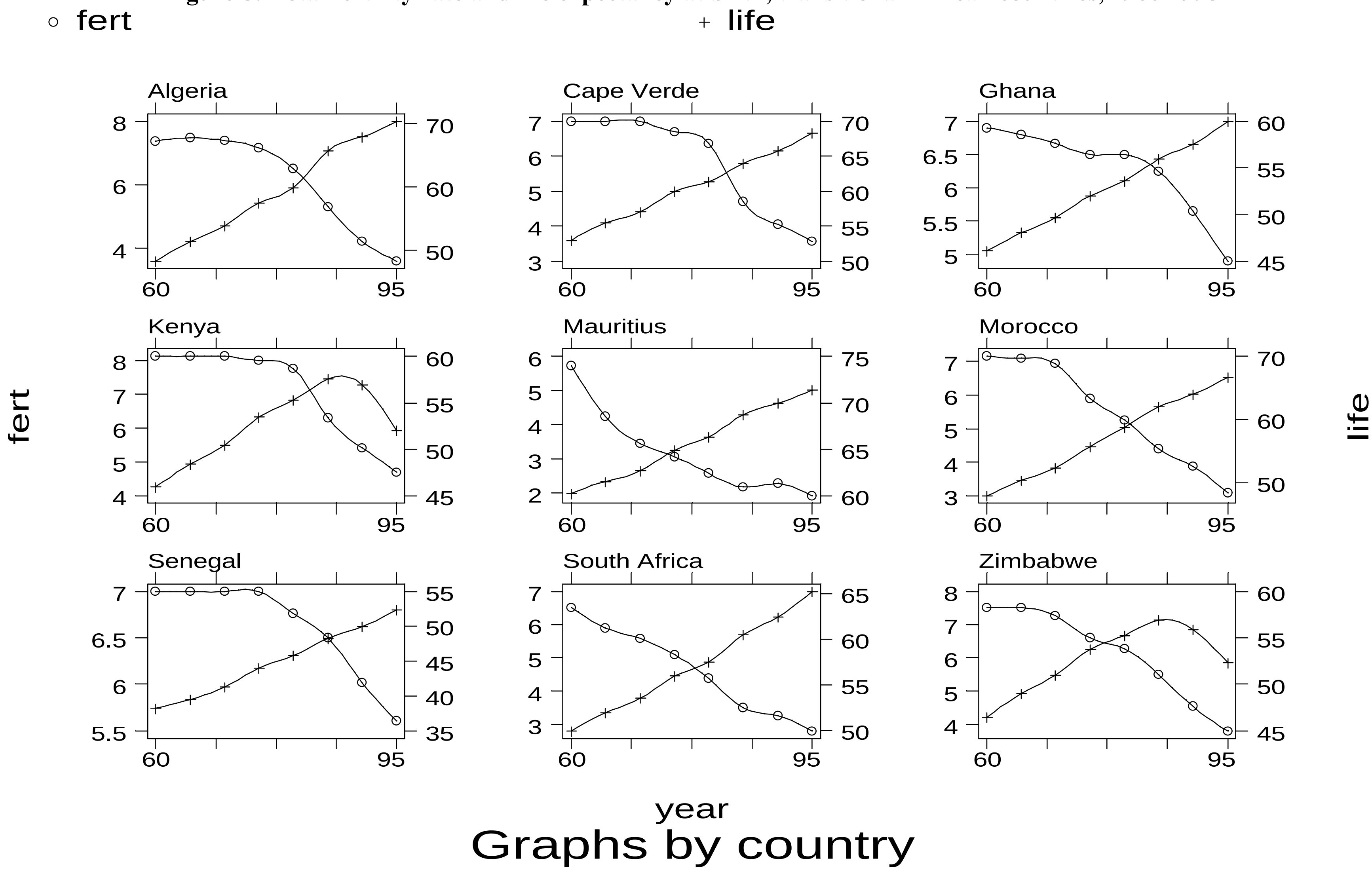

Figure 9: Total fertility rate and life expectancy at birth, Asian countries, 1960-1995

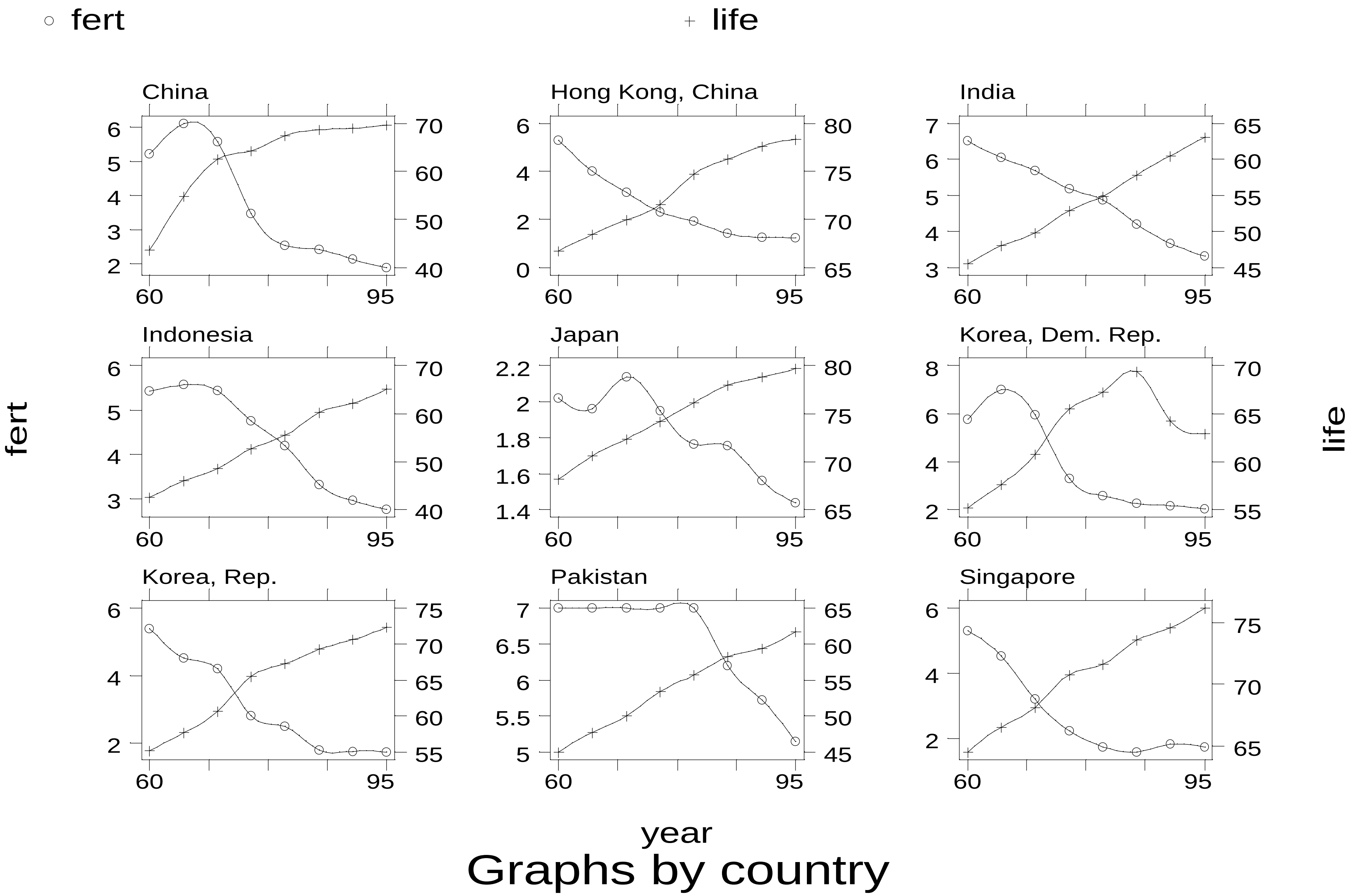

Figure 10: Total fertility rate and life expectancy at birth, Latin American countries, 1960-1995

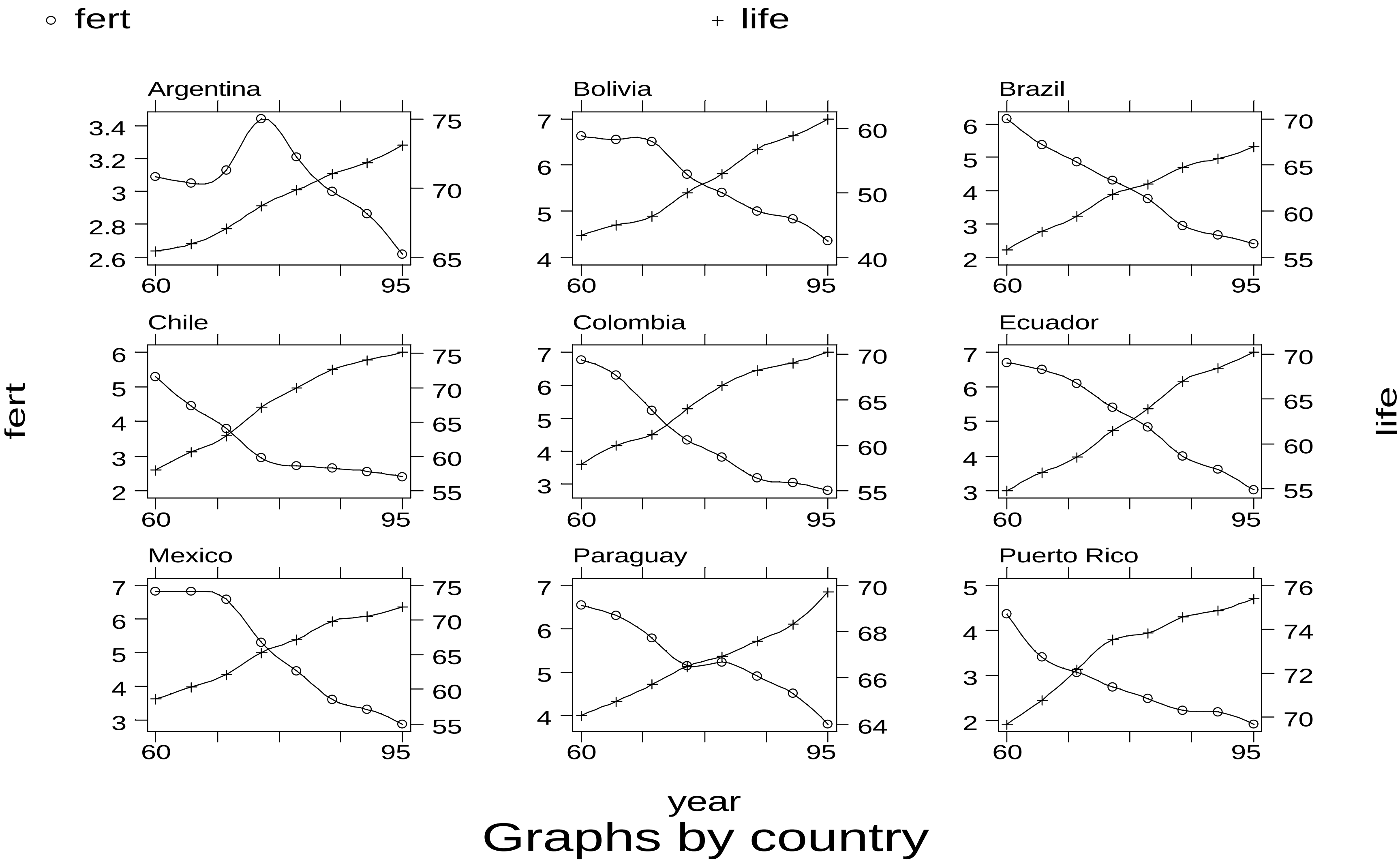

Figure 11: Total fertility rate and life expectancy at birth, non-transitional African countries, 1960-1995

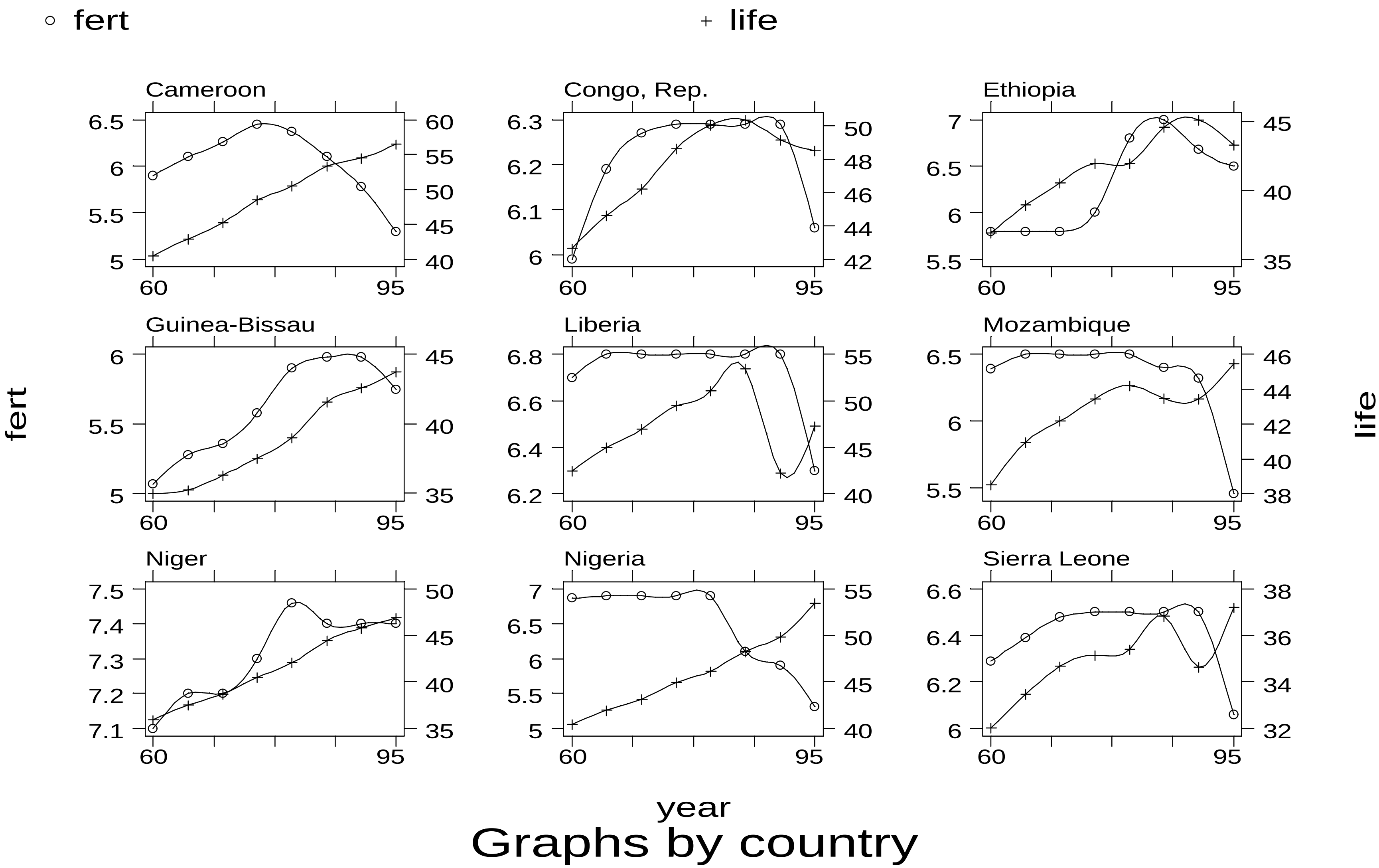

Figure 12: Fertility Before and After the Year when Life Expectancy at Birth Reached 50

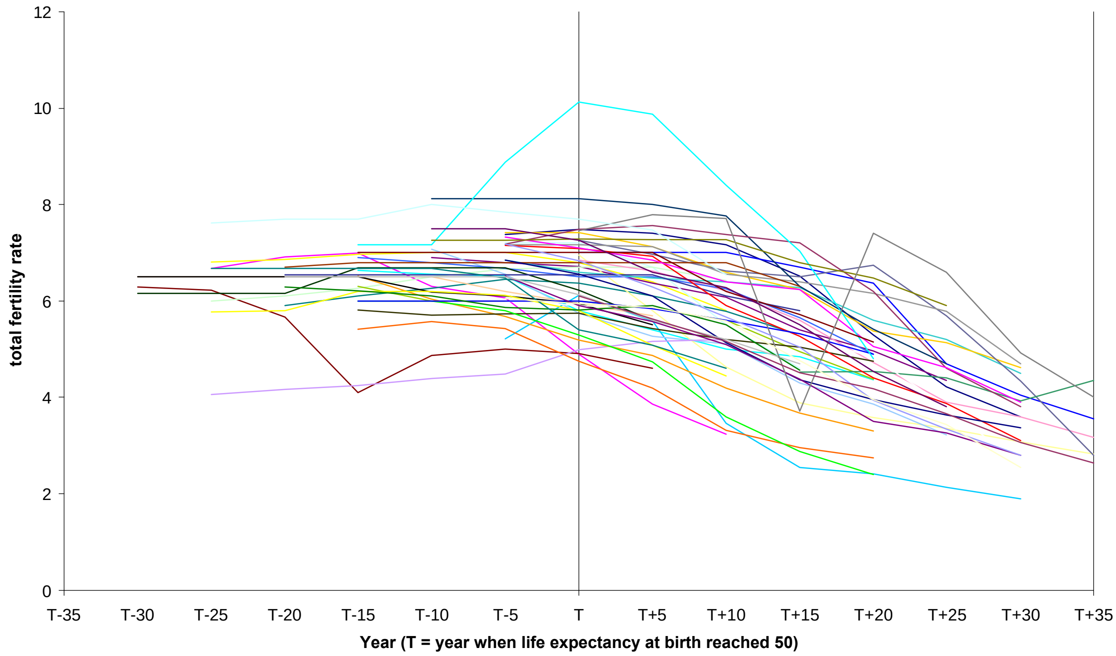

Figure 13: Fertility Deviation from Initial Transitional Level Before and After the Year when Life Expectancy at Birth Reached 50

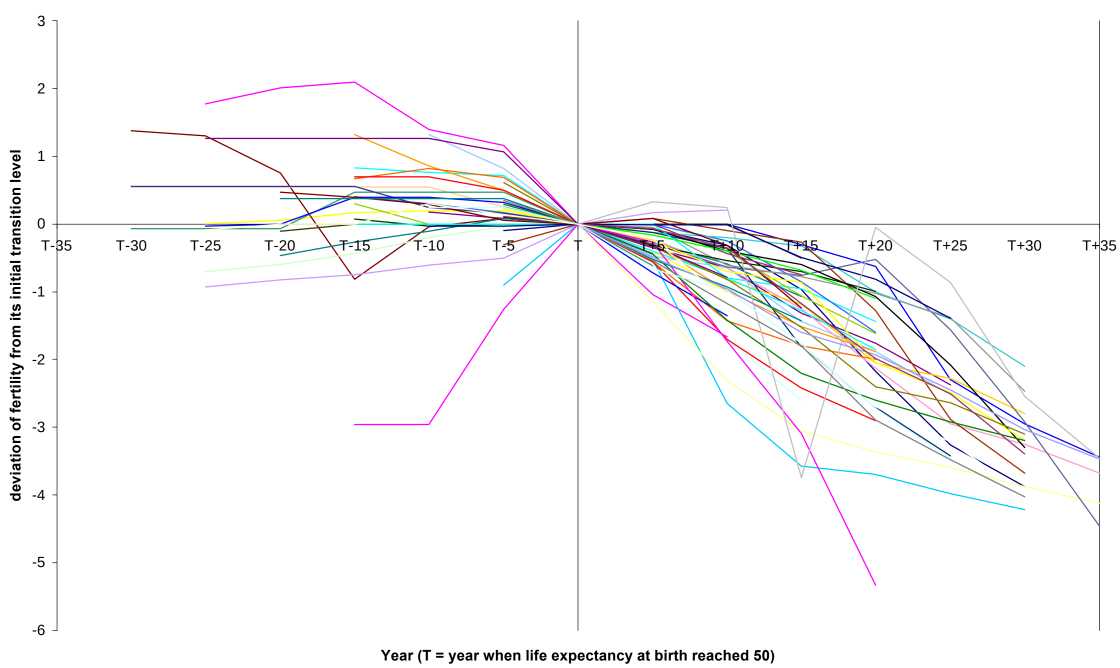

Figure 14: Schooling Before and After the Year when Life Expectancy at Birth Reached 50

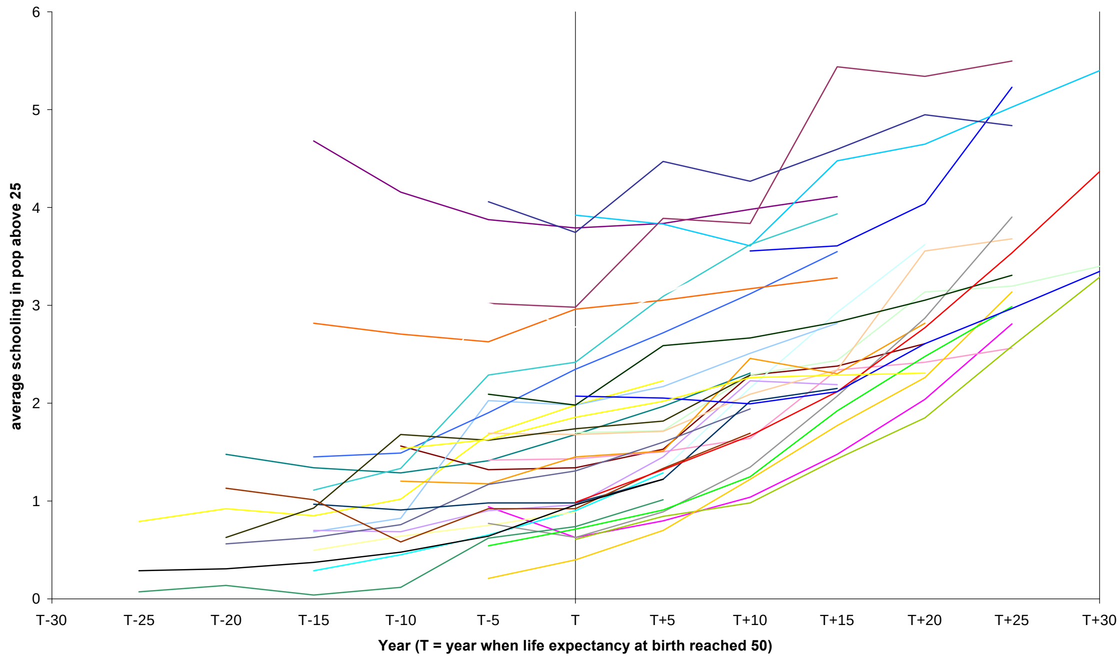

Figure 15: Schooling Deviation from Initial Transitional Level Before and After the Year when Life Expectancy at Birth Reached 50