Electoral control in the presence of gridlocks*

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Abstract

This article presents a game-theoretic partisan model of voting and political bargaining. In a two-period setup, voters first elect an executive incumbent and the legislators from a pool of candidates belonging to different parties. Once elected, the executive and the legislature bargain over a budget. Party origin and a relevant parameter of the economy, the state of the world, influence the bargaining cost, such that political gridlocks may occur. At the end of the first period voters observe the outcome of bargaining but do not observe the true estate of the world, and decide whether or not to reelect the same parties for the Executive and the Legislature. The model confirms the very recent literature by showing that voters tend to have more flexible reelection criteria when they believe the true state of the world is likely to be unfavorable. On the other hand, when voters believe the true state of the world is likely to be favorable, they become more demanding in order to reelect the incumbents. In particular, there will be government shutdown with positive probability in equilibrium. Gridlocks occur due to the imperfect information of voters and they constitute indeed an information revelation mechanism that improves electoral control in the second period.

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1 Introduction

The literature on voting highlights two instruments of electoral control: reelection and vote splitting. The reelection approach to electoral control establishes that, in order to be reelected, an incumbent is required to produce a minimum level of social output, according to a rule that is optimally set by voters, in an electoral game. Barro (1973) uses this approach to conclude that voters can restrict the incumbent level of overspending in a general equilibrium model where government tax citizens in order to finance its expenditure. Ferejohn (1986) shows that the reelection connection may induce the incumbent into putting a higher personal effort level –which is costly– into his administration.

Therefore, reelection models usually focus on the period just before a new term. On the other hand, the vote-splitting literature focuses in what happens after a politician is elected, and incorporates particular features of the decision making process once election has occurred, like bargaining structures. Fiorina's articles (1986, 1988, 1992) show that the rigidity of political parties' ideological positions makes it best for the voters at the center of the political spectrum to split their ticket, inducing a more moderate political outcome. This argument is made more precise in Alesina and Rosenthal's studies (1989, 1995, 1996), where a very elegant game theoretic model of probabilistic voting concludes on the optimality of vote splitting. Finally, Bugarin (1999) focuses on the role of uncertainty and shows that, even though voters may prefer a party over another in a deterministic world, when a stochastic process affects voters' utilities, then vote splitting may be optimal in order to insure society against very extreme policies in bad states of the world.

Although the above instruments are both available to voters in real world elections, the traditional literature fails to analyze them in an integrated framework. A recent exception is Bugarin (2000) which studies a model where voters recognize that an Executive incumbent has to bargain with the Legislature in order to pass a budget. The incumbent receives "political income" from overspending, but passing an expensive budget is costly, the more so the stronger is the opposition in the Legislature. The true competitive cost of the budget is a random variable which is observed only by the Executive incumbent. The main result of the article is that voters tend to be more demanding (on the level of social output generated by government) in order to reelect an incumbent when they expect the true state of the world to be "favorable", whereas they become more flexible if they believe the true state of the world is "unfavorable". Moreover, vote splitting will be optimal in the good state of the world whereas unified government will be the choice of the electorate in the bad state.

The referred article highlights the basic trade-offs between the two instruments of electoral control: reelection and vote splitting. Nevertheless, many features of real world political negotiation are abstracted from the model. In particular it assumes that the executive incumbent can always pass the budget, if he pays for its political costs. However, political gridlocks do occur when a government cannot have its budget proposal supported by the Legislature, as it happened during the first Clinton administration in the United States. What are the effects of political gridlocks on voters' bahavior? How do gridlocks affect the trade-off between reelection and vote splitting? Is the basic model in Bugarin (2000) robust to the presence of this type of political immobility?

The objective of the present article is to explore that issue. The next section extends the basic Bugarin (2000) model allowing for political gridlocks. The electoral control game is then solved in section 3, showing that political gridlocks occur with positive probability. Gridlocks arise in equilibrium as an information revelation device, that permits voters to determine the true state of the world, before they take their reelection decisions. Moreover, the possibility of gridlocks reduces the scope of vote splitting as an electoral control mechanism. Section 4 analyses the role of political "efficiency" of the incumbent, showing that, in the present context, having more "capable" politicians may not be good for society. Section 5 explores the equilibrium effects of considering a more general framework, the multidimensional case, where politicians can choose among a number of different projects for the budget proposal. Finally, section 6 presents some concluding remarks.

2 The electoral-control game

There are two periods. Voters elect the Executive and the Legislature at each period. Once elected, the Executive incumbent proposes a budget of the form (n, p) where p is the unit cost and n is the number of copies of a project to be implemented. Then the Executive officeholder and the legislators bargain over the proposed project.[1]

The project has a real cost r, corresponding to competitive factors of production, which is the private information of the proposer. If $p > r$, then there is an overpayment $e = p - r$ from which the Executive extracts political income. A factor $\beta \in (0, 1)$ describes this political income in that $n\beta e$ is the utility gain to the Executive from project (n, p). The coefficient β can be interpreted as the Executive corruption factor associated to project (n, p). The choice of proposal (n, p) is restricted by a budget constraint which requires total spending not to be higher than a certain amount B, *i.e.*, $np \leq B$.

Voters elect the Executive incumbent and the Legislators from two identical parties, I and II. The bargaining process among elected officials is summarized by a cost function c to the proposer, which depends both on the proposed cost p and the representation $1 - \pi$ of the Executive incumbent's party in the Legislature, *i.e.*, $c = c(p, \pi)$ where π is the proportion of the Legislative opposition to the Executive incumbent's party.

Therefore, if the incumbent passes a budget (n, p), her resulting utility is:

$$v(n, p, \pi) = n\beta(p - r) - c(p, \pi) \tag{1}$$

The cost function c is assumed to be multiplicatively separable on its arguments:

[1] A more general model would assume a multidimensional budget including many different projects: $(N, P) = \{(n_1, p_1), \ldots, (n_k, p_k)\}$ where p_i is the unit cost and n_i is the number of copies of project i, $i = 1, \ldots, k$. Most of the present article abstracts from multidimensionality for tractability reasons; that simpler approach can be supported by two different arguments: first, the electoral campaign may be polarized around one main issue; second, the political decision making in the Legislature may be constrained by a committee structure where each committee decides on a particular project, as in the structure induced equilibria literature (see Shepsle, 1979 and Shepsle & Weingast, 1981). Section 5 presents a preliminary exploration of the multidimensional case.

$c(p, \pi) = f(p) \cdot g(\pi)$. The function f represents the general opposition of the Legislature to costly projects, whereas the function g captures the cost of the ideological opposition of competing parties in the Legislature. The functions f and g are assumed to be strictly increasing, strictly convex and continuously differentiable functions with $f(0) = 0$.

Note that, depending on the final cost of passing budget (n, p), the incumbent may prefer not to submit that project. This will happen if $v(n, p, \pi) < 0$. If the cost $c(p, \pi)$ is so high that the incumbent has negative utility $v(n, p, \pi)$ for every project (n, p), then she will prefer not to submit any project at all. This corresponds to a political gridlock, in which case a basic reversionary budget is implemented. For simplicity, the model assumes without loss of generality that neither the executive incumbent, nor the voters derive any utility from this reversionary budget.

Voters are uncertain about the true unit cost of the projects, r, but know that it must be one of two possible values: $r = l$ or $r = h$ with $l < h$. The cost $r = l$ corresponds to a "good" state of the world, in that it is cheap to implement the projects, whereas $r = h$ corresponds to a "bad" state in that the projects are costly. Voters assign probability $\rho \in (0, 1)$ to the state $r = l$ and $1 - \rho$ to $r = h$.

At each period, voters derive utility from the social return of the implemented budget (n, p), which is measured in terms of the number of projects implemented. This social return is given by the real-valued function $\varphi(n)$, which is assumed to be concave and strictly increasing with $\varphi(0) = 0$. Therefore, voters want to induce the Executive incumbent to pass a budget with a high value for n.

Figure 1 presents an extensive form of the electoral-control game played by the voters and the politicians. At the beginning of period 1 the state of the world r is realized but not observed by the voters, who elect the Executive incumbent (from party I or II) and the proportion of the opposing party in the Legislature, π_1. The elected Executive observes r and decides whether to pass a budget or not. If a budget is not passed, a reversionary policy rule applies and all agents receive (normalized) zero utilities; in the figure this corresponds to the choice $n_1 = p_1 = 0$. If the budget (n_1, p_1) is passed, the corresponding first period utilities are derived according to expression (1). The index 1 in the figure corresponds to the first period.

Period 2 starts with voters deciding whether to reelect the Executive incumbent or not, as well as the proportion of the opposition party in the Legislature, π_2. Then, like in period 1, the elected Executive decides whether to pass a budget or not. If a budget is not passed –in which case $n_2 = p_2 = 0$– a reversionary policy rule applies and all agents receive (normalized) zero utilities. If the budget (n_2, p_2) is passed, the corresponding second period utilities are derived according to (1). Finally, the game concludes.

The utility of a representative voter is $\varphi(n_1) + \delta\varphi(n_2)$ where $\delta \in (0, 1)$ is the intertemporal discount factor, common to all agents.

The utility of a reelected Executive incumbent is $v(n_1, p_1, \pi_1) + \delta v(n_2, p_2, \pi_2)$. The utility of an incumbent that is elected only at period 1 is $v(n_1, p_1, \pi_1)$ and the present value of the utility of an Executive that is elected only at period 2 is $\delta v(n_2, p_2, \pi_2)$.

For simplicity, the game tree presents one generic choice among infinitely many at each node; for example, when voters (V) elect an incumbent of party I in period 1, they can choose any proportion of party II representatives in the Legislature: $\pi_1 \in [0, 1]$.

3 Solving the electoral-control game

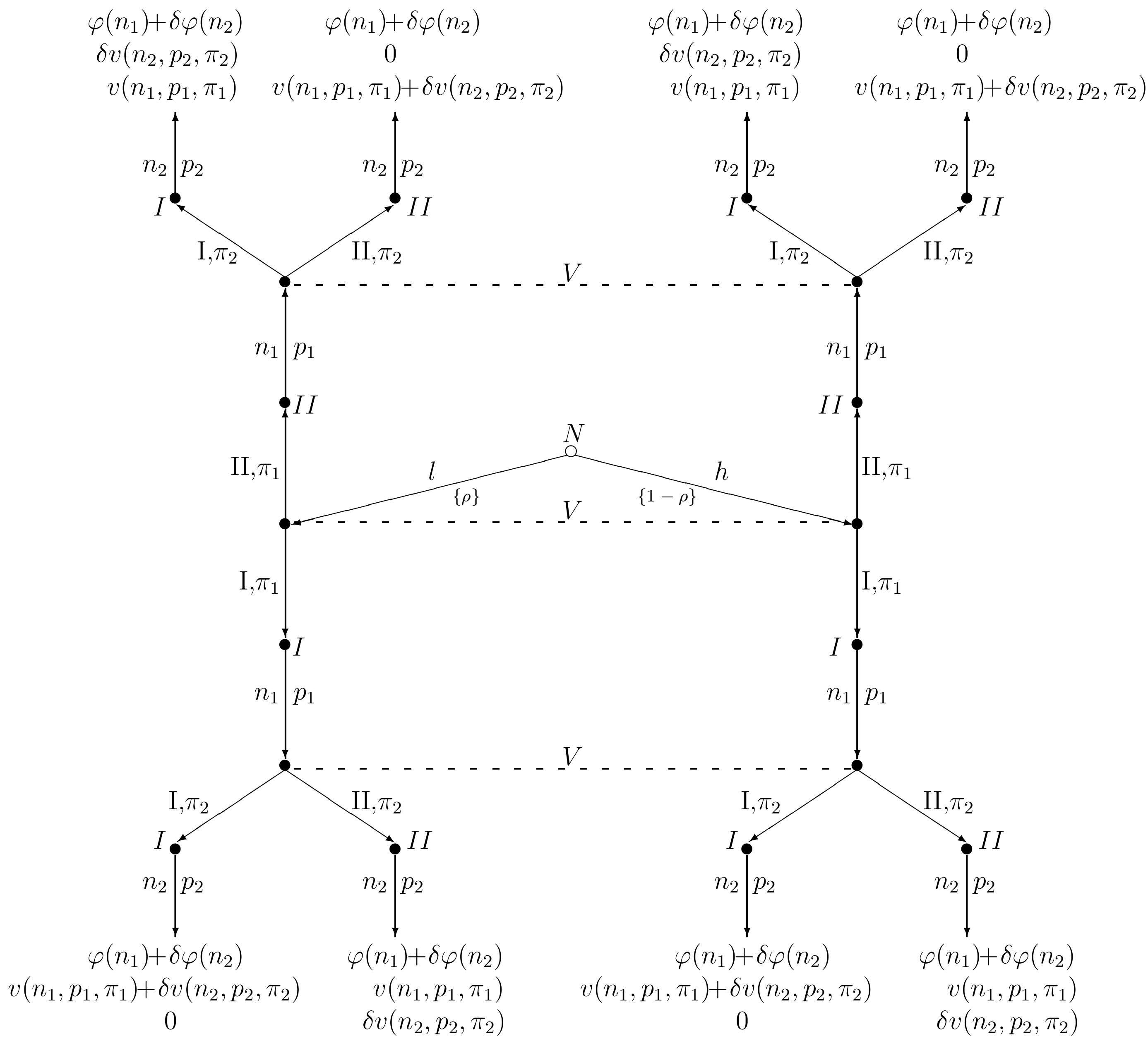
The natural solution concept for this model is perfect Bayesian equilibrium, given the existence of imperfect information. Therefore, the solution process starts looking for sequentially rational strategy profiles.

3.1 The second period

In period 2 an elected incumbent solves the following maximization problem.

$$\max_{n,p} v(n, p, \pi_2) = n\beta(p - r) - c(p, \pi_2)$$
$$\text{s.t. } np \leq B$$

Figure 1: The extensive form of the electoral-control game



In the above problem, a proposal $(n,p) \neq (0,0)$ with $np < B$ is strictly dominated by the feasible proposal (n',p) where $n' = B/p$, *i.e.*, in equilibrium the budget constraint holds with equality. Therefore, writing $e = p - r$, the Executive incumbent's maximization problem can simply be written as:

$$\max_{e \geq 0} v(r,e,\pi_2) = \beta B \frac{e}{r+e} - c(r+e,\pi_2)$$

The notation in the previous problem stresses the dependence of the utility of the incumbent on the true estate of the world r and on the overspending level e, rather than on n and p as before. Is is straightforward to show that there exists a unique solution, $\hat{e}(r,\pi_2)$ for the above problem.[2] The incumbent's optimal choice is then:

$$\hat{n}_2(r,\pi_2) = \frac{B}{r+\hat{e}(r,\pi_2)}, \quad \hat{p}_2(r,\pi_2) = r + \hat{e}(r,\pi_2) \qquad \text{if} \qquad v(r,\hat{e}(r,\pi_2),\pi_2) \geq 0,$$

$$\hat{n}_2(r,\pi_2) = 0 = \hat{p}_2(r,\pi_2) \qquad \text{(no proposal at all)} \qquad \text{otherwise.}$$

It is assumed in this article that there is always an interior solution $\hat{e}(r,\pi_2) > 0$ to the incumbent's aximization problem. The following proposition characterizes that solution as a function of the opposition in the Legislature, π_2. Proofs to all propositions in this article are presented in the appendix.

Proposition 1 *The optimal overspending level $\hat{e}(r,\pi_2)$ of an incumbent at period 2 is a strictly decreasing function of π_2.*

In order to determine the optimal level of vote splitting in period 2, it is necessary to establish how the indirect utility function $v(r,\hat{e}(r,\pi_2),\pi_2)$ changes as π_2 changes. The following proposition answers that question.

Proposition 2 *The indirect utility function $v(r,\hat{e}(r,\pi_2),\pi_2)$ is a decreasing function of the opposition level π_2.*

[2] Note that $h(e) = \beta B \frac{e}{r+e}$ is strictly concave and $k(e) = c(r+e,\pi_2)$ is strictly convex; moreover, $h'([0,+\infty)) = (0, \beta B/r]$ and $k'(e)$ is unbounded.

Therefore, in period 2 voters have two basic concerns.

On one hand, by Proposition 1, they want to choose the highest possible level of opposition, since increasing opposition reduces the overspending $\hat{e}$ which in turn increases $\hat{n} = \frac{B}{r+\hat{e}}$ and consequently increases voters' utility $\varphi(\hat{n})$.

On the other hand, Proposition 2 states that as π_2 increases, the corresponding indirect utility decreases. If it becomes negative, then the incumbent will choose $(n, p) = (0, 0)$ and gridlock will result, yielding zero utility to voters. Therefore, if voters could observe the true state of the world, r, they would choose $\pi_2 \equiv \pi_r$ to be the highest opposition in the Legislature that gives nonnegative utility to the incumbent. Thus, one of the two situations below must arise.

$\pi_r = 1 \qquad$ if $\qquad v(r, \hat{e}(r, 1), 1) \geq 0$

$\pi_r < 1 \qquad$ if $\qquad v(r, \hat{e}(r, 1), 1) < 0$. In this case:

$$\begin{array}{lll} v(r, \hat{e}(r, \pi_2), \pi_2) \geq 0 & \text{for} & \pi_2 \leq \pi_r \\ v(r, \hat{e}(r, \pi_2), \pi_2) < 0 & \text{for} & \pi_2 > \pi_r \end{array}$$

In order to determine the effect of the state of nature on the optimal level of vote splitting at period 2 one needs to be able to compare π_h to π_l. The following proposition resolves that issue.

Proposition 3 *The indirect utility function $v(r, \hat{e}(r, \pi_2), \pi_2)$ is a strictly decreasing function of r:*

$$v(l, \hat{e}(l, \pi_2), \pi_2) > v(h, \hat{e}(h, \pi_2), \pi_2)$$

The previous proposition shows that the incumbent has always a utility advantage in the good state if compared to the bad state of nature. In particular, if $\pi_h < 1$ then,

$$v(l, \hat{e}(l, \pi_h), \pi_h) > v(h, \hat{e}(h, \pi_h), \pi_h) = 0$$

Therefore, by Proposition 2, $\pi_l > \pi_h$. Clearly, if $\pi_h = 1$ then also $\pi_l = 1$.

The case where $\pi_h = \pi_l = 1$ is an extreme solution where total vote-splitting is optimal, regardless of the state of the world. This will happen, for example, if there is a significant unbalance of political power towards the Executive incumbent, in such a way that it can always pass a utility-enhancing budget even if the entire Legislature is opposing him. That case reflects some institutional weakness in society in the sense that the Legislature in unable to impose strong opposition costs to the Executive incumbent, even when totally dominated by opposing parties. This is typically the case in authoritarian regimes, and may explain why in the seventies in Brazil, the military regime witnessed an incredible increase in the seats occupied by the opposing party, the *MDB: Movimento Democrático Brasileiro.* That situation has been explored in a previous article (Bugarin (2000)) and will not be considered here. Instead, in what follows it is assumed that total vote splitting is not always a dominant strategy, *i.e.*, $\pi_h < 1$ and $\pi_l = 1$. This more intuitive assumption leads to the conclusion that voters will impose more opposition to the Executive incumbent when they know the state of the world is favorable. Equivalently, voters tend to be more sympathetic towards the incumbent when a bad state of the world is realized. The corollary below summarizes the above results.

Corollary 1 *If voters could observe the true state of the world in period 2, they would choose the level of vote splitting according to the following rule:*

(i) $\pi_2 = \pi_l = 1 \qquad \text{if} \qquad r = l.$

(ii) $\pi_2 = \pi_h < 1 \qquad \text{if} \qquad r = h.$

An important consequence of this result is that it is optimal for the voters to reinforce the opposing party in the Legislature in the last period, *i.e.*, some degree of vote splitting at $t = 2$ is optimal regardless of the state of nature. That result is intuitive: in the last period reelection cannot be used in order to induce a more favorable outcome, therefore vote splitting becomes the only control mechanism voters are left with. However, the possibility of gridlock caused by a too strong opposition in the Legislature reduces the optimal level of vote splitting in the last period, in the bad state $r = h$.

Notice that voters are indifferent at period 2 between an Executive from party I or from party II, as long as the proper opposition in the Legislature is chosen. Therefore, they can credibly make their decisions of reelecting an incumbent contingent on that incumbent's choice n_1 at period 1. Given this strategic opportunity, voters' optimal reelection strategy will be given by a threshold number n^* such that the Executive incumbent is reelected if and only if $n_1 \geq n^*$.

3.2 The first period

Given that parties are essentially identical, any choice for the incumbent's party at period 1 is optimal, as long as π_1 and n^* are chosen properly. The choice of π_1 and n^* will depend upon voters' beliefs ρ about the state of the world, as described below.

Suppose voters could observe $r = h$. Then sequential rationality would require voters to choose $\pi_2 = \pi_h$, i.e. partial vote splitting, at period 2. In that case, an incumbent would have zero utility at the second period, so that the reelection connexion would be lost. Therefore, voters would choose also $\pi_1 = \pi_h$ in the first period. Given that level of opposition, the incumbent would pass the budget $\hat{n}_h = \dfrac{B}{h + \hat{e}(h, \pi_h)}$, $\hat{p}_h = h + \hat{e}(h, \pi_h)$, derive zero utility in the first period, and would not be concerned about satisfying any reelection criterion n^* since, if reelected, he would have zero utility in the second period as well.

On the other hand, suppose voters could observe $r = l$. Then sequential rationality would require voters to choose $\pi_2 = 1$, i.e. total vote splitting, at period 2. Therefore, if reelected, the incumbent would obtain a positive utility in the last period: $v(l, \hat{e}(l, 1), 1) > 0$. Hence, given π_1, voters would be able to induce a minimal level of overpayment $\tilde{e}(l, \pi_1)$ by setting $n^* = \dfrac{B}{l + \tilde{e}(l, \pi_1)}$ where $\tilde{e}(l, \pi_1)$ is the minimal value of e such that:

$$v(l, e, \pi_1) + \delta v(l, \hat{e}(l, 1), 1) = v(l, \hat{e}(l, \pi_1), \pi_1)$$

That is, e is selected in such a way that an incumbent is indifferent between choosing her optimal $\hat{e}(l, \pi_1)$ at period $t = 1$ and not being reelected or choosing the lower[3] $\tilde{e}(l, \pi_1)$ at $t = 1$, being reelected, and choosing $\hat{e}(l, 1)$ at the second period. By differentiating the

[3]By construction $v(l, e, \pi)$ is a strictly concave, nonmonotonic function of e, hence it is single peaked.

above equation and using the properties of the cost function c, the following proposition can be proved.

Proposition 4 *When the economy is in a good state, i.e., $r = l$, the induced overspending $\tilde{e}(l, \pi_1)$ is a strictly decreasing function of π_1.*

Therefore, if voters could observe the true state of the world, their optimal choices would be clear:

$$\begin{array}{lll} \pi_1 = \pi_2 = \pi_h, & \text{any } n^* & \text{if } r = h. \\ \pi_1 = \pi_2 = 1, & n^* = \tilde{n}_l = \dfrac{B}{l + \tilde{e}(l,1)} & \text{if } r = l. \end{array}$$

However, voters do not observe r. Therefore, they will make their optimal electoral decision based on their beliefs about the true state of the world and the information they can acquire at the end of period 1. Note that, since the incumbent gains no extra utility if reelected in the bad state $r = h$, voters will find it optimal to focus their reelection rule on controlling the incumbent in the good state $r = l$. Let $\bar{n}_l = \dfrac{B}{l + \tilde{e}(l, \pi_h)}$; then $\hat{n}_h < \bar{n}_l < \tilde{n}_l$, and the unique perfect Bayesian equilibria of the game are stated in the following proposition.

Proposition 5 *For each value of the ex-ante probability ρ, there exists a unique perfect Bayesian equilibrium to the electoral control game, which is described below.*

(i) If $\rho > \frac{\varphi(\hat{n}_h)}{\varphi(\tilde{n}_l) - \varphi(\bar{n}_l) + \varphi(\hat{n}_h)}$, then in the unique perfect Bayesian equilibrium of the game voters choose divided government in period 1: $\pi_1 = 1$, and $n^* = \tilde{n}_l$.

If the realized state of the world is $r = l$, the incumbent will choose $n = \tilde{n}_l$, will be reelected, and will face total opposition in the Legislature in period 2: $\pi_2 = 1$.

If the realized state is $r = h$, there will be gridlock in the first period, the incumbent will not be reelected and the new incumbent will face partial opposition in the Legislature in period 2: $\pi_2 = \pi_h$.

(ii) If $\rho < \frac{\varphi(\hat{n}_h)}{\varphi(\tilde{n}_l) - \varphi(\bar{n}_l) + \varphi(\hat{n}_h)}$, then in the unique perfect bayesian equilibrium of the game voters choose partial divided government in period 1: $\pi_1 = \pi_h$, and $n^* = \bar{n}_l$.

If the realized state of the world is $r = l$, the incumbent will choose $n = \bar{n}_l$, will be reelected, and will face total opposition in the Legislature in period 2: $\pi_2 = 1$.

If the realized state is $r = h$, the incumbent will choose $n = \hat{n}_h$ and will not be reelected. The new incumbent will face partial opposition in the Legislature in period 2: $\pi_2 = \pi_h$.

Notice that in both cases the equilibrium is a separating one, with full revelation of the true state of the world at the end of period 1. This explains why voters can always chose the optimal level of vote splitting at period 2. Moreover, because of the full information revelation, it becomes a trivial exercise to check for Bayesian consistency of beliefs in the second period, on the equilibrium path. Also notice that there are no pooling equilibria.

The main difference between the two cases is related to the way voters view the true state of nature *ex-ante*. If voters believe that the good state is very likely, they will totally divide their political ticket and be very demanding for reelection, even though they understand this may induce a government shutdown. On the other hand, if they believe the bad state is very likely, then they will avoid gridlocks by selecting less opposition in the Legislature and they will adopt a less demanding reelection criterion, even though they understand that the incumbent will not produce the optimal level of social output if the realized state of nature is indeed good.

An important consequence of the proposition is that, when voters believe the underlying state of nature is likely to be good, then gridlocks occur with positive probability. Therefore, the inefficient outcome of government shutdown arises here as an equilibrium behavior of the electoral-control game, due to the incomplete information of voters.

4 The effect of the incumbent's political efficiency

The parameter β reflects how able the incumbent is in transforming overspending into an argument of his own utility. This could be seen as a measure of the political efficiency of a system, since the part $(1 - \beta)e$ of the overspending does not go to any of the agents in the game. In particular, one may view β as the *type* of the incumbent, so that more efficient incumbents (higher β's) will be able to aquire a higher part of the overspending e.

A natural question that arises then is: what is the effect to society of having incumbents with different β's? Should society prefer more efficient incumbents? The next proposition shows that, in fact, less incumbent efficiency is desirable in the context of the present electoral-control game.

Proposition 6 *Let $\hat{e}(r, \pi, \beta)$ be the solution to the one-period incumbent maximization problem, where the dependency on the parameter β is made explicit. Then $\hat{e}(r, \pi, \beta)$ is a strictly increasing function of β.*

Therefore, the higher the parameter β, the higher will be the level of overespending chosen by an incumbent. The main insight from the above proposition is that, the higher the parameter β, the higher the potential gains from overspending, *ceteris paribus.* Therefore, the higher the incentives an incumbent has for deviating from the optimal budget. This result may be vaguely related to the recent debate on term limits. Indeed, if one believes that the parameter β is not only a characteristic of the incumbent, but rather a learning variable, that may increase as the incumbent acquires administrative experience, then a limit on the possibility of reelection may be another instrument of control of the politicians, by restraining them from becoming *experts* in the precise sense of deviating a higher percentage of the overspending to their personal benefit. Viewing β as corruption, the previous proposition confirms that expertise in that area tends to have undesirable effects to society.

Nota bene, the present model is not intended to model corruption opportunities, which are probably much more sophisticated and diversified than what appears here. Moreover, even in the context of the model, a higher β does not necessarily imply a higher level of overspending in equilibrium, since in the particular case of a bad state of the world, the extra atractiveness of the budget may be compensated with a higher opposition in the Legislature. The model, however, highlights the role of the legislative opposition in reducing corruption in a world of incomplete information.

5 The multidimensional case: A first exploration

Suppose now that an executive incumbent presents a budget proposal $(N, P) = \{(n_1, p_1), \ldots, (n_k, p_k)\}$ where p_i is the unit cost and n_i is the number of copies of project i, $i = 1, \ldots, k$. Then the incumbent's second period maximization problem becomes:

$$\max_{N,P} v(N, P, \pi) = \sum_{i=1}^{k} n_i \beta_i (p_i - r_i) - c(p_1, \ldots, p_k, \pi)$$

$$\text{s.t. } NP \leq B$$

The main difficulty that arises in this context is that we do not have a concave problem anymore, which invalidates the use of first order conditions. It may be shown, however, that the optimal choice for the incumbent involves the choice of exactly one type of project, reverting to the one dimensional case.[4] That result might explain why some politicians tend to focus on a few very expensive projects, as it the case of America's president George W. Bush's nuclear protection system.

Given that solution, voters can decide on the optimal level of vote splitting. Results similar to those of the unidimensional case can be proves, showing in particular that voters will impose more opposition in the Legislature when the good state of the world is revealed.

The analysis gets more demanding, however, when it comes to the first period solution. In order to solve the problem, this author needed to assume that the social utility function $\varphi(N) = \varphi(n_1, \ldots, n_k)$ is additively separable in the inputs $n_i, i = 1, \ldots, k$, and symmetric.

In that particular case, it remains optimal to the incumbent to select one type of project and a result similar to the one-dimensional case obtains. However, the assumption on φ is strong and hardly intuitive, which suggests that much work remains to be done

[4]The proofs can be obtained from the author upon request.

in the multidensional case.

6 Concluding remarks

The present article is part of a program of research that tries to build formal models of political processes in which all agents are rational, act strategically, and in which institutions observed in the real world arise as consequences of the equilibrium behavior of the agents. A previous work (Bugarin, 2000) has shown how vote splitting can be added to reelection strategies in order to increase voters' control of politicians. The main result in that article stresses the flexibility of voters, which tend to be less demanding for reelection when they believe the incumbent may not be sole responsible for a weak social outcome, i.e., when they believe the world is in a "bad state". The article, however, limitates the analysis by assuming out the possibility of gridlocks, i.e., situations in which the Executive and the Legislators do not reach an agreement on the budget proposal. Moreover, the bargaining process between the Executive and the Legislature assumes the existence of a cost function with some very specific properties.

The present article extends that previous one, by adding the possibility of gridlocks, and by relaxing one strong hypothesis on the bargaining process. The main result, however, seems robust: voters do take their decisions based on their beliefs about the underlying state of nature, and are more flexible when they believe that the state is unfavorable. Moreover, it shows that the inefficient phenomenon of gridlock may occur in equilibrium with positive probability.

This study may be extended in many directions, in order to further test the robustness of the main results. First, heterogeneity of voter may be considered: what happens if some voters do have specific preferences on the candidates, in such a way that they are not totally symmetric in the second period? Second, the full multidimensional analysis needs to be deepened: what is the result of the electoral game when the budget takes the form $(N, P) = \{(n_1, p_1), \ldots, (n_k, p_k)\}$, where $k > 1$ is the number of different projects available? Is there a natural solution which does not depend on strong hypothesis on the shape of φ?

Third, the number of periods may be altered: what if there are infinitely many reelection periods? In that case, how should the role of uncertainty be extended? One way to do this is, following Bugarin (1999), is to think of the uncertainty as a stationnary Markov process and study how voters update their beliefs; an alternative approach that is followed in Persson & Tabellini (2000, chapter 4) is to postulate a moving average process for the uncertainty, resulting in a model of electoral cycles. Finally, the previous section suggests that there may be a positive relationship between term limits and electoral control, a result that diverges from the usual literature on voting and therefore deserves a more careful analysis.

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Appendix

Proof of Proposition 1

The optimal overspending level $\hat{e}(r, \pi_2)$ is the solution e to the following first order condition:

$$\beta Br = (r+e)^2 \frac{\partial}{\partial p} c(r+e, \pi_2) = (r+e)^2 f'(r+e) g(\pi_2)$$

Note that the left hand side of the above expression does not depend on π_2. Since the function f' is strictly increasing in p and the function g is strictly increasing in π_2, if π_2 increases, the corresponding solution e must decrease.

Proof of Proposition 2

Taking first order derivatives of the expression for v yields:

$$\frac{\partial v}{\partial \pi_2}(l, \hat{e}(r, \pi_2), \pi_2) = \frac{\partial v}{\partial e}(r, \hat{e}(r, \pi_2), \pi_2) \frac{\partial \hat{e}}{\partial \pi_2}(r, \pi_2) + \frac{\partial v}{\partial \pi}(l, \hat{e}(r, \pi_2), \pi_2).$$

By the optimality of $\hat{e}$, $\frac{\partial \hat{e}}{\partial \pi_2}(r, \pi_2) = 0$.

Moreover, $\frac{\partial v}{\partial \pi}(r, \hat{e}(r, \pi_2), \pi_2) = -f(r + \hat{e}(r, \pi_2)) g'(\pi_2) < 0$.

Therefore, $\frac{\partial v}{\partial \pi_2}(r, \hat{e}(r, \pi_2), \pi_2) < 0$.

Proof of Proposition 3

Let $e_l = \hat{e}(l, \pi_2)$ and $e_h = \hat{e}(h, \pi_2)$. First notice that $l + e_l < h + e_h$. Indeed, $e_r, r = l, h$ is the solution to the first order condition:

$$\beta Br = (r+e)^2 \frac{\partial}{\partial p} c(r+e, \pi_2) = (r+e_r)^2 f'(r+e_r) g(\pi_2)$$

Let $\Delta = h - l > 0$. Then $\beta Bh = \beta Bl + \beta B\Delta$. Therefore,

$(h+e_h)^2 f'(h+e_h) g(\pi_2) = (l+e_l)^2 f'(l+e_l) g(\pi_2) + \beta B\Delta > (l+e_l)^2 f'(l+e_l) g(\pi_2)$.

Since the right hand side function is strictly increasing, it follows that $h + e_h > l + e_l$. Therefore, $v(h, e_h, \pi_2) = \beta B \frac{e_h}{h+e_h} - f(h+e_h) g(\pi_2) = \beta B \frac{e_h}{l+(e_h+\Delta)} - f(l + (e_h + \Delta)) g(\pi_2) < \beta B \frac{e_h + \Delta}{l + (e_h+\Delta)} - f(l + (e_h+\Delta)) g(\pi_2) = v(l, e_h + \Delta, \pi_2)$.

Now $e_h + \Delta = e_h + (h - l) > e_l$. Therefore, by the optimality of the solution e_l, we have: $v(l, e_h + \Delta, \pi_2) < v(l, e_l, \pi_2)$, and the result follows.

Proof of Proposition 4

Suppose that voters observe the state of nature is l. Then, the function $\tilde{e}(l, \pi)$ is the smaller solution e to the following problem:

$$v(l, e, \pi_1) + \delta v(l, \hat{e}(l, 1), 1) = v(l, \hat{e}(l, \pi_1), \pi_1)$$

Taking first order derivatives of that equation yields:

$$\frac{\partial v}{\partial \pi}(l, \tilde{e}(l, \pi_1), \pi_1) + \frac{\partial v}{\partial e}(l, \tilde{e}(l, \pi_1), \pi_1)\frac{\partial \tilde{e}}{\partial \pi_1}(l, \pi_1) =$$

$$\frac{\partial v}{\partial \pi}(l, \hat{e}(l, \pi_1), \pi_1) + \frac{\partial v}{\partial e}(l, \hat{e}(l, \pi_1), \pi_1)\frac{\partial \hat{e}}{\partial \pi_1}(l, \pi_1)$$

By definition of $\hat{e}$, $\frac{\partial v}{\partial e}(l, \pi_1, \hat{e}(l, \pi_1)) = 0$. Moreover, since $\tilde{e}(l, \pi) < \hat{e}(l, \pi_1)$, we have $K := \frac{\partial v}{\partial \pi}(l, \tilde{e}(l, \pi_1), \pi_1) > 0$

Finally, since $\frac{\partial v}{\partial \pi}(l, \pi, e) = -\frac{\partial c}{\partial \pi_1}(r+e, \pi_1) = f(r+e)g'(\pi_1)$, the above expressin yields:

$$\frac{\partial \tilde{e}}{\partial \pi_1}(l, \pi_1) = \left[f(l + \hat{e}(l, \pi_1), \pi_1) - f(l + \tilde{e}(l, \pi_1), \pi_1)\right] g'(\pi_1)$$

Since f and g are strictly increasing functions, and since $\tilde{e}(l, \pi) < \hat{e}(l, \pi_1)$, it follows from the above expression that $\frac{\partial \tilde{e}}{\partial \pi_1}(l, \pi) < 0$, i.e. $\tilde{e}(l, \pi)$ is a strictly decreasing function of π_1.

Proof of Proposition 5

First notice that, at period 1, an incumbent has a dominant strategy if $r = h$. She will select:

$\hat{n}_h(\pi_1) = \frac{B}{h+\hat{e}(h,\pi_1)}$, $\hat{p}_h(\pi_1) = h + \hat{e}(h, \pi_1)$ if $\pi_1 \leq \pi_h$

$n = 0 = p$ if $\pi_1 > \pi_h$

On the other hand, in the good state $r = l$, the incumbent cares about reelection, and has a dominant strategy as well. She will select:

n^*, $p^* = \frac{B}{n^*}$	if	$n^* \geq \hat{n}_l(\pi_1) = \frac{B}{l+\hat{e}(l,\pi_1)}$	and	$n^* \leq \tilde{n}_l(\pi_1) = \frac{B}{l+\tilde{e}(l,\pi_1)}$
$\hat{n}_l(\pi_1) = \frac{B}{l+\hat{e}(l,\pi_1)}$, $\hat{p}_l(\pi_1) = \frac{B}{\hat{n}_l(\pi_1)}$	if	$n^* \leq \hat{n}_l(\pi_1) = \frac{B}{l+\hat{e}(l,\pi_1)}$	or	$n^* \geq \tilde{n}_l(\pi_1) = \frac{B}{l+\tilde{e}(l,\pi_1)}$

Therefore, any strategy profile (π_1, n^*) for voters in period 1 with $\pi_1 < \pi_h$ is strictly dominated be the strategy profile (π_h, n^*). Similarly, any strategy profile (π_1, n^*) for voters in period 1 in which $\pi_1 \in (\pi_h, 1)$ is strictly dominated be the strategy profile $(1, n^*)$. Hence, there are only two possible choices for π_1 in a perfect Bayesian equilibrium: $\pi_1 = \pi_h$ or $\pi_1 = 1$.

If voters choose $\pi_1 = \pi_h$, inefficient gridlocks will be avoided in period 1. Given that choice, the best possible selection for n^* is then $\bar{n}_l$. In that case, the expected utility of voters in period 1 is $E_h = \rho\varphi(\bar{n}_l) + (1-\rho)\varphi(\hat{n}_h)$.

On the other hand, if voters choose $\pi_1 = 1$, then the best possible voters' choice for n^* is $\tilde{n}_l$. In that case the expected utility of voters at period 1 is $E_l = \rho\varphi(\tilde{n}_l) + (1-\rho)\varphi(0) = \rho\varphi(\tilde{n}_l)$.

Now, $E_l > E_h \Leftrightarrow \rho > \frac{\varphi(\hat{n}_h)}{\varphi(\tilde{n}_l) - \varphi(\bar{n}_l) + \varphi(\hat{n}_h)}$.

Therefore, if $\rho > \frac{\varphi(\hat{n}_h)}{\varphi(\tilde{n}_l) - \varphi(\bar{n}_l) + \varphi(\hat{n}_h)}$, voters obtain the highest possile expected utility at period 1 by choosing $(\pi_1, n^*) = (1, \tilde{n}_l)$.

Conversely, if $\rho < \frac{\varphi(\hat{n}_h)}{\varphi(\tilde{n}_l) - \varphi(\bar{n}_l) + \varphi(\hat{n}_h)}$, voters obtain the highest possile expected utility at period 1 by choosing $(\pi_1, n^*) = (\pi_h, \bar{n}_l)$.

Finally, given any of the above strategies, voters learn the underlying state of nature by the choices of the incumbents, so that they can extract the maximum possible utility at period 2, by selecting the complete information optimum $\pi_2 = \pi_h$ if $r = h$ and $\pi_2 = 1$ if $r = l$.

Proof of Proposition 6

Recall that $\hat{e}(\beta, r, \pi_2)$ is the solution e to the following first order condition:

$$\beta B r = (r+e)^2 \frac{\partial}{\partial p} c(r+e, \pi_2) = (r+e)^2 f'(r+e) g(\pi_2)$$

If β increases, so does the right hand side of the above equation. Now, since the functions $(r+e)^2$ and $f'(r+e)$ are strictly increasing functions of e, when β increases the corresponding e has to increase as well. Therefore, the optimal incumbent choice of overspending $\hat{e}(\beta, r, \pi_2)$ is a strictly increasing function of β.