Pricing and Hedging of Oil Futures
- A Unifying Approach -

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Abstract

We develop and empirically test a continuous time equilibrium model for the pricing of oil futures. The model provides a link between no-arbitrage models and expectation oriented models. It highlights the role of inventories for the identification of different pricing regimes. In an empirical study the hedging performance of our model is compared with five other one- and two-factor pricing models. The hedging problem considered is related to Metallgesellschaft´s strategy to hedge long-term forward commitments with short-term futures. The results show that the downside risk distribution of our inventory based model stochastically dominates those of the other models.

In the mid eighties highly liquid spot markets for crude oil superseded the integrated contract system of the major oil companies. As prices in the spot market tend to be highly volatile, risk management became an increasingly important issue in the oil business. This is reflected in the success of oil futures contracts at the New York Mercantile Exchange (NYMEX) and the International Petroleum Exchange (IPE). For example, in 2000 about 37 million crude oil futures contracts were traded on NYMEX, which represents a volume of 37 billion barrels, more than the worldwide oil production.

An effective use of futures contracts in risk management requires an understanding of the factors determining futures prices and of the price sensitivities with respect to these underlying risk factors. In particular, the change of oil futures markets from backwardation into contango and vice versa needs to be captured.

In the more recent literature on commodity futures pricing, two approaches have mainly been followed. The first one is based on the notion of a "convenience yield", defined as the benefit which accrues to the owner of the commodity but not to the owner of the futures contract (Brennan (1991)).[1] Important examples of this approach are the models of Brennan and Schwartz (1985), Gibson and Schwartz (1990), Brennan (1991), Schwartz (1997), Model 3, and Hilliard and Reis (1998). Despite their varying complexity all these models share the following feature: Oil futures prices are determined by the *current* oil price and the costs and benefits of *storing* oil.

The second approach to valuing oil futures was put forward in the recent literature by Ross (1997) and Schwartz (1997), Model 1, and extended by Schwartz and Smith (2000) and Sørensen (1999). This approach builds on the idea that a replication of futures contracts by storing or short selling the physical commodity is impracticable or impossible due to market frictions. Thus, futures prices cannot be deduced from *current* spot prices and the costs and benefits of *storage*. Instead they are essentially determined by the *expected* spot price at maturity of the contract.

Both valuation approaches take extreme views of the structure of the spot market. In the first approach, potential arbitrageurs trade without transaction costs and can build long or short positions in the physical commodity without limits. In the second approach, the no-arbitrage link between current spot prices and futures prices is cut completely due to market frictions. Figure 1 illustrates that both approaches do not capture an important aspect of oil futures pricing. It shows the prices of the closest to six-month NYMEX crude oil futures plotted against the corresponding spot price using weekly data form July 1986 to November 1996. The dark line depicts theoretical futures prices resulting from Model 1 of Schwartz (1997), and the light line shows theoretical futures prices from a simple cost-of-carry model. While the cost-of-carry model describes futures prices well when spot prices are low, Schwartz´s model fits well for high oil prices. Figure 1 suggests that – depending on the price level of oil – two pricing regimes exist, one where prices are well explained by the *current* spot price and the costs and benefits of *storage* and another where prices are well explained by the *expected* spot price at maturity. This relates to the observation of Routledge et al. (2000) that oil partly behaves like an asset, partly like a consumption good.

(Insert Figure 1)

Our paper contributes to theoretical and empirical literature on oil futures. On the theoretical side we develop an equilibrium model with a representative investor which provides a connection between the two approaches followed in the literature and identifies the determinants of oil futures prices in different market regimes. While the model presented is simple enough for practical use in pricing and hedging, it highlights that costs and benefits of storage, the current spot price, the characteristics of the spot price process, the level of inventories and risk premia all play a role in oil futures pricing.

Our model has a number of interesting theoretical implications. A first key insight is that the characteristics of futures prices change fundamentally as soon as no discretionary[2] inventories are available. If the spot price of oil is low and inventories are high, the market price of oil risk is positive and futures prices coincide with those of a simple cost-of-carry model. If, on the other hand, spot prices are high and inventories are zero, the market price of oil risk becomes zero and futures prices equal the spot prices expected for the expiration dates. Second, both backwardation and contango situations can be explained endogenously without using a convenience yield variable. As expected, backwardation occurs for high oil prices and contango for low oil prices. Third, the oil price sensitivity of futures prices strongly depends on the oil price level and the level of discretionary inventories. If the spot oil price is high, the sensitivity of the futures price is low; if the oil price is low, the sensitivity can be greater than one. This result has important consequences for hedging strategies.

Schöbel (1992) developed an inventory based continuous time equilibrium model for commodity futures pricing which shares some important features of our model. Using the concept of a state dependent convenience yield, the state space of Schöbel´s model captures two regimes: a complete market regime, where the spot instrument is available for arbitrage as long as the convenience yield is nil, and an incomplete market regime, where arbitrage is impossible due to a relative scarcity of the spot, whenever the convenience yield becomes positive. While Schöbel´s model is driven by an exogenous level of inventories, which results in a mean-reverting spot price process, we determine inventories endogenously and model the spot price process directly. Because we use a short sale restriction for the inventories held by the representative investor, we do not require to introduce a convenience yield.

Our model has qualitatively similar implications as the discrete time equilibrium model of Routledge et al. (2000). However, the underlying economic mechanisms are quite different. In Routledge et al. (2000) risk neutral agents determine futures prices according to the expected spot price at the maturity of the contract. The agents´ storage decisions become important, as they influence the endogenously determined spot price process. In our approach, risk averse traders can engage in spot and futures positions. The level of inventories held affects the oil price sensitivity of the traders´ wealth and in turn changes the risk premium demanded for oil futures. One advantage of our approach is that because of its simplicity it fits nicely into the framework of standard continuous time valuation models and can easily be extended to a multi-factor setting. It provides a rich modeling framework which allows a consistent integration of different stochastic factors determining the costs and benefits of storage as well as the oil price dynamics.

The empirical contribution of the paper has two aspects. First, we test the hedging performance of our pricing model in comparison with five alternative models representing the two valuation approaches. Second, we look a the problem of hedging long-term delivery commitments with short-term futures, which is interesting in itself due to the case of the Metallgesellschaft AG (MG).[3] Here, we try to answer the question what kind of model based strategy MG should have been followed to hedge its exposure. As our models leads to oil price dependent hedge-ratios, we want to check in particular whether a link between current market conditions and hedge-ratios offers some empirical improvement.

In the basic case we consider a forward contract with ten years to maturity, which is hedged by successively rolling over short-term futures. For each hedging strategy we determine the probability distribution of the hedged positions terminal value using NYMEX futures data and a bootstrap methodology. Under ideal conditions this value should be zero with probability one. The most important empirical result is that for each of the five hedging strategies the distribution of terminal losses is first order stochastically dominated by the strategy derived from our Two-Regime-Pricing (TRP) model.

An extensive stability analysis supplements the basic case. This analysis quantifies the errors introduced by our bootstrap approach, the parameter sensitivity of the hedging strategies and the impact of an “extremal event” like the Gulf Crisis. In addition the study is repeated with a forward data set provided by Enron. This data set covers a relatively short time period but contains forward prices with maturities of up to nine years, which can be used instead of forward prices derived form the pricing rule of MG. It turns out that the two-factor models perform better than the TRP model in some cases, but are very sensitive to parameter and data variations. None of the models under consideration always stochastically dominates all other models.

Neuberger (1999) also analyzes the performance of different strategies to hedge long-term exposures with short-term futures. The main difference to our investigation is that we consider, as the studies of Brennan and Crew (1997) and Ross (1997), hedging strategies based on no-arbitrage or equilibrium models which imply that the long-term commitment can be hedged perfectly. Instead, Neuberger assumes a linear relationship between the prices of currently traded futures and the price at which new futures are expected to open. He imposes no additional restrictions on the stochastic development of futures prices and does not assume that the long-term contract can be replicated by a sequence of short-term futures. The resulting strategy turns out to be robust and performs well empirically. Purely data driven hedging strategies are proposed by Edwards and Canter (1995) and Pirrong (1997). As a main focus of our study is to compare and empirically test different no-arbitrage and equilibrium valuation models by means of their hedging performance, these alternative approaches are not pursued any further.

The remaining part of the paper is organized as follows: Section I develops the basic version of the model. Section II illustrates the model characteristics, reports some results of a comparative static analysis and points at some possible model extensions. Section III summarizes the hedging strategies derived from different models used in the empirical study. Section IV includes the empirical analysis of the hedging performance and Section V concludes.

I. Model Setup

In this section we develop our basic continuous time partial equilibrium model of oil futures prices. The interest rate and the dynamics of the oil price are taken as exogenous, the dynamics of futures prices are endogenous. These simplifying assumptions are made to highlight the economic reasoning leading to different pricing regimes in our approach while keeping the model as close as possible to the standard pricing models mentioned in the introductory section. This of course precludes feedback effects as in Routledge et al. (2000). The individual investor is assumed to have time-additive preferences of the form

$$E_0\left[\int_0^{\hat{T}} e^{-\varphi t} \ln(C(t))\, dt\right], \tag{1}$$

where $E_0[.]$ denotes the expectations operator conditional on the information in $t = 0$, $C(t)$ represents time t consumption flow and $\hat{T}$ is the planning horizon of the individual investor. The investor receives only capital income and can choose between immediate consumption and three investment alternatives. First, there is the possibility to buy and store oil, paying a constant rate K of storage costs per barrel, as well as to sell oil. Second, long or short positions in oil futures can be taken and third, long or short positions in an asset earning a riskless rate of r are possible. In the basic model, r is assumed to be constant.[4]

The investor can be interpreted as a trader who is active both in the spot as also the futures market for oil. This is in line with the empirical findings of Ederington and Lee (2000), who report that the dominant market participants in the heating oil futures market also take considerable positions in the spot oil market.

The oil price S is given exogenously and its logarithm $\ln S$ is driven by an Ornstein-Uhlenbeck process with positive mean-reversion parameter γ, stationary mean $\Theta - (\sigma^2 / 2\gamma)$ and positive volatility σ. This results in the following price process:

$$dS = \gamma(\Theta - \ln S)\, S\, dt + \sigma S\, dz \ , \qquad (2)$$

which is well supported empirically[5] and ensures that prices will always be positive. The futures price F is assumed to be at most a function of the investor´s wealth W, the oil price S and $T - t$, the time to expiration of the contract. It is further assumed that the futures price follows a diffusion process:

$$dF = \beta_F(W, S, t)dt + \sigma_F(W, S, t)dz\,, \qquad (3)$$

with drift β_F and diffusion coefficient σ_F, and that the futures contract is continuously marked to market. Hence, dF equals the linearized gains and losses of one contract long held over the time period dt.

The investor is a price taker and trades in continuous markets. There are no transaction costs and no limits on the storage capacity.

The individual investor´s decision problem is to maximize (1) subject to the budget constraint:

$$dW = \left[aW\left(\gamma(\Theta - \ln S) - K/S\right) + (1-a)Wr + n\beta_F - C\right]dt + \left[aW\sigma + n\sigma_F\right]dz\,, \qquad (4)$$

where a is the proportion of wealth, after consumption, held in inventories of oil and n is the number of futures contracts taken. Long or short positions in futures can initially be taken and rebalanced without any cash consequences. The investor´s choice variables are $C \geq 0$, a and n.

The well known first order conditions for a and n, as e.g. given in Cox et al. (1985a), p. 370, are

$$\gamma(\Theta - \ln S) - K/S - r - a\sigma^2 - n\sigma\sigma_F / W = 0 \ , \qquad (5)$$

$$\beta_F - a\sigma\sigma_F - n\sigma_F^2 / W = 0 \ . \qquad (6)$$

In equilibrium, we have to consider that futures contracts are in zero net supply and that the aggregated discretionary inventories must be non-negative. Therefore, if the investor is representative for the futures market, in equilibrium the aggregated number of futures contracts, n^*, must be equal to zero. Moreover, the representative investor cannot short positions in discretionary inventories, i.e. the proportion of aggregated wealth invested in oil, a^*, must be non-negative. Intuitively speaking, whenever a short position in oil is attractive for the representative investor, no discretionary inventories are available in the market and thus no oil can be borrowed to execute a short sale.

Using the conditions $n^* = 0$ and $a^* \geq 0$ together with equation (5), we receive the following proportion a^* of wealth optimally invested in oil:

$$a^*(S) = \max\left[\frac{\gamma(\Theta - \ln S) - K/S - r}{\sigma^2}\ , 0\right]. \tag{7}$$

a^* has a unique maximum at $\hat{S} = K/\gamma$. In the following we assume that $a^*(\hat{S})$ is positive, i.e. $\gamma(\Theta - \ln(K/\gamma)) - \gamma - r > 0$. Then it can be shown that there exist critical positive oil prices $\tilde{S}$ and $\bar{S}$ such that the discretionary inventory is zero for $S < \tilde{S}$ and $S > \bar{S}$.

Equation (7) states that the representative investor stores oil whenever an instantaneous positive return, net of storage and financing costs, is expected.[6] Otherwise the storage is zero. The optimal storage a^* increases with the expected return received from holding inventories and decreases with the risk in terms of σ^2.

Assuming that the futures price in equilibrium is a sufficiently smooth function of the oil price and time, from (6) and Ito´s lemma for β_F and σ_F the fundamental partial differential equation is obtained:

$$\frac{\partial F}{\partial S}(\gamma(\Theta - \ln S) - \sigma\lambda(S))\, S + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2 = 0 \tag{8}$$

where,

$$\lambda(S) = a^*(S)\,\sigma = \max\left[\ \frac{\gamma(\Theta - \ln S) - K/S - r}{\sigma}, \ 0\right] \tag{9}$$

is the market price of oil risk[7]. For our analysis of the futures price in Section II it is important to note that the market price of risk is positive for low oil prices and zero for high oil prices.

The fundamental valuation equation (8) has to be solved for the futures price subject to the terminal condition $F_T(T, S) = S(T)$. No analytical solution to (8) is known, but numerical solutions based on finite-difference methods or Monte-Carlo methods can be obtained easily.

II. Model Analysis and Extensions

We begin our analysis of futures prices resulting from (8) with two special cases. These allow us to illustrate the economic intuition behind the model and its relation to other models. First, assume that the current oil price $S(t)$ is very high compared to the critical oil price $\overline{S}$ and that $\text{Prob}(S(\tau) \le \overline{S};\, t \le \tau \le T)$ and thus $\text{Prob}(a^*(S(\tau)) > 0;\, t \le \tau \le T)$ are negligible with respect to the physical probability measure of the oil price. When no inventories are held, the representative investor holds the riskless security only and the covariance of changes in the oil price with changes in the optimally invested wealth is zero. For the logarithmic utility function this implies that the risk premium is zero. With $\lambda = 0$, equation (8) together with the terminal condition can be solved analytically and we get the following expression for the futures price:

$$F_T(t,S) = \exp\left[e^{-\gamma(T-t)} \ln S(t) + \left(\Theta - \frac{\sigma^2}{2\gamma} \right)(1 - e^{-\gamma(T-t)}) + \frac{\sigma^2}{4\gamma}(1 - e^{-2\gamma(T-t)}) \right]. \tag{10}$$

This pricing formula is identical to the one resulting from Model 1 of Schwartz (1997). It equals the expected spot price at maturity of the contract, given the current spot price and the oil price dynamics in (2). Thus, in situations where the spot price is very high and no inventories are likely to be held over the life of the contract, the futures price is driven exclusively by expectations about the spot price dynamics.

Second, assume that the current oil price $S(t)$ is so low that $\text{Prob}(S(\tau) \le \overline{S};\, t \le \tau \le T)$ is close to one.[8] Then, with probability close to one, the storage of oil has a positive expected instantaneous return throughout the life of the contract and both the inventories $a^*(S(\tau))$ and the market price of oil risk $\lambda(S(\tau))$, $t \le \tau \le T$, will be strictly positive. Thus, the fundamental valuation equation (8) becomes

$$\frac{\partial F}{\partial S}(K + rS) + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2 = 0, \tag{11}$$

with the following solution for the futures price:

$$F_T(t,S) = e^{r(T-t)} S(t) + \frac{K}{r}\left[e^{r(T-t)} - 1 \right]. \tag{12}$$

The price in (12) is identical to the one obtained from a simple cost-of-carry model, i.e. it depends only on the current spot price and the costs of storage. The intuition behind this result is as follows: When the investor´s optimal strategy would always lead to positive inventories during the life of the futures contract, any deviation from (12) can be exploited by standard arbitrage strategies. This is obvious if the futures price would exceed (12). If the futures price were lower than in (12), the investor could sell some oil from inventories, take a long position in the futures and buy the oil back at the expiration date. This would provide some additional, riskless income compared to the original strategy a^*, i.e. the original strategy cannot be optimal.[9]

Looking from another perspective, equation (12) results from a specific risk adjustment. If the market price of oil risk $\lambda = a^* \sigma$ is positive, it is proportional to the fraction of the investor´s wealth held in inventories. The more oil the representative investor stores, the higher is the instantaneous covariance between changes in aggregate wealth and changes in the oil futures price. As the investor is risk averse, changes in aggregate wealth will be hedged with short positions in the futures in order to smooth the consumption stream. Thus, when inventories are high, short positions in futures become relatively more attractive, which drives futures prices downwards.

The special cases (10) and (12) are also useful reference cases for the following comparative static analysis of the futures price. Our basic parameter scenario is defined by the following values:[10] The futures contract has six months to maturity, $\Theta = \ln(20.5)$ [11], $\gamma = 2.5$, $\sigma = 0.35$, $K = 4$ and $r = 0.05$. For these parameter values the two critical oil prices $\tilde{S}$ and $\overline{S}$ are $\tilde{S} = \$0.41$/barrel and $\overline{S} = \$\,18.42$/barrel, i.e. the discretionary inventories $a^*(S)$ are positive if $\$0.41 < S < \$18.42$, and zero otherwise.

First, we consider a variation of the current spot price $S(t)$. Both pricing equations (10) and (12) provide upper bounds for the futures price resulting from our simple Two-Regime-Pricing (TRP) model. For relatively high current spot prices $S(t)$, futures prices converge to the prices as if oil could not be stored. For low current spot prices they are close to those given by the simple cost-of-carry model. Figure 2 illustrates this behavior of the futures price. The futures prices of the TRP model have been obtained by a Monte Carlo method based on the antithetic variable technique and a control variable taken from the Schwartz model.

(Insert Figure 2)

As Figure 2 shows, the futures price obtained from the TRP model always increases with the spot price. However, the oil price sensitivity is quite different for high and low spot prices.

This will be important for delta hedging strategies based on the model, as fairly different hedge-ratios may result in situations of high and low oil prices.

Next consider the consequences of a parameter variation. K and r determine the costs of storing oil. When K and r grow, less oil will be stored and the market price of oil risk decreases. Thus, futures prices tend to increase and move towards those of the Schwartz model[12], in which storage of oil is not possible. The futures price is also an increasing function of the mean level parameter Θ of the log oil price process. An increasing mean-reversion parameter γ can move futures prices upwards or downwards, depending on whether the current spot price is below or above its average level. Finally, an increase in σ leads to lower futures prices. As can be deduced from (10), the expected spot price at expiration of the futures decreases with σ, whereas the risk premium $\lambda\sigma S$ remains unchanged.

Another important issue refers to the endogenous term-structure of futures prices. Figure 3 presents three different term structures for the same parameters as used in Figure 2 and maturities of up to twelve months. They differ only in the value of the current oil price, which is \$15/barrel, \$19/barrel and \$25/barrel respectively. For the low oil price, the term structure is upward-sloping, for the medium one it is slightly humped and for the high one it is downward-sloping. Thus, for low oil prices there is a contango and for high ones a backwardation situation. As in the purely expectation-based models the concept of a convenience yield is not needed to explain backwardation. Backwardation simply occurs when the oil price expected for the expiration date is declining with the maturity of the futures contracts. By the mean-reversion property of log oil prices this is the case if the current oil price is relatively high so that no discretionary inventories are held and backwardation cannot be exploited by arbitrage trading.

(Insert Figure 3)

Another interesting point to notice is that the term structure does not react symmetrically to changes in the difference between the current spot price and its mean level. When the spot price is below its mean level, the short end of the term structure becomes almost linear, reflecting the similarity to model (12). For high spot prices, there can be a considerable curvature at the short end of the term structure, which is because of the similarity to model (10).

Several extension of the basic model can easily be incorporated.[13] When looking at the "storage cost side" of the model, instead of considering costs only, one could add potential benefits of storage, i.e. introduce a convenience yield. The convenience yield could either be a constant proportion of the oil price as in Brennan and Schwartz (1985) or a stochastic variable as in Brennan (1991) or Gibson and Schwartz (1990). Stochastic interest rates, as in Schwartz (1997), Model 3, provide a further possible model extension. As far as the "expectations side" of the model is concerned, a more complex oil price process could be considered. For example, the mean level of the oil price process could be modeled as stochastic, like in Schwartz and Smith (2000), or a stochastic volatility could be introduced. Equilibrium market prices of risk for all additional stochastic factors can be easily obtained analytically. Their sign and magnitude depends crucially on the instantaneous correlation between the changes in the risk factor and the changes in the representative investor´s wealth.

III. Risk-Minimal Hedging Strategies

In order to evaluate the empirical performance of our model, we consider the problem of hedging long-term forward commitments with short-term futures contracts. This "MG problem" has received much attention in the literature.[14] There is still a controversy on the appropriate hedging strategy and whether a continuation of MG´s strategy would have resulted in lower losses than the closing of all contracts.

We compare the hedge results of our model with those of five other models proposed in the literature. We include three one-factor and two two-factor models which serve as representatives of the no-arbitrage-based and the expectations-based approaches discussed in the introduction. The one-factor models considered are a simple cost-of-carry model, the model of Brennan and Schwartz (1985) and Model 1 of Schwartz (1997). The futures price in the Brennan and Schwartz model is given by

$$F_T(t,S) = e^{(r-y)(T-t)} S(t), \tag{13}$$

where y denotes the constant net convenience yield rate. The futures prices resulting from the other two models are provided in (10) and (12).

Within these models the forward commitment can be hedged perfectly by a single futures. This is the case if at every point of time the sensitivity of the discounted forward price equals the sensitivity of the futures portfolio with respect to the oil price. The appropriate number of futures contracts h_t^1 to hedge one forward contract is given by equation (14), where T_1 and T are the expiration dates of the current futures in this hedge and the forward, respectively.[15]

$$h_t^1 \frac{\partial F_{T_1}}{\partial S} = e^{-r(T-t)} \frac{\partial F_T}{\partial S}. \tag{14}$$

Substituting the prices from (10), (12) and (13) into (14) provides the following values for the hedge-ratio h_t^1:

$$h_t^1 = e^{-r(T_1-t)}, \quad \text{for the cost-of-carry model,} \tag{15}$$

$$h_t^1 = \frac{e^{-r(T-t)} F_T}{F_{T_1}}, \quad \text{for the Brennan-Schwartz model,} \tag{16}$$

$$h_t^1 = e^{-\gamma(T-T_1)} \frac{e^{-r(T-t)} F_T}{F_{T_1}}, \quad \text{for the Schwartz model.} \tag{17}$$

The cost-of-carry model implies a hedge-ratio close to one, which corresponds to the strategy followed by MG. A hedge-ratio slightly lower than one results because of the continuous mark-to-market of the futures contracts which requires to tail the hedge over the relatively short period until the short-term futures expires. Note that there is no long-term tailing-the-hedge effect as a forward commitment and not a spot position in oil is hedged. The hedge-ratio of the Brennan-Schwartz model is smaller than one when the discounted forward price lies below the futures price. This will be the case when the market is in backwardation or when the effect of discounting dominates. The latter is likely for forwards with several years to expiration. Hedge-ratios resulting from the Schwartz model are even smaller. They are identical to those in (16) except for a multiplicative factor smaller than one. This factor decreases with the time to maturity of the forward and with the mean-reversion parameter γ of the spot price process.

For the TRP model hedge-ratios have to be obtained numerically. Figure 4 shows the number of one-month futures needed to hedge a six-month forward for varying spot prices. The model parameters are identical to those in Figures 2 and 3. For comparison reasons, hedge-ratios of the cost-of-carry model and the one-factor model of Schwartz (1997) are included in the figure.

(Insert Figure 4)

As Figure 4 shows, the relation between the TRP model, the cost-of-carry model (12) and Model 1 of Schwartz (1997) also translates into the hedging strategies. For low spot prices, the hedge-ratio of the TRP model is close to one. For high oil prices it approaches the hedge-ratio of the Schwartz model, which is considerably lower. More importantly, Figure 4 clearly shows the strong spot price dependence of a hedging strategy based on the TRP model.

The two-factor model considered in this study are the no-arbitrage based model of Gibson and Schwartz (1990) and the expectation oriented model of Schwartz and Smith (2000). Gibson and Schwartz generalize the Brennan and Schwartz model by allowing for a stochastic convenience yield. Futures prices take the following form:

$$F_T(t,y,S) = S(t)\cdot \exp\left[-y(t)\left[1-e^{-\alpha(T-t)}\right]/\alpha + A(\cdot)\right], \tag{18}$$

where $\alpha > 0$ is the mean reversion parameter of the convenience yield dynamics, $y(t)$ is the current convenience yield rate and $A(\cdot)$ is for fixed parameters a function of time only. To hedge the forward commitment perfectly, a dynamically rebalanced portfolio of two futures contracts with different times to maturity are needed. The appropriate numbers h_t^1 and h_t^2 of these futures contracts can be determined by the following linear system of equations:

$$\begin{aligned} h_t^1 \frac{\partial F_{T_1}}{\partial S} + h_t^2 \frac{\partial F_{T_2}}{\partial S} &= e^{-r(T-t)} \frac{\partial F_T}{\partial S} \ , \\ h_t^1 \frac{\partial F_{T_1}}{\partial y} + h_t^2 \frac{\partial F_{T_2}}{\partial y} &= e^{-r(T-t)} \frac{\partial F_T}{\partial y} \ . \end{aligned} \tag{19}$$

Substituting the prices according to (18) into (19), the following hedging positions in the futures contracts with expiration dates T_1 and T_2 $(T_1 < T_2 < T)$ result.

$$\begin{aligned} h_t^1 &= \left[1 - \frac{1-e^{-\alpha(T-T_1)}}{1-e^{-\alpha(T_2-T_1)}}\right] \frac{e^{-r(T-t)}F_T}{F_{T_1}} \ , \\ h_t^2 &= \left[\frac{1-e^{-\alpha(T-T_1)}}{1-e^{-\alpha(T_2-T_1)}}\right] \frac{e^{-r(T-t)}F_T}{F_{T_2}} \ . \end{aligned} \tag{20}$$

Note that the futures contract with maturity T_1 is always shorted if $T_2 < T - t$, whereas always long positions are held in the second futures. The net position $h_t^1 + h_t^2$ is close to (16), the number of futures contracts taken in the corresponding one-factor model.

The model of Schwartz and Smith (2000) generalizes Model 1 of Schwartz (1997) as the mean level parameter Θ of the log oil price process is stochastic and follows a Brownian motion with drift.[16] The corresponding futures prices are given by (21), where $B(\cdot)$ is for fixed parameters a function of time only.

$$F_T(t,S,\Theta) = \exp\left[e^{-\gamma(T-t)}\ln(S(t)) + \left(\Theta(t) - \frac{\sigma^2}{2\gamma}\right)(1-e^{-\gamma(T-t)}) + B(\cdot)\right]. \tag{21}$$

The hedge portfolio for the Schwartz and Smith model results again from (19), using the price function (21) and replacing the convenience yield rate in (19) by Θ. Explicitly the numbers of futures contracts are

$$\begin{aligned} h_t^1 &= \left[1 - \frac{1-e^{-\gamma(T-T_1)}}{1-e^{-\gamma(T_2-T_1)}}\right]\frac{e^{-r(T-t)}F_T}{F_{T_1}}, \\ h_t^2 &= \left[\frac{1-e^{-\gamma(T-T_1)}}{1-e^{-\gamma(T_2-T_1)}}\right]\frac{e^{-r(T-t)}F_T}{F_{T_2}}. \end{aligned} \tag{22}$$

A comparison of (22) with (20) shows that the hedge positions have the same structure. This is due to the formal equivalence of the two models.[17] The only difference is that the mean-reversion parameter α of the convenience yield process in (20) is replaced by the mean reversion parameter γ of the oil price process in (22). However, the economic basis of the two models is very different. This difference will carry over to the empirical implementation of the two models and to the size of the hedging portfolios.

IV. Empirical Comparison of Hedging Strategies

A. Methodology

The MG problem of hedging forward commitments of up to ten years is both an interesting and difficult one. As short-term futures have to be rolled over many times, hedging strategies can be strongly exposed to basis risk. A long hedge horizon also complicates the empirical evaluation of different hedging strategies as the available time series of data do not allow the generation of a sufficient number of independent hedge results for quantifying both the expected return and the risk of a strategy. Here, we follow a bootstrap methodology similar to Ross (1997) and Bollen and Whaley (1998) to simulate hedge portfolios. The structure of our empirical study is depicted in Figure 5.

(Insert Figure 5)

We begin by specifying a data model which captures the main features of historical spot and futures prices for oil. This model allows us to simulate different oil price scenarios. The second step comprises the calibration of the valuation models to the current term-structure of futures prices. In the third step 20.000 time series of spot and futures prices are simulated for the hedge period of ten years. For each time series the dynamic hedge strategy of each of the models and the hedge results are determined. In the final step the performance of the models is assessed.

B. Data

The futures data set available for the empirical study consists of daily settlement prices of the NYMEX crude oil futures contract over the period 7/1/86 to 11/25/96. The contract is settled by physical delivery of 1,000 barrels of West Texas Intermediate (WTI) crude in Cushing, Oklahoma. Up to 1989 the longest maturity contracts were 12 months. Currently, contracts for the next 84 consecutive months are traded. However, trading activity concentrates on the shortest maturity contracts. Trading terminates on the third business day prior to the 25th calendar day of the month preceding the delivery month.

In addition to the NYMEX data set, Enron made available some proprietary data containing daily forward prices for the period 1/18/93 to 8/30/96. The main disadvantage of the data set is the relatively short time period covered. The main advantage, however, lies in its complete coverage of contracts maturities from one month to nine years. The Enron data set is used for the stability analysis described in Subsection IV.F.

Daily spot prices for WTI crude in Cushing were provided by Platt´s, the leading oil price information service. Prices are available – as for the NYMEX data – from 7/1/86 to 11/25/96.

As we change the hedge positions once a month when the short-term futures are rolled over, monthly observations are sufficient for the analysis. Thus, for each month of the data period we selected the spot price and the futures prices for maturities of up to twelve months on the third business day prior to the 25th calendar day.[18] This provides us with a total of 13 time series with 125 observations each.

(Insert Figure 6)

Figure 6 shows the time series of the oil price and the six month futures basis, defined as the spot price minus the futures price for delivery in six months. The oil price exhibits a considerable variability with an annualized return volatility of 33 percent. The period of the

Gulf Crisis between July 1990 and February 1991 can be clearly identified, where the oil price reached a level of up to $ 35/barrel. As the visual impression suggests, spot price and basis are positively correlated with a correlation coefficient of 0.73.

C. Data Model

To generate plausible price scenarios, it is important to capture the main features of spot and futures prices. This leads to the following conditions:

(i) Prices should always be positive.

(ii) Futures prices should be tied to the spot price to avoid unrealistic deviations.

(iii) There should be a positive correlation between the oil price level and the basis.

The simplest way to fulfill the first condition is to model and simulate logarithmic prices. A link between futures prices and spot prices is established when futures prices are generated indirectly via a model of the basis. Moreover, if such a model uses the relative basis, i.e. the basis divided by the oil price, the absolute deviation between the oil price and the futures price is likely to be smaller for low oil prices than for high oil prices. The third condition can be fulfilled by allowing for cross sectional correlation between the innovation terms of the stochastic processes describing the log spot price and the relative basis in the data model.

Our data model builds on the log spot price and the relative basis for futures with up to twelve months to maturity. The spot price, the one-month, two-month, five-month and six-month basis are needed to generate the results for the hedge instruments, which are the two-month and six-month futures. Futures prices for maturities of up to twelve months only serve as inputs for the calculation of long-term forward prices, following the pricing rule of MG for firm-fixed contracts.[19] According to this rule the ten-year forward price is set to be the average price of the one-month to twelve-month futures plus a surcharge of $2.1/barrel.

All thirteen time series exhibit a significant autocorrelation, which, however, declines quickly with growing lags. Augmented Dickey-Fuller tests indicate the stationarity of the time series. Non-stationarity can be rejected on a 1 percent significance level for most of the series, and on a 5 percent significance level for all of them.[20]

The data model consists of one equation for the log oil price and one equation for each of the twelve relative bases. The relative basis BAS_t^k of the k-month futures at time t is explained by the relative basis of the (k+1)-month futures at time $t-1$, the (k+2)-month futures at time $t-2$ etc.. This means that lagged values of the same contract are used as regressors.[21] The number of lags was determined separately for each equation with the information criterion of

Schwarz (1978). As a final specification of the data model we obtain the system of equations given below, where $u_{0t},\ldots,u_{12t}$ are error terms with an expectation of zero.

$$\ln S_t = a_0 + b_0 \ln S_{t-1} + c_0 \ln S_{t-2} + u_{0t} \tag{23}$$

$$BAS_t^1 = a_1 + b_1\, BAS_{t-1}^2 + u_{1t} \tag{24}$$

$$BAS_t^2 = a_2 + b_2\, BAS_{t-1}^3 + u_{2t} \tag{25}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$BAS_t^5 = a_5 + b_5\, BAS_{t-1}^6 + u_{5t} \tag{28}$$

$$BAS_t^6 = a_6 + b_6\, BAS_{t-1}^7 + c_6\, BAS_{t-2}^8 + u_{6t} \tag{29}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$BAS_t^{12} = a_{12} + b_{12}\, BAS_{t-1}^{12} + c_{12}\, BAS_{t-2}^{12} + u_{12t}\,, \qquad \text{with} \quad t = 3,\ldots,125 \tag{35}$$

Results from the OLS-estimations of equations (23)-(35) are provided in Table I. Together with the parameter estimates some diagnostic tests are shown.

(Insert Table I)

The Ljung-Box tests give no indication of autocorrelation in the residuals of the data model. Moreover, there is little evidence for ARCH effects in the residuals. For none of the equations are ARCH effects significant on a 1 percent significance level, only for some of the equations on a 5 percent level. In summary, if there is any time series dependence at all in the residuals, it is only weak. This allows us to simulate price paths by standard bootstrapping methods. Whole residual vectors $[u_{0t},\ldots,u_{12t}]$ are drawn to maintain the strong cross sectional correlation between the residuals of different equations at the same point of time.

The simulation procedure starts with the observed values for the log spot price and relative bases on 7/22/92. This day was selected for two reasons. First, MG was already active in the futures market. However, the volume was small so that no price effect has to be expected. Second, at that time the term structure of futures prices was in a typical backwardation situation with a spot price of \$21.80/barrel and futures prices for delivery in one, six and twelve months of \$21.55/barrel, \$21.09/barrel and \$20.48/barrel respectively. A residual vector is drawn and the simulated log spot price and basis for the next month are calculated according to the data model. Starting from these new values, the subsequent residual vector is drawn etc.. In this way paths for a time horizon of up to ten years are generated. Corresponding paths for the futures prices are easily obtained from the simulated values of the log spot price and the relative bases. Finally, the simulation of the ten-year price paths is replicated 20.000 times. This provides us with a sufficient number of scenarios to investigate the risk and return of the different hedging strategies.

D. Implementation of Hedging Strategies

In order to implement the hedging strategies as described in Section III, we first have to specify which futures contracts to use as hedge instruments. Since our investigation should be based on highly liquid contracts while not picking up expiration effects we chose to use futures with two months to maturity for the one-factor strategies.[22] For the two-factor strategies we use the six-month future as second hedging instrument.[23]

For the calculation of the hedge-ratios $h_t = e^{-r(T_1 - t)}$ derived from the cost-of-carry model, only the time to expiration $T_1 - t$ of the short-term futures and the fixed interest rate r of 5 percent p.a. are required. As in our empirical study hedge positions are rolled over once a month at the last trading day of the expiring contract and are not rebalanced inbetween, we obtain constant hedge-ratios $h_t = e^{-0.05/6} \approx 0.9917$. For all other strategies prices of the two-month futures, the six-month futures (two-factor models only) and the long-term forward are needed to calculate hedge-ratios. For the model of Brennan and Schwartz (1985) information on prices and the interest rate is sufficient. The hedging strategies based on the models of Schwartz (1997), Schwartz and Smith (2000) and Gibson and Schwartz (1990) demand the additional knowledge of either the mean-reversion parameter γ or α. Since for the TRP model, where futures prices are not available in closed form, hedge-ratios have to be obtained numerically, the current spot price, the parameters γ, Θ and σ, the storage costs K and the interest rate are required.

Price information entering the hedge-ratios in (16), (17), (20) and (22) is obtained from the data model. Simulated futures prices for contracts of up to twelve months to maturity are directly available. The forward price for delivery in ten years is set according to the pricing rule of MG as the average of the one- to twelve-month futures prices plus \$2.1. As time elapses over the hedge horizon, forward prices for intermediate maturities are needed to calculate the hedge-ratios according to the Brennan-Schwartz, Schwartz, Gibson-Schwartz and Schwartz-Smith strategies. These forward prices are calculated by linear interpolation of the one-year futures price and the ten-year forward price. MG´s pricing rule was used since one of the objectives of this study is the assessment of MG´s hedging policy It might be suspected, however, that the hedge results strongly depend on the pricing rule and thus influence the model comparison. In particular, a strictly decreasing volatility of futures prices with increasing time to maturity is not compatible with the MG pricing rule. This objection will be tested in Subsection IV.F by means of the Enron data set, which covers only a relatively short time period but contains long-term forward prices.

The parameters of the Ornstein-Uhlenbeck process underlying the oil price dynamics in the one-factor model of Schwartz (1997) and the TRP model were estimated from the time series of logarithmic spot prices by the method of maximum likelihood. Monthly observations form July 1986 to July 1992 were used, thus the estimates utilize only information which was available at the beginning of our hedging period. The resulting parameter estimates for the log oil price process are $\hat{\gamma} = 2.71$, $\hat{\Theta} = 3.02$ and $\hat{\sigma} = 0.36$, with standard errors of 0.71, 0.05 and 0.03, respectively. The storage costs K per barrel of oil were set equal to $ 4.[24]

The one-factor models are, in general, not flexible enough to fit even the current two-month futures and ten-year forward prices correctly. This deficiency causes a problem for the comparison between the hedging strategies. Under ideal model conditions and time continuous hedging the corresponding strategies would all result in riskless hedged positions. This provides us with a natural reference point for the empirical comparison of different models. However, if the two-month futures or the ten-year forward are not correctly priced initially, some hedging error is introduced even under ideal conditions and no common reference point exists. To resolve this problem we modified the one-factor models slightly and introduced calendar time dependent parameters. Time dependent storage costs K were used in the cost-of-carry model, a time dependent convenience yield rate y in the Brennan-Schwartz model and a time dependent Θ in the Schwartz and in the TRP model. These parameters can be chosen to fit the initial term structure of futures prices correctly. The procedure is identical with the fitting of spot rate models to the current term structure of interest rates as suggested by Cox et al. (1985b) and implemented by Hull and White (1990, 1993). Interestingly, the time dependence of the parameters does not change the hedge-ratio formulas (15), (16) and (17).

The two-factor model of Gibson and Schwartz (1990) is flexible enough to fit three futures prices and the price of the ten-year forward exactly for an appropriate choice of the identified model parameters.[25] One of these identified parameters is the mean-reversion parameter α of the convenience yield rate process. We calibrated the model parameters to the prices of the one-month, two-month, six-month and ten-year futures contracts prevailing on 22 July 1992, the starting point of the hedging periods. This leads to a value for α of 5.62. Of course, more sophisticated methods to estimate α exist.[26] However, since our focus lies on the hedging strategies, we prefer to analyze the sensitivity of the hedging results for a wide range of parameter values – including those obtained from other estimation methods in the literature – instead of relying on only one parameter estimate. This sensitivity analysis will be presented in Subsection IV.F.

As Schwartz and Smith (2000) have shown, their model is structurally equivalent to the one of Gibson and Schwartz (1990). Thus, it is impossible to differentiate between the two mean-reversion parameters α and γ if they are estimated implicitly, though the two parameters have a different economic meaning. Whereas the implicitly estimated value of 5.62 for the convenience yield process is in line with estimates reported in the literature[27], it is unrealistically high for the mean-reversion parameter of the log oil price process. Therefore, we proxy the γ-parameter of the Schwartz-Smith model by its estimated value from the time-series data, i.e. we use a value of 2.71. With γ set equal to 2.71 the remaining free model parameters were then chosen such that the prices of the two- and six-month futures and the ten-year forward are fit correctly.

Given the specifications discussed above, Figure 7 shows how the hedge-positions h_t^1 for the one-factor models evolve over time when the hedging period is initially ten years. The figure provides the average hedge-ratios, averaged over the 20.000 simulation paths.

(Insert Figure 7)

Considerable differences are visible between the strategies. While the cost-of-carry model leads to a constant hedge-ratio of almost one, the Brennan-Schwartz model starts with a hedge-ratio of about 0.6, which gradually increases towards a value of one at the end of the hedging period. The hedging positions taken according to the Schwartz model are negligible most of the time. This is due to the significant mean-reversion of the oil price. Only in the last year of the hedging period are considerable positions in the two-month futures taken. The strategy resulting from the TRP model is similar to that of the Schwartz model, though slightly more futures contracts are held in the last two years of the hedging period. We want to stress, however, that the hedge-ratios of the TRP model strongly depend on the current oil price, which is not visible from the average hedge-ratios presented here. As Figure 4 showed, in a contango situation considerably higher hedge-ratios could result.

Figure 8 plots the evolution of hedge-positions resulting from the Gibson-Schwartz model and the Schwartz-Smith model over time. As in Figure 7, the average hedge-positions over the 20.000 price paths are depicted. This figure confirms the result that the investor always holds a long position in the six-month futures and a short position in the two-month futures when the maturity of the contract to be hedged exceeds the maturity of the hedging instruments. The net hedge position, i.e. the sum of h_t^1 and h_t^2 is identical for both models and is close to the hedge-ratio of the Brennan-Schwartz model shown in Figure 7. Due to the different values for α and γ, the net hedge position is spread differently between the two hedging instruments, however. In general, the absolute values of h_t^1 and h_t^2 decrease if α or γ increases.

(Insert Figure 8)

E. Hedging Results

Given the specifications discussed in the previous subsections, the hedge strategies can now be evaluated for the 20.000 simulated price paths. We hedged a forward commitment to deliver one barrel of oil in ten years[28] and calculated the $-value of the hedged position at the end of the hedge horizon. Futures contracts were rolled over once a month on the last trading day of an expiring contract into contracts with the same initial maturity. During the month the hedge portfolio was not rebalanced. This introduces a discretization error for all but the cost-of-carry and the Brennan-Schwartz strategies.[29] Intermediate payments from the futures are either invested or financed assuming a fixed interest rate of 5 percent p.a.. Summary statistics for the distribution of the hedge results, based on the 20.000 observations, are shown in Table II. The results refer to one barrel of oil with an initial ten-year forward price of $23.12 resulting from the pricing rule of MG. If the models would work perfectly, the hedge results should have a mean and a standard deviation of zero. For comparison reasons, results for the unhedged forward commitment are added in the last column of the table.

(Insert Table II)

The table provides a number of interesting insights. First, all hedging strategies lead to gains on average. These are highest for the cost-of-carry model, with a mean of $31.97 or 147 percent of the spot price on 22 July 1992. A comparison with the results for the unhedged position shows that only $3.32 of these gains can be attributed to the forward commitment.[30]

The remaining gains have been earned by rolling over the short-term futures contracts. Such a strategy is expected to be profitable when the market is predominantly in backwardation and the oil price has no downward trend. This was the case for the data period 1986 to 1996.

Second, the hedge-ratio close to one as it results from the cost-of-carry model leads not only to the highest mean but also to the highest standard deviation of all strategies. When some measure of downside risk, such as the loss probability or the mean loss, is considered, only the Schwartz-Smith strategy is riskier. These results support the view that the strategy of MG was a speculative one, highly profitable on average, but also highly risky. If MG would have continued its strategy at the end of November 1993 level of about 200 million barrels, in the worst case they would have suffered a loss of about $4 billion or more than three times the loss they had to cover until end of December 1993.

Third, if the mean and the standard deviation of the hedged positions value are considered, the one-factor models fall into two groups. The first group consists of Schwartz´s model and the TRP model with standard deviations of about half the size of the unhedged position. The second group contains the cost-of-carry and the Brennan-Schwartz model, which lead to much higher means and standard deviations than an unhedged position. These results can be explained by two observations. First, the hedge strategies derived from the models of the second group have consistently higher positions in futures than those of the first group. The roll over of these futures contracts exposes the hedged position to a considerable basis risk. Second, due to the mean-reversion of the spot price, price fluctuations will average out over the long hedge period of ten years. This explains the relatively low risk of the unhedged position and of the models of the first group, as these models lead to smaller futures positions.

The two-factor models lie somewhere between the two groups of one-factor models. Surprisingly, both two-factor models do not seem to lead to an improvement compared to the one-factor models of the first group. In particular, the two-factor models perform worse than the TRP model with respect to all three risk measures. The reason might be a lack of robustness of the two-factor models with respect to the specification of α and γ or the prices entering the hedge-positions. An indication for parameter sensitivity are the quite different results of the Gibson-Schwartz and the Schwartz-Smith strategies, since these strategies differ only by the values for α and γ. The robustness of the two-factor strategies is further explored in the next subsection.

Finally, it turns out that the hedge strategy derived from the TRP model first order stochastically dominates both the unhedged position and the strategy based on the Schwartz model. This is demonstrated in Figure 9, which shows the cumulative distribution function of the terminal value resulting from the unhedged forward commitment and the two hedge strategies.

(Insert Figure 9)

Especially interesting is the behavior of the different strategies in the loss region of the distribution function. Of course the first order stochastic dominance relations between the TRP strategy, the Schwartz strategy and the unhedged position shown in Figure 9 still hold. Interestingly, the TRP strategy dominates also the other four strategies if one considers only this region of the value distribution. This strong result is shown in Figure 10.

(Insert Figure 10)

F. Stability Analysis

Overall, none of the model based hedging strategies provides fully satisfactory results. This may be for different reasons:

(i) One potential problem may arise from the use of a data model to simulate representative price scenarios. It is possible that the valuation models explain futures prices well but the data structure is not appropriately captured by the data model. Therefore, the hedging quality appears to be too low.

(ii) The hedge portfolios are adjusted only once a month. This introduces discretization errors for some models compared to a theoretically correct continuous adjustment of the hedge portfolio.

(iii) The hedge-ratios depend on unknown model parameters. If these parameters are estimated with error the hedging performance is reduced.

(iv) The valuation models may not appropriately explain observed futures prices in all market scenarios. They might work well in normal periods but eventually collapse in extreme situations like the Gulf Crisis.

(v) For all but one strategies long-term forward prices enter the hedge-ratios. If the pricing rule of MG does not capture the behavior of long-term prices correctly, these hedge-ratios will be misspecified. In particular, the two-factor models are suspected to be quite sensitive to misspecification.

These five possible explanations are now discussed in turn. The errors introduced by the data model and the monthly adjustments of the hedging strategies can be isolated and quantified when the whole analysis is replicated with model consistent data. Therefore, we replaced the historical futures prices by a ”new set of futures data” calculated according to the Gibson-Schwartz model from the historical spot prices and the calibrated model parameters. This ”new data set” was then used together with the historical spot prices to reestimate the data model. Finally, the whole simulation study was repeated for a ten-year hedge horizon. The resulting hedging errors of the Gibson-Schwartz strategy give some indication on the joint effect of an inappropriate data model and the discrete hedge adjustments. It turns out that the standard error of the hedge results lies below 0.2, which is small compared to the value of 3.26 given in Table II. This shows that only a small proportion of the hedging errors can be attributed to a misspecified data model and to the non-continuous rebalancing of the hedging positions.

The effect of an imprecise estimation of the parameters γ and α can best be quantified by a sensitivity analysis. As far as the γ parameter in the Schwartz model is concerned, the study already covers the extreme cases of the parameter range. For $\gamma \to 0$ the hedging strategy converges to that of the Brennan-Schwartz model and for very large values of γ one essentially obtains an unhedged position. If γ decreases gradually, starting from a value of 2.71, the general tendency in the results of Table II is confirmed: Decreasing values of γ result in increasing hedge-ratios, means and standard deviations of the hedge results.[31]

Within the group of the two-factor models the results for the structurally equivalent Gibson-Schwartz and Schwartz-Smith models, as shown in Table II, must be a consequence of the different values used for the two mean-reversion parameters α and γ. Table III provides hedging results for a wide range of different parameter values. The smallest value of 1.49 was the γ-estimate obtained by Schwartz and Smith (2000) from time series and cross section data of futures prices by means of the Kalman filter technique.[32] The maximum value of 15 is close to the α-estimate of Gibson and Schwartz (1990).

(Insert Table III)

Again, lower parameter values below the implicitly estimated mean-reversion parameter of the convenience yield ($\alpha = 5.62$) worsen the hedge performance with respect to all three risk measures. The minimum of the risk measures is attained for parameter values of about 12.

For this “optimal” parameter value, which cannot be extracted from the data, the loss probability and the mean loss improves considerably, leading to better results than all one-factor strategies. The volatility of the position value is still close to the one of a unhedged position, however. In conclusion, a two-factor model in principle is flexible enough to achieve an improvement compared to all one-factor models but can also lead to much worse results, depending on the parameter values.

In order to investigate the influence of the Gulf Crisis on the hedging performance of different strategies, we excluded the period between July 1990 and February 1991 from the data set, estimated the data model with the remaining observations and repeated the hedging study. As the time of the Gulf Crisis was a period of highly volatile spot prices and bases, we expected lower standard deviations and lower mean values of the hedging results.

(Insert Table IV)

Table IV provides the hedging results. As expected, five of six strategies show a lower mean value compared to the results of Table II. The differences are considerable for the cost-of-carry strategy and the Brennan-Schwartz strategy, which take relatively large positions in the two-month futures. However, the risk of these strategies does not change as expected, as the standard deviations of the hedge results even increase. This result is driven by two effects. First, the exclusion of the extreme observations during the Gulf War reduces the volatility of the basis and the risk of the roll-over hedging strategies. Second, the mean-reversion of the relative basis becomes smaller, which leads to an increase in the volatility of the basis. It turns out that the second effect compensates the first one. For the cost-of-carry model, the Brennan-Schwartz model and the TRP model it even dominates. Overall, the “extremal event” of the Gulf Crisis cannot explain the high risk of some of the analyzed hedging strategies. The general characteristics of the different strategies remain unaffected by the exclusion of the Gulf Crisis period from the data set.

Finally, we address the problem that the MG pricing rule might not properly reflect characteristics of long-term forward prices and therefore leads to misspecified hedge-ratios. This can only be checked by means of additional price information for long-term contracts, which was made available to us by Enron. Unfortunately, no clean comparison is possible. As the Enron data set covers only the period from 1/18/93 to 8/30/96, we have to consider at least two important effects. First, different hedge qualities as compared to Subsection IV.E can result from the use of Enron forward prices instead of NYMEX futures prices and the MG pricing rule. Second, they can results from the different time periods used for the specification of the data model.

Our methodology is basically the same as applied to the NYMEX data. We estimated an enlarged data model with Enron forward prices instead of NYMEX futures prices, where estimation equations like (24) to (35) were added for relative bases of up to nine years to maturity. Beyond maturities of nine years a flat forward curve was assumed. Since no prices are available for the 22 July 1992 we chose the 20 April 1993 as a new starting date. On that day prices of one- to twelve-month Enron forwards were closest to those of one- to twelve-month NYMEX futures on 22 July 1992.[33] For the new starting date we calibrated the models to the then prevailing forward curve, simulated 20.000 price paths and finally obtained the terminal $-value of a hedged ten-year forward commitment resulting from the different strategies. The parameter values α and γ of the Gibson-Schwartz and Schwartz-Smith models were initially chosen as in Subsection IV.E for comparison reasons and varied later to cover a wide range of values.

(Insert Table V)

Hedging results for the Enron data set are summarized in Table V. As in Subsection IV.E, two groups of one-factor models can be distinguished. The first group shows high means and standard deviations and consists of the cost-of-carry model and the Brennan-Schwartz model. Compared to the results in Table II, the risk of these strategies increases, even though the risk of an unhedged forward commitment decreases. This means that a higher basis risk incurred by repeatedly rolling over short-term contracts overcompensates a lower oil price risk. The reduction in oil price risk can be understood from inspection of Figure 6, when comparing the period January 1993 to August 1996 with the whole period July 1986 to November 1996. While the annualized spot price volatility is 33 percent for the whole period, a volatility of only 23 percent is obtained for the period covered by the Enron data.

The second group of one-factor models again consists of the Schwartz model and the TRP model, both leading to relatively low means and standard deviations. For the Enron data the Schwartz model improves relative to the TRP model and none of the models stochastically dominates the other. Its generally lower hedge position seems to works in favor of the Schwartz strategy in a period when oil price volatility is low and basis risk is high.

Most astonishing is the bad performance of the two-factor strategies. Thus, the hypothesis that unrealistic forward prices obtained from the MG pricing rule can explain why a two-factor model does not clearly outperform a one-factor model is certainly not confirmed. Although, it might be argued that there is an interaction between unrealistic long-term forward prices and badly estimated parameter values. To check this, Table VI provides hedge results of the two-factor models for a wide variety of parameter values α and γ. It turns out that even for the "optimal" value of about 12 the two-factor models are outperformed by the Schwartz and TRP strategies with respect to all three risk measures. This result might be due to the high net hedge position of the two-factor models, which is close to the hedge-ratio of the Brennan-Schwartz model, irrespective of the α or γ parameter. Such a high net hedge-position may not be favorable in a period when oil price risk is low and basis risk is high.

(Insert Table VI)

V. Conclusion

In this paper we develop a continuous time partial equilibrium model for the pricing of oil futures. This model provides a link between two important valuation approaches and offers a unifying framework to nests different kinds of models for futures prices: storage-based explanations, models using the concept of a convenience yield and pure expectation-based models in which oil is a non-traded asset. Therefore, the model is able to identify the roles played by different stochastic factors which drive the costs and benefits of storage as well as the oil price dynamics.

Our model has a number of interesting implications. First, the characteristics of futures prices depend crucially on the amount of discretionary inventories available. If the spot price of oil is low and inventories are high, the market price of oil risk becomes positive and futures prices coincide with those of simple cost-of-carry models. If, on the other hand, spot prices are high and inventories are zero, the market price of oil risk is zero and futures prices are equal to the spot prices expected for the expiration dates. Second, both backwardation and contango situations can be explained endogenously without the conception of a convenience yield. Backwardation occurs for high oil prices and contango for low oil prices. Third, the oil price sensitivity of futures prices strongly depends on the spot price level and the related level of discretionary inventories. If the spot oil price is high, the sensitivity is low, if the oil price is low, the sensitivity high. This result has important implications for delta hedging strategies derived from the model.

In the empirical part of the paper we apply our model to the problem of hedging long-term forward commitments with short-term futures. We investigate the dynamic strategy implied by our model and compare its performance with five other strategies derived from different one- and two-factor models. The empirical results show that for a hedge horizon of ten years the downside risk distribution of the strategy resulting from our model stochastically dominates those of the other models.

Even though the hedge results of the basic one-factor version of our model are encouraging, the potential of the proposed modeling framework needs to be explored further. First, the stability analysis of the hedge performance of our model could be generalized by varying the hedge horizon and by considering interest rate risk. Second, it needs to be explored as to which possible risk factors are most important and what kind of specification works best for which kind of application. For example, it is unlikely that a one-factor model is flexible enough to yield a high explanatory power in a pricing study.

Another interesting and empirically testable implication of our model which could be examined in future work concerns the relation between current futures prices and future spot prices. Theoretically, in our model, futures prices are downward biased predictors of future spot prices in general, but the magnitude of this bias strongly depends on the current oil price and the level of inventories. The bias will be highest when the current oil price is low since this implies a high market price of oil risk.

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1 The theory of storage relates the benefits of holding inventories to the level of inventories. It dates back to Working (1949), Telser (1958) and Brennan (1958).

2 As Routledge et al. (2000), we use the term discretionary inventories for inventories which are not directly committed to production.

3 The main contributions to the debate surrounding the MG case are collected in Culp and Miller (1999).

4 Extensions to stochastic interest rates are easily obtained and shortly discussed at the end of Section II.

5 The stationarity of oil prices has been documented in the literature, e.g. in Bessembinder et al. (1995). Pilipovic (1997), p. 74 ff. compares several models of the oil price dynamic. An Ornstein-Uhlenbeck process of the log oil price provides the best explanation of the statistical properties of observed prices.

6 It seems natural to consider an upper limit on the storage capacity also. However, this more general approach is not followed here. Contrary to the short sale restriction a barrel limit on the storage capacity can not be expressed as a restriction on the *proportion* of wealth invested in oil (a^*). To check whether the limit is reached, knowledge of both the *level* of aggregate wealth invested in oil and the oil price is needed. Thus a barrel limit on the storage capacity leads to futures prices which depend on two state variables, one of which is unobservable. Moreover, it is far from obvious how to set the limit on discretionary inventories to be stored when the model has to be implemented.

7 The valuation equation (8) can be considered as a special case of the fundamental valuation equation for derivatives in Cox et al. (1985a).

8 For reasonable parameter values, as empirically determined in Section IV, $\tilde{S}$ becomes \$0.41/barrel. Even if the current oil price were as low as \$5/barrel, the probability for an oil price below $\tilde{S}$ in one year were less than 10^{-100}. Therefore, the event $\left\{S(\tau) \le \tilde{S};\ t \le \tau \le T\right\}$ is neglected in the following argumentation.

9 When there is a positive probability for a stock out during the life of the futures contract, selling oil from the optimal inventory and buying it back later via a futures contract is no longer a utility increasing strategy because of two effects. First, there is an additional gain resulting from selling current stocks and buying them back later. Second, it is necessary to adjust the original strategy in the case of a stock out, implying a loss in utility. As both effects have to be taken into account, (12) provides only an upper bound for the futures price.

10 These parameter values are in accordance with the empirical results of Section IV.

11 For $\Theta = \ln(20.5)$ the stationary mean of the log oil price process equals $\ln(20)$.

12 With $K \to \infty$ oil will become a non-storable good. In this case futures prices are as in (10).

13 For details see Bühler et al. (2000), Section III.

14 See the collection of papers in Culp and Miller (1999).

15 We do not have to distinguish between forward prices and futures prices here, as none of the models analyzed assumes stochastic interest rates. See Cox et al. (1981).

16 In Schwartz and Smith (2000) the log oil price is the sum of two unobservable stochastic factors. Here, we chose an equivalent formulation with the oil price and Θ as stochastic factors.

17 See Schwartz and Smith (2000), Section 4.

18 Before 1989, contracts with maturities of more than nine months were occasionally not traded. In these cases the prices of the longest maturity available have been substituted.

19 See C&L (1995), p. 33. The impact of this pricing rule on the results is can be analyzed by means of the Enron data set, which has long-term forward prices available. For the results see Subsection IV.F.

20 Detailed results from the preliminary data analysis and the Dickey-Fuller tests are available from the authors. Note that the stationarity of the log oil price series supports the corresponding assumption of our valuation model.

[21] If values of the (k+1)-month basis are not available, as for the twelve-month futures, we used lagged values of the k-month basis instead.

[22] Results do not change qulitatively, when one-month futures are used. For details see Bühler et al. (2000)

[23] Results worsen for the two-factor strategies when one-month and two-month futures are used as hedge instruments. It seems that the maturities of the hedge instruments should not be to close as to capture the variation of a second risk factor. See Bühler et al. (2000) for details.

[24] Ross (1997), p. 388, estimates the storage costs per barrel and year to be 20% of the current spot price. For a spot price of $20/barrel this would be $4 per year.

[25] Here, we interpret the unobserved state variable $y(t)$ as another free parameter.

[26] See Schwartz (1997), p. 932 f.

[27] Compare the values in Gibson and Schwartz (1990), p. 966, Table II and Brennan and Crew (1997), p. 177, Figure 4.

[28] MG had to hedge a whole portfolio of forward commitments with different maturities. Here, we concentrate on a single component in order to demonstrate the effects more clearly.

[29] The impact of these discretization errors on the hedging results is analyzed in Subsection IV.F for the exemplary case of the Gibson-Schwartz model.

[30] There is a gain on the unhedged position on average, because the ten-year forward price at the beginning of the hedging period lies above the mean level of the oil price.

[31] Detailed results are available from the authors.

[32] See Schwartz and Smith (2000), p. 903, Table 2.

[33] The similarity was measured by the sum of the squared price differences.

Figure 1. Prices of NYMEX crude oil futures with a maturity close to six months plotted against the spot price of West Texas Intermediate (WTI) crude oil. Weekly data is used for the period 7/1/86 to 11/25/96. In addition, theoretical futures prices are depicted. The dark line shows prices of futures with six months to maturity according to Model 1 of Schwartz (1997); the light line shows prices resulting from a simple cost-of-carry model with constant storage costs per barrel. The parameter values used are provided in Section IV.

Figure 2. Theoretical prices of futures with six months to maturity for different values of the current spot oil price. Three pricing models are considered: The Two-Regime-Pricing (TRP) model, Model 1 of Schwartz (1997) and a cost-of-carry model with constant storage costs per barrel. The following parameter values are used: A mean parameter of the log oil price process (Θ) of ln(20.5), a mean-reversion parameter of the log oil price process (γ) of 2.5, a volatility (σ) of 0.35, storage costs (K) of $4 per barrel and year and a riskless interest rate (r) of 5 percent p.a..

Figure 3. Term structures of futures prices arising from the Two-Regime-Pricing (TRP) model for different current oil prices. The figure shows three term structures referring to current oil prices of $15/barrel, $19/barrel and $25/ barrel. The following parameter values are used: A mean parameter of the log oil price process (Θ) of ln(20.5), a mean-reversion parameter of the log oil price process (γ) of 2.5, a volatility (σ) of 0.35, storage costs (K) of $4 per barrel and year and a riskless interest rate (r) of 5 percent p.a..

Figure 4. Initial hedge-ratios for different current spot oil prices derived from three valuation models when a one-month futures is used to hedge a six-month forward. The pricing models considered are the Two-Regime-Pricing (TRP) model, Model 1 of Schwartz (1997) and a cost-of-carry model with constant storage costs per barrel. The following parameter values are used: A mean parameter of the log oil price process (Θ) of ln(20.5), a mean-reversion parameter of the log oil price process (γ) of 2.5, a volatility (σ) of 0.35, storage costs (K) of $4 per barrel and year and a riskless interest rate (r) of 5 percent p.a..

Figure 5. Structure of the empirical study. Only data information was used to obtain simulated price paths. Hedge-ratios were determined from the theoretical pricing models based on parameter values estimated from the data.

Figure 6. Spot price of West Texas Intermediate (WTI) crude oil in Cushing and six-month basis. The six-month basis is defined as the difference between the spot price and the price of the six-month NYMEX future, for the period from January 1986 to November 1996.

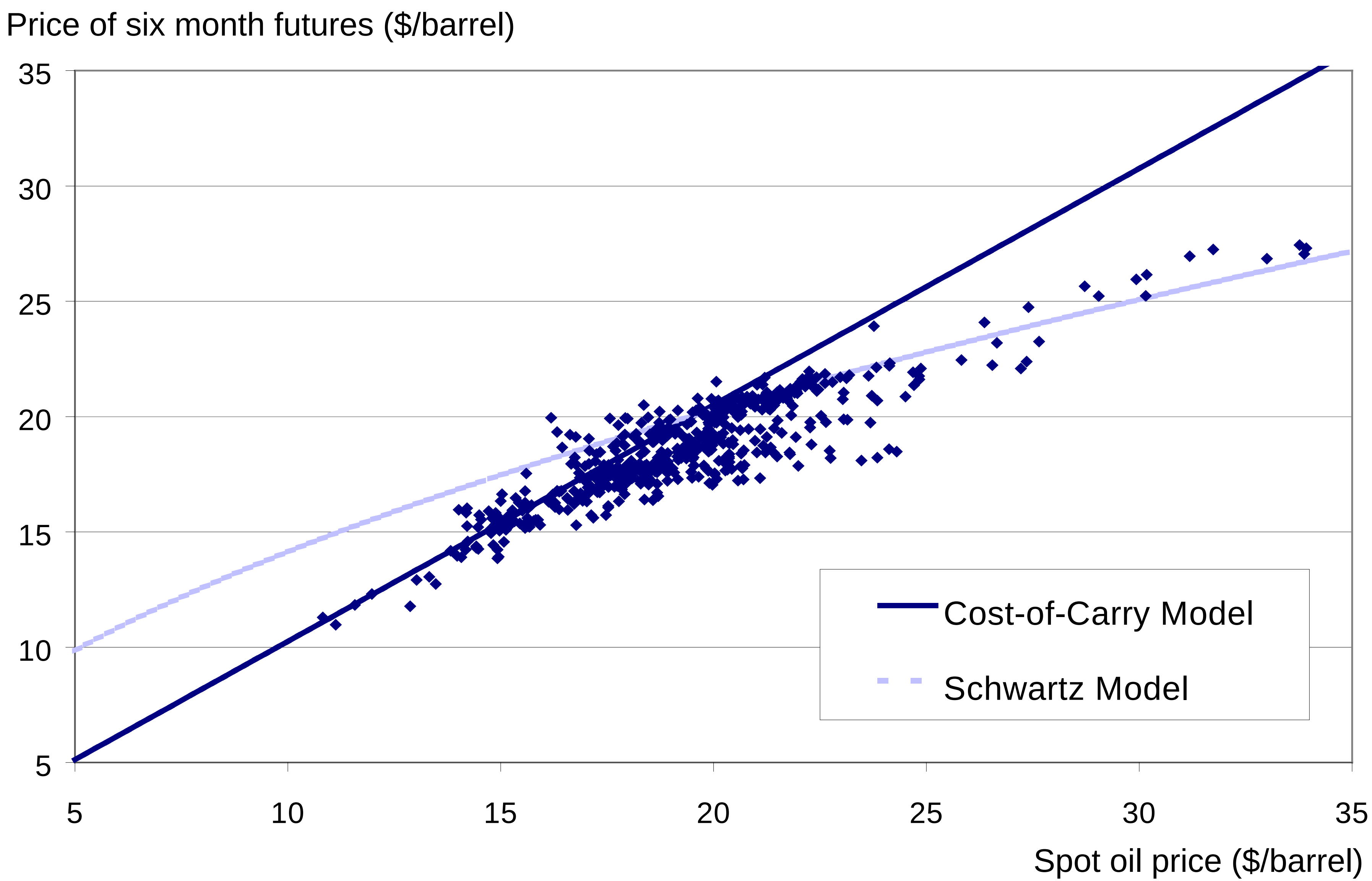
Figure 7. Average hedge-ratios calculated from 20.000 simulated price paths for four strategies derived from one-factor pricing models. It is shown how many two-month futures are taken over the hedging period when one short forward contract with initial time to maturity of ten years is hedged. The pricing strategies considered correspond to a cost-of-carry model with constant storage costs per barrel, the model of Brennan and Schwartz (1985), the Two-Regime-Pricing (TRP) model and Model 1 of Schwartz (1997).

Figure 8. Average hedge-positions calculated from 20.000 simulated price paths for two strategies derived from two-factor pricing models. It is shown how many two-month futures and how many six-month futures are taken over the hedging period when one short forward contract with initial time to maturity of ten years is hedged. The pricing models considered are the ones of Gibson and Schwartz (1990) and Schwartz and Smith (2000).

Figure 9. Cumulative distribution function of the hedge results at the end of a ten-year hedge horizon for two model based hedging strategies and an unhedged delivery commitment to deliver one barrel of oil in ten years. Model based strategies refer to the Two-Regime-Pricing (TRP) model and Model 1 of Schwartz (1997). Hedge results are calculated as the sum of the accumulated proceeds from the monthly mark-to-market of two-month futures contracts and the final pay-off of the ten-year forward contract. The figure is based on the hedge results obtained from 20.000 simulated price scenarios.

Figure 10. Loss region of the cumulative distribution function of the hedge results at the end of a ten-year hedge horizon for four model based strategies to hedge a commitment to deliver one barrel of oil in ten years. Model based strategies refer to a cost-of-carry model with constant storage costs per barrel, the model of Brennan and Schwartz (1985), the model of Gibson and Schwartz (1990) and the Two-Regime-Pricing (TRP) model. Hedge results are calculated as the sum of the accumulated proceeds from the monthly mark-to-market of two-month futures contracts, six-month futures contracts (Gibson-Schwartz model only) and the final pay-off of the ten-year forward contract. The figure is based on those of the hedge results obtained from 20.000 simulated price scenarios which lead to losses. The cumulative distribution function for the strategy derived from the model of Schwartz and Smith (2000) is omitted, as it markedly exceeds all other cumulative distribution functions in the loss region.

Figure 1.



Price of six month futures ($/barrel)
35
30
25
20
15
10
5
5
10
15
20
25
30
35
Spot oil price ($/barrel)
Cost-of-Carry Model
Schwartz Model

Figure 2.

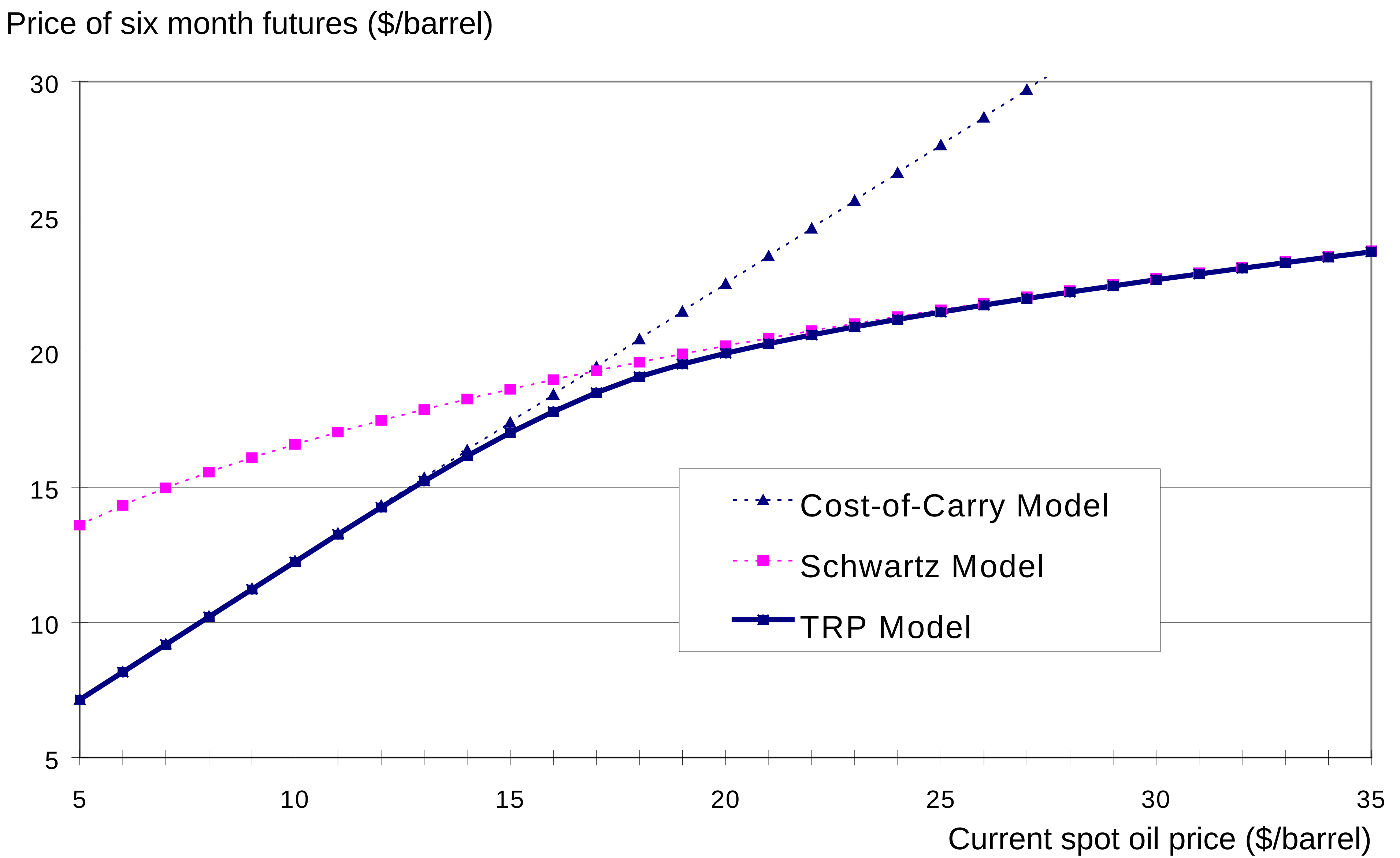

Price of six month futures ($/barrel)
30
25
20
15
10
5
5
10
15
20
25
30
35
Current spot oil price ($/barrel)
Cost-of-Carry Model
Schwartz Model
TRP Model

Figure 3.

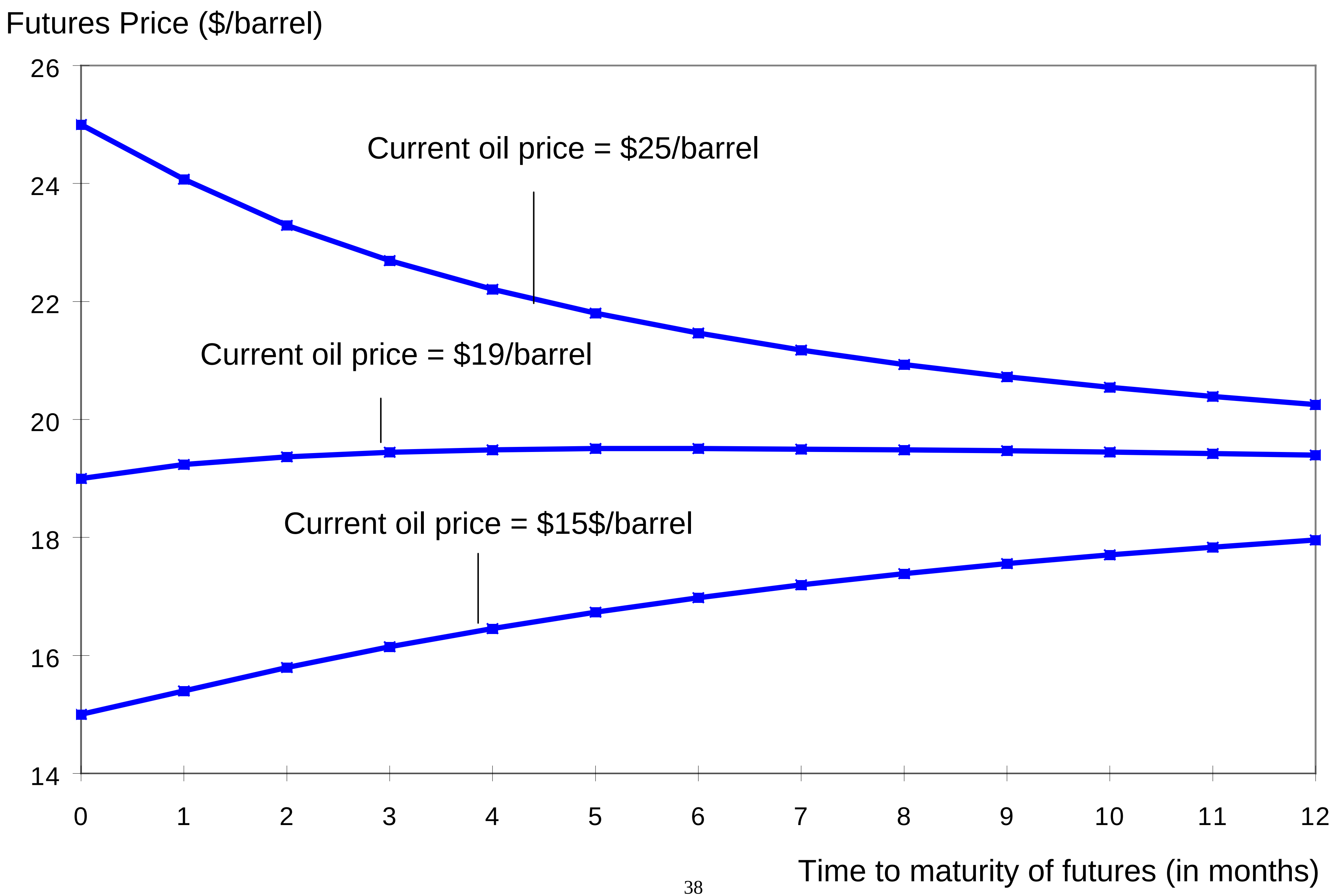
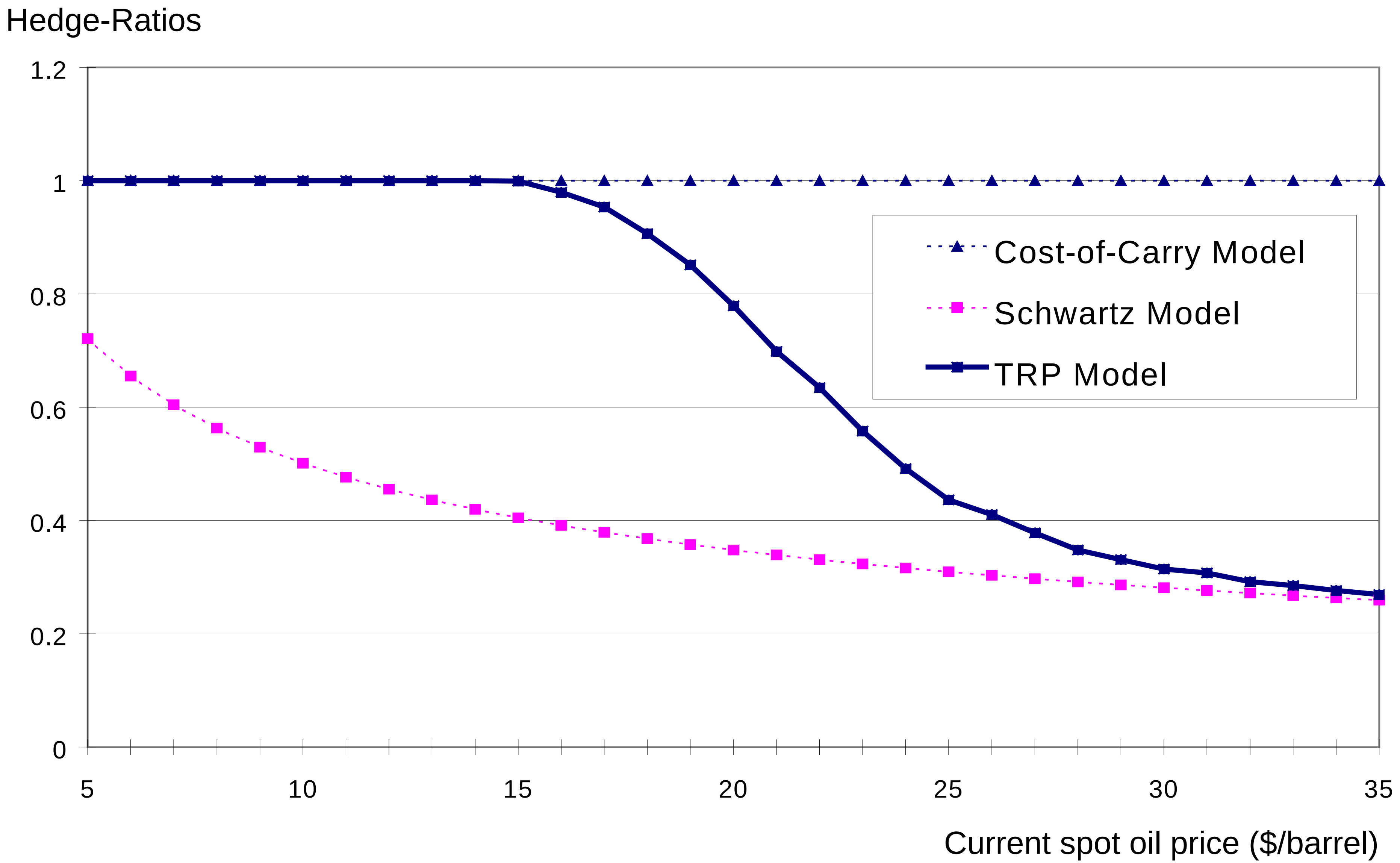

Futures Price ($/barrel)
26
24
22
20
18
16
14
Current oil price = $25/barrel
Current oil price = $19/barrel
Current oil price = 15/barrel
0
1
2
3
4
5
6
7
8
9
10
11
12
Time to maturity of futures (in months)

Figure 4.



Hedge-Ratios
1.2
1
0.8
0.6
0.4
0.2
0
Cost-of-Carry Model
Schwartz Model
TRP Model
5
10
15
20
25
30
35
Current spot oil price ($/barrel)

Figure 5.

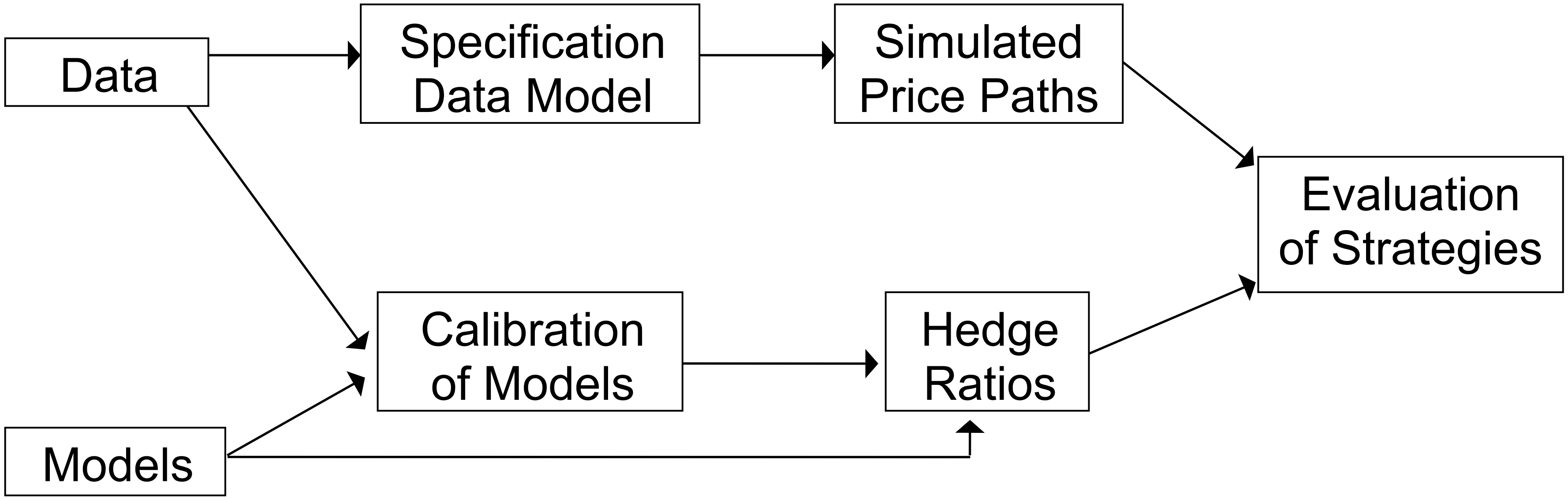

Data
Specification
Data Model
Simulated
Price Paths
Evaluation
of Strategies
Calibration
of Models
Hedge
Ratios
Models

Figure 6.

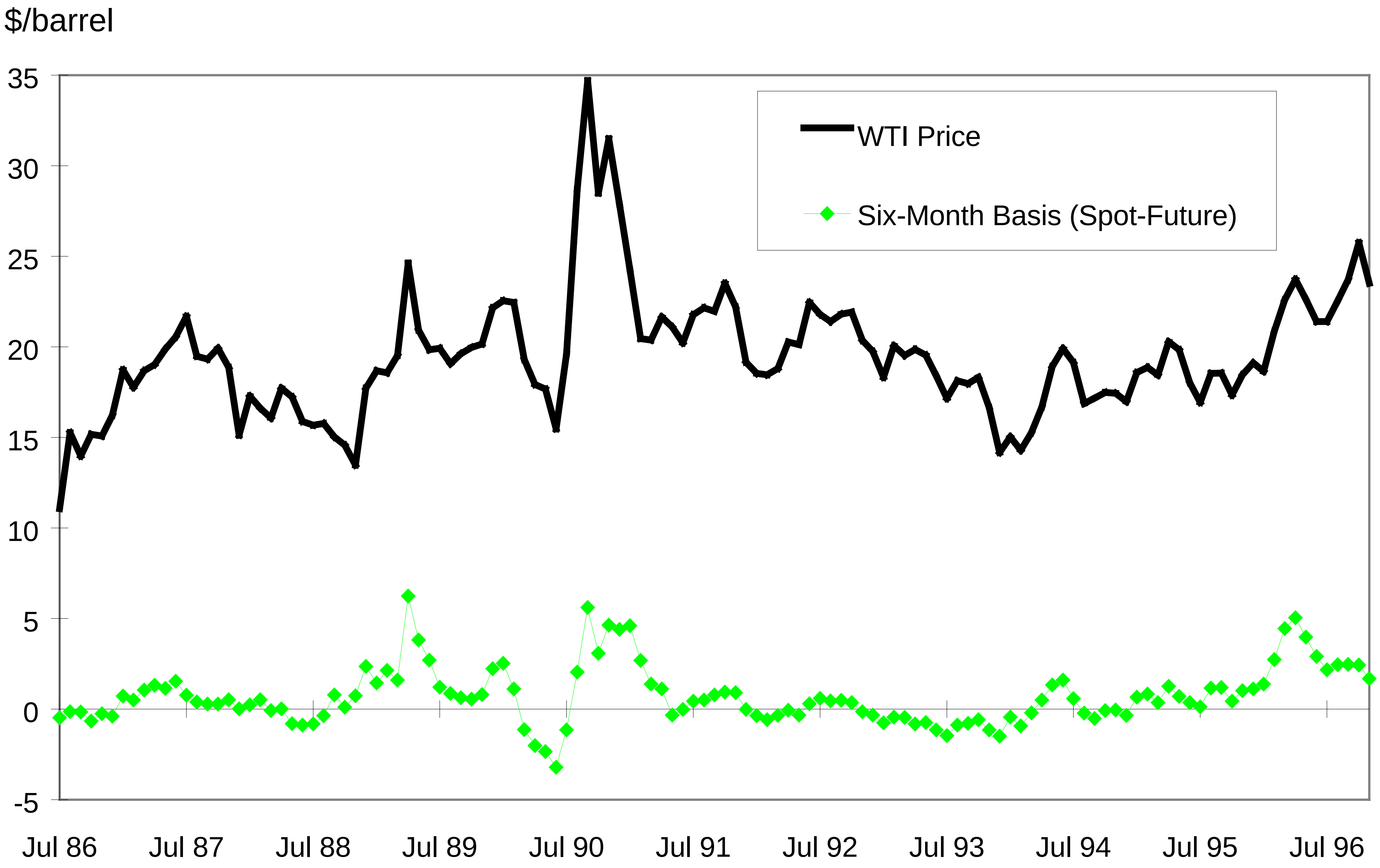

$/barrel
35
30
25
20
15
10
5
0
-5
WTI Price
Six-Month Basis (Spot-Future)
Jul 86
Jul 87
Jul 88
Jul 89
Jul 90
Jul 91
Jul 92
Jul 93
Jul 94
Jul 95
Jul 96

Figure 7.

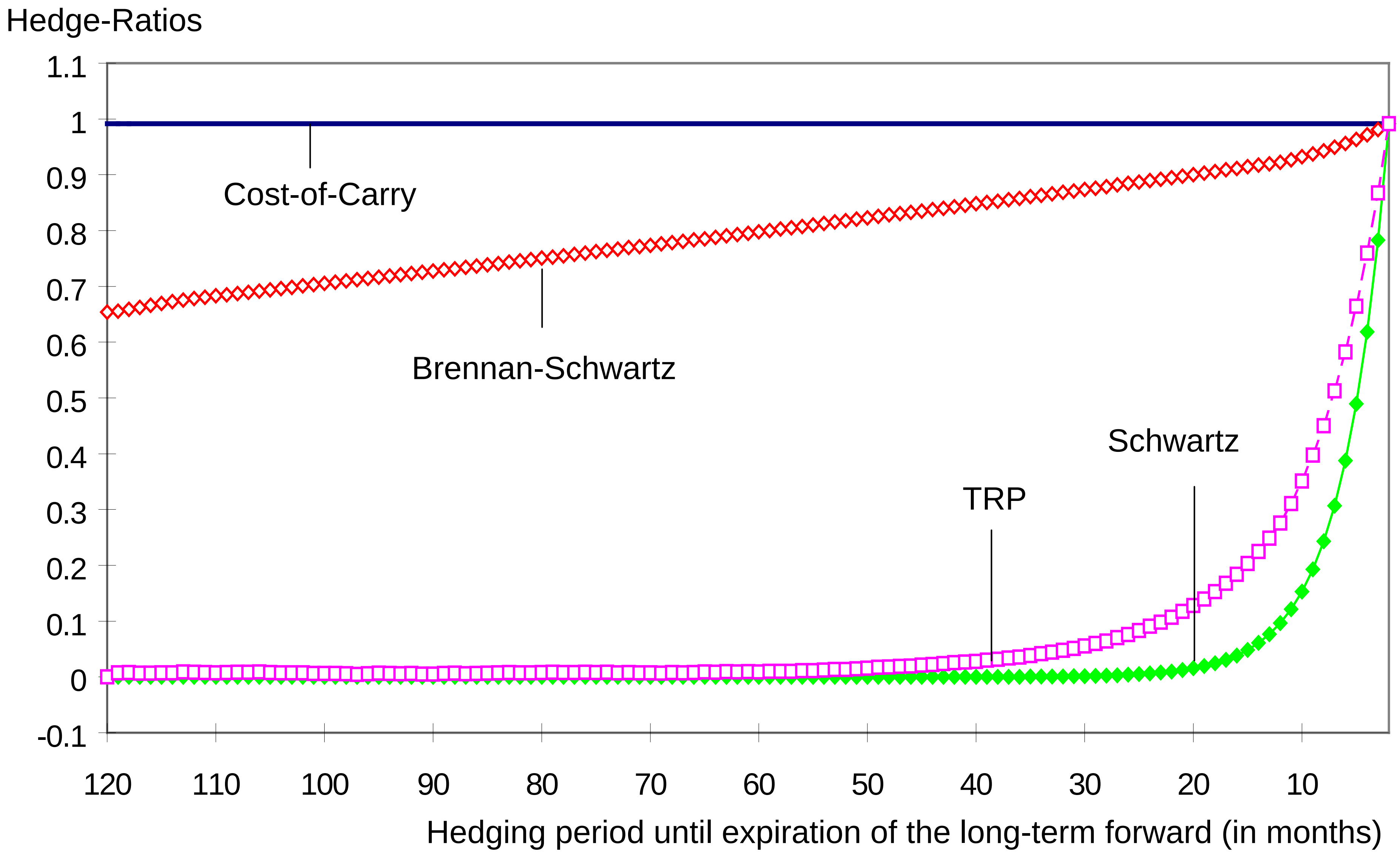

Hedge-Ratios
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
Cost-of-Carry
Brennan-Schwartz
Schwartz
TRP
120
110
100
90
80
70
60
50
40
30
20
10
Hedging period until expiration of the long-term forward (in months)

Figure 8

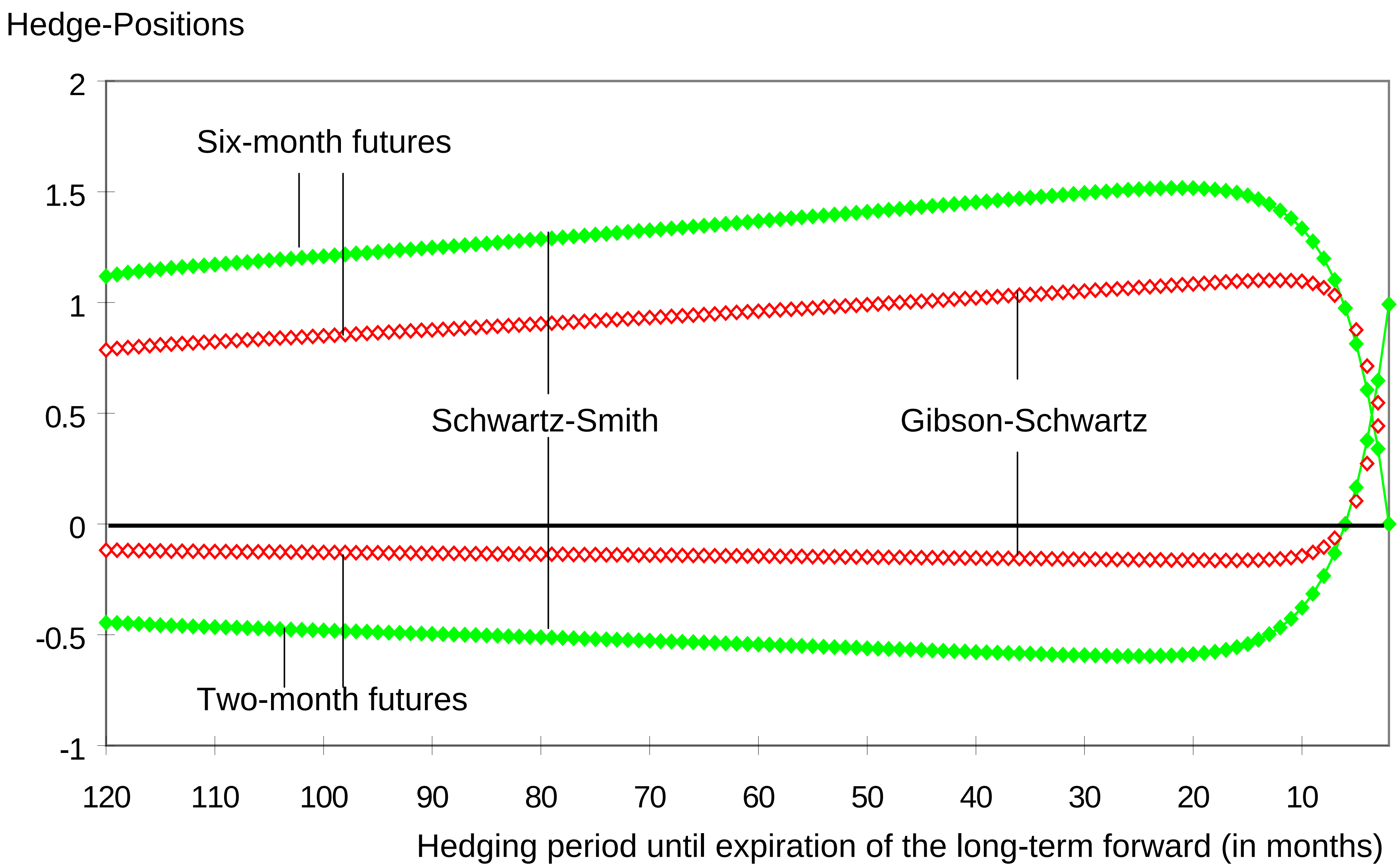

Hedge-Positions
2
1.5
1
0.5
0
-0.5
-1
Six-month futures
Schwartz-Smith
Gibson-Schwartz
Two-month futures
120
110
100
90
80
70
60
50
40
30
20
10
Hedging period until expiration of the long-term forward (in months)

Figure 9.

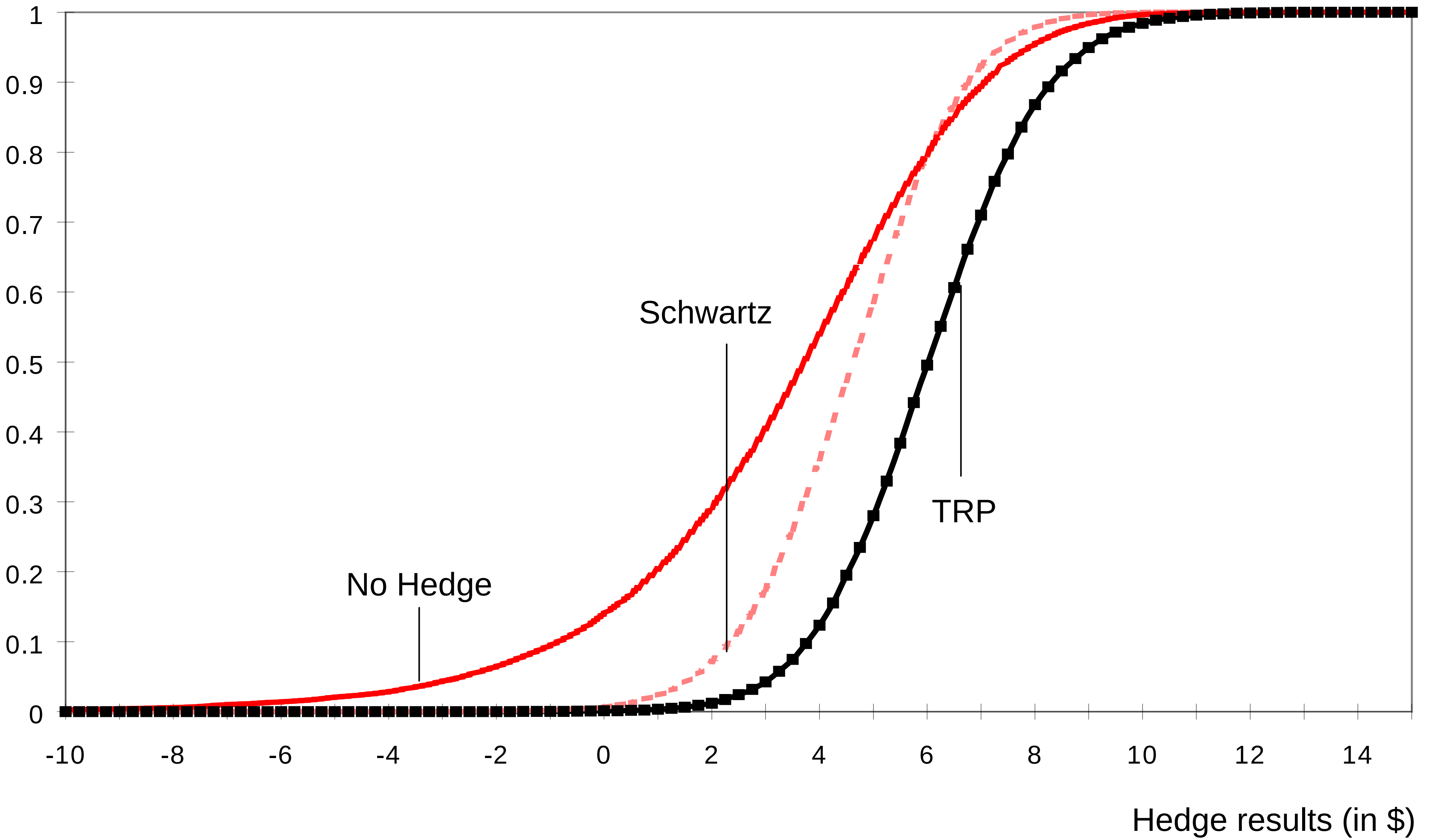

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Schwartz
TRP
No Hedge
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
Hedge results (in $)

Figure 10.

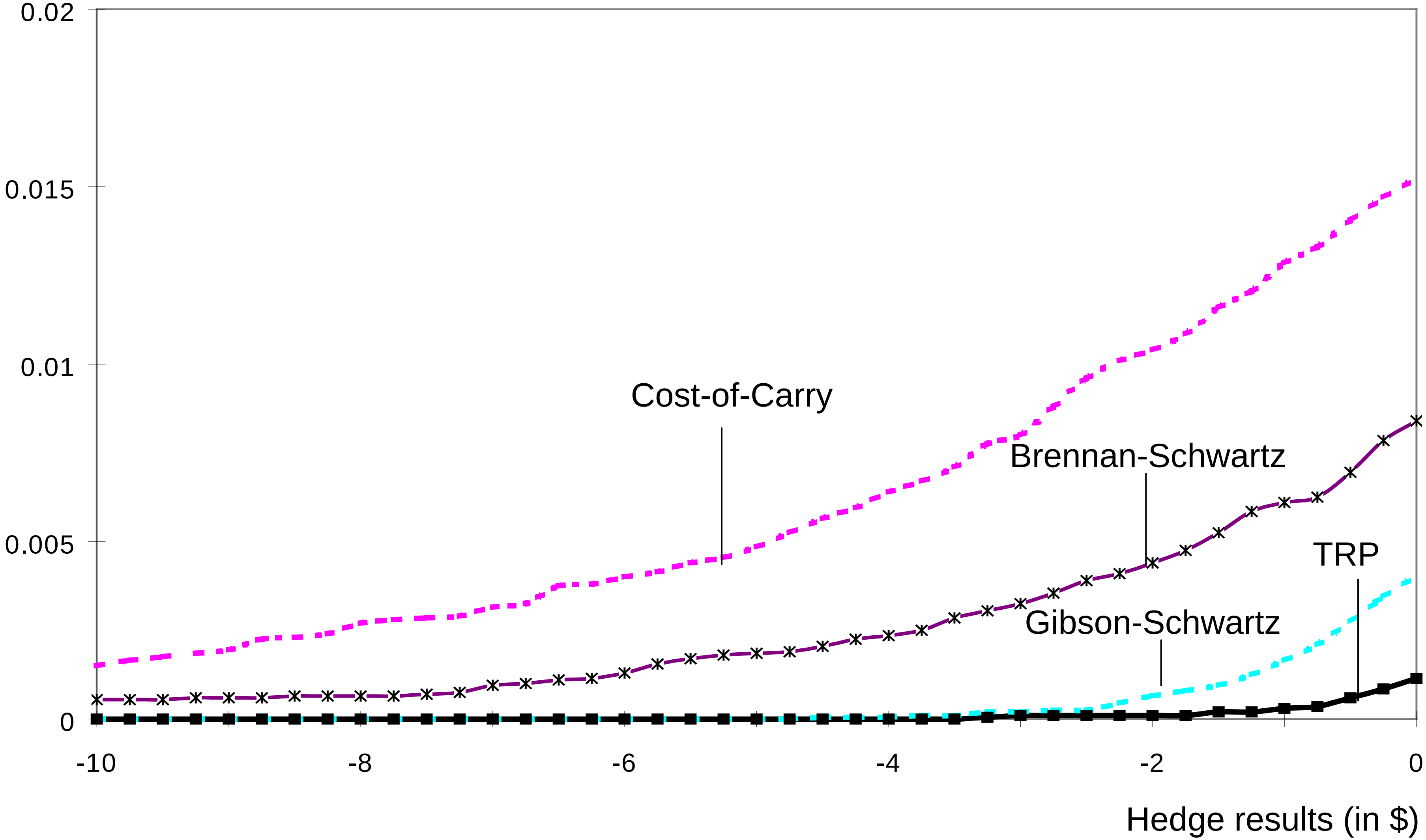

0.02
0.015
0.01
0.005
0
-10
-8
-6
-4
-2
0
Cost-of-Carry
Brennan-Schwartz
TRP
Gibson-Schwartz
Hedge results (in $)

Table I

Estimation Results for the Data Model

This table shows the OLS parameter estimates of the data model (equations 23 to 35) used to generate price scenarios. Estimation is based on monthly observations of spot and futures prices obtained from Platt´s and NYMEX, respectively, for the period July 1986 to November 1996. In the third row the results for the log spot price are given. The fourth to fifteenth row show the results for the relative futures basis, defined as the difference between spot and futures price divided by the spot price, for futures maturities between one and twelve months. The last two columns provide some test statistics for residual autocorrelation (Ljung-Box-Test) and ARCH-effects in the residuals (LR-Test for ARCH).

Equation	Parameter			R^2	Ljung-Box-Test (12 Lags)	LR-Test for ARCH (12 Lags)
	$\hat{a}$ (St. dev.)	$\hat{b}$ (St. dev.)	$\hat{c}$ (St. dev.)		Test Statistic (p-value)	Test Statistic (p-value)
(23)	0,542 (0,150)	0,942 (0,088)	-0,124 (0,086)	0,71	16,93 (0,152)	23,86 (0,021)
(24)	0,001 (0,002)	0,464 (0,046)	-	0,46	17,43 (0,134)	9,38 (0,670)
(25)	0,002 (0,003)	0,636 (0,047)	-	0,61	18,49 (0,101)	10,52 (0,570)
(26)	0,003 (0,003)	0,709 (0,048)	-	0,65	17,31 (0,138)	11,98 (0,447)
(27)	0,004 (0,004)	0,745 (0,048)	-	0,66	16,20 (0,182)	13,94 (0,305)
(28)	0,005 (0,004)	0,768 (0,048)	-	0,67	16,57 (0,166)	16,71 (0,161)
(29)	0,007 (0,004)	0,944 (0,085)	-0,186 (0,081)	0,69	11,48 (0,488)	15,34 (0,223)
(30)	0,007 (0,004)	0,960 (0,086)	-0,194 (0,083)	0,70	10,98 (0,531)	17,33 (0,137)
(31)	0,008 (0,005)	0,974 (0,086)	-0,202 (0,084)	0,70	10,71 (0,554)	19,70 (0,073)
(32)	0,009 (0,005)	0,986 (0,086)	-0,211 (0,085)	0,71	10,41 (0,580)	22,35 (0,034)
(33)	0,009 (0,005)	0,993 (0,087)	-0,212 (0,085)	0,71	10,60 (0,564)	24,75 (0,016)
(34)	0,009 (0,005)	0,994 (0,087)	-0,212 (0,087)	0,71	10,85 (0,541)	24,28 (0,018)
(35)	0,009 (0,005)	1,014 (0,089)	-0,213 (0,090)	0,71	11,65 (0,474)	24,13 (0,020)

Table II

Descriptive Statistics of the Hedge Results: NYMEX Data

This table presents descriptive statistics of the hedge results at the end of a ten-year hedge horizon for six model based hedging strategies and an unhedged delivery commitment to deliver one barrel of oil in ten years. Model based strategies refer to a cost-of-carry model with constant storage costs per barrel, the model of Brennan and Schwartz (1985), Model 1 of Schwartz (1997), the Two-Regime-Pricing (TRP) model, the model of Gibson and Schwartz (1990) and the model of Schwartz and Smith (2000). Hedge results are calculated as the sum of the accumulated proceeds from the monthly mark-to-market of two-month futures contracts and six-month futures contracts (Gibson-Schwartz and Schwartz-Smith models only), and the final pay-off of the ten-year forward contract. The figure is based on the hedge results obtained from 20.000 simulated price scenarios. The data model used to simulate price scenarios was estimated with monthly spot oil prices and NYMEX futures prices covering the period 7/1/86 to 11/25/96.

	Hedging Strategies						
	Cost-of-Carry	Brennan-Schwartz	Schwartz (Model 1) $\gamma = 2.71$	TRP	Gibson-Schwartz $\alpha = 5.62$	Schwartz-Smith $\gamma = 2.71$	No Hedge
Mean (in $)	31.97	26.90	4.58	6.03	8.52	1.14	3.32
St. dev. (in $)	16.09	12.10	1.74	1.80	3.26	6.75	3.28
Minimum (in $)	-21.16	-16.41	-3.78	-3.46	-4.62	-25.97	-17.35
25% Quantile (in $)	20.67	16.47	3.44	4.84	6.30	-3.30	1.55
50% Quantile (in $)	31.25	18.48	4.62	6.02	8.50	1.34	3.72
75% Quantile (in $)	42.26	26.49	5.75	7.21	10.71	5.79	5.58
Maximum (in $)	105.66	78.20	10.55	13.81	22.81	26.55	12.42
Loss Probability	1.53%	0.84%	0.64%	0.12%	0.41%	42.09%	13.99%
Mean Loss (in $)	-0.07	-0.03	-0.01	-0.00	-0.00	-2.17	-0.36

Table III

Hedge Results for the Gibson-Schwartz and Schwartz-Smith Models for Different Values of the Mean-Reversion Parameter: NYMEX Data

This table presents descriptive statistics of the hedge results obtained from strategies based on the models of Gibson and Schwartz (1990) and Schwartz and Smith (2000) for varying values of the mean-reversion parameters α and γ. Hedge results are calculated as the sum of the accumulated proceeds from the monthly mark-to-market of two-month futures contracts and six-month futures contracts, and the final pay-off of the ten-year forward contract. The figure is based on the hedge results obtained from 20.000 simulated price scenarios. The data model used to simulate price scenarios was estimated with monthly spot oil prices and NYMEX futures prices covering the period 7/1/86 to 11/25/96.

	Hedge Results					
	Gibson-Schwartz; Schwartz-Smith					
	$\alpha = \gamma = 1.49$	$\alpha = \gamma = 2.71$	$\alpha = \gamma = 5.62$	$\alpha = \gamma = 9$	$\alpha = \gamma = 12$	$\alpha = \gamma = 15$
Mean (in $)	-11.10	1.14	8.52	10.50	11.07	11.34
St. dev. (in $)	14.97	6.75	3.26	3.23	3.38	3.57
Minimum (in $)	-72.63	-25.97	-4.62	-2.07	-1.47	-2.10
25% Quantile (in $)	-20.88	-3.30	6.30	8.29	8.73	8.89
50% Quantile (in $)	-10.50	1.34	8.50	10.44	10.99	11.24
75% Quantile (in $)	-0.74	5.79	10.71	12.64	13.31	13.68
Maximum (in $)	40.16	26.55	22.81	25.74	26.55	29.57
Loss Probability	76.70%	42.09%	0.41%	0.03%	0.03%	0.04%
Mean Loss (in $)	-12.97	-2.17	-0.00	-0.00	-0.00	-0.00

Table IV

Hedge Results With the Gulf Crisis Period Excluded

This table presents descriptive statistics of the hedge results at the end of a ten-year hedge horizon for six model based hedging strategies and an unhedged delivery commitment to deliver one barrel of oil in ten years. Model based strategies refer to a cost-of-carry model with constant storage costs per barrel, the model of Brennan and Schwartz (1985), Model 1 of Schwartz (1997), the Two-Regime-Pricing (TRP) model, the model of Gibson and Schwartz (1990) and the model of Schwartz and Smith (2000). Hedge results are calculated as the sum of the accumulated proceeds from the monthly mark-to-market of two-month futures contracts and six-month futures contracts (Gibson-Schwartz and Schwartz-Smith models only), and the final pay-off of the ten-year forward contract. The figure is based on the hedge results obtained from 20.000 simulated price scenarios. In contrast to the results in Table II data points referring to the Gulf Crisis period (July 1990 to February 1991) were excluded from the data set before estimating the data model.

	Hedging Strategies						
	Cost-of-Carry	Brennan-Schwartz	Schwartz (Model 1) $\gamma = 2.71$	TRP	Gibson-Schwartz $\alpha = 5.62$	Schwartz-Smith $\gamma = 2.71$	No Hedge
Mean (in $)	15.98	13.83	4.50	5.28	8.33	6.12	3.92
St. dev. (in $)	18.28	14.09	1.62	1.94	2.92	5.33	2.53
Minimum (in $)	-45.12	-35.43	-1.81	-1.60	-3.04	-16.00	-9.42
25% Quantile (in $)	3.20	4.09	3.43	3.97	6.33	2.60	2.35
50% Quantile (in $)	15.35	13.52	4.51	5.25	8.30	6.16	4.06
75% Quantile (in $)	27.91	23.14	5.58	6.56	10.28	9.73	5.67
Maximum (in $)	100.36	74.90	10.40	13.13	21.52	25.30	12.10
Loss Probability	19.19%	16.32%	0.36%	0.25%	0.18%	12.37	6.52%
Mean Loss (in $)	-1.74	-1.13	-0.00	-0.00	-0.00	-0.35	-0.10

Table V

Descriptive Statistics of the Hedge Results: Enron Data

This table presents descriptive statistics of the hedge results at the end of a ten-year hedge horizon for six model based hedging strategies and an unhedged delivery commitment to deliver one barrel of oil in ten years. Model based strategies refer to a cost-of-carry model with constant storage costs per barrel, the model of Brennan and Schwartz (1985), Model 1 of Schwartz (1997), the Two-Regime-Pricing (TRP) model, the model of Gibson and Schwartz (1990) and the model of Schwartz and Smith (2000). Hedge results are calculated as the sum of the accumulated proceeds from the monthly mark-to-market of two-month futures contracts and six-month futures contracts (Gibson-Schwartz and Schwartz-Smith models only), and the final pay-off of the ten-year forward contract. The figure is based on the hedge results obtained from 20.000 simulated price scenarios. In contrast to the results in Table II the data model used to simulate price scenarios was estimated with monthly spot oil prices and Enron forward prices covering the period 1/18/93 to 8/30/96.

	Hedging Strategies						
	Cost-of-Carry	Brennan-Schwartz	Schwartz (Model 1) $\gamma = 2.71$	TRP	Gibson-Schwartz $\alpha = 5.62$	Schwartz-Smith $\gamma = 2.71$	No Hedge
Mean (in $)	34.21	29.30	4.03	6.00	2.70	-7.92	2.45
St. dev. (in $)	19.43	14.70	1.25	2.04	4.02	4.27	2.22
Minimum (in $)	-32.84	-25.94	-0.90	-4.07	-15.83	-26.72	-6.44
25% Quantile (in $)	20.69	19.32	3.19	4.63	-0.03	-10.77	0.98
50% Quantile (in $)	33.62	29.28	4.03	6.04	2.64	-7.91	2.54
75% Quantile (in $)	47.32	39.47	4.87	7.40	5.38	-5.03	4.01
Maximum (in $)	126.85	89.93	9.17	13.47	19.55	10.36	9.88
Loss Probability	3.32%	2.38%	0.11%	0.24%	25.28%	97.01%	13.78%
Mean Loss (in $)	-0.22	-0.12	-0.00	-0.00	-0.59	-7.97	-0.17

Table VI

Hedge Results for the Gibson-Schwartz and Schwartz-Smith Models for Different Values of the Mean-Reversion Parameter: Enron Data

This table presents descriptive statistics of the hedge results obtained from strategies based on the models of Gibson and Schwartz (1990) and Schwartz and Smith (2000) for varying values of the mean-reversion parameters α and γ. Hedge results are calculated as the sum of the accumulated proceeds from the monthly mark-to-market of two-month futures contracts and six-month futures contracts, and the final pay-off of the ten-year forward contract. The figure is based on 20.000 hedge results obtained from 20.000 simulated price scenarios. In contrast to the results in Table III the data model used to simulate price scenarios was estimated with monthly spot oil prices and Enron forward prices covering the period 1/18/93 to 8/30/96.

	Hedge Results					
	Gibson-Schwartz; Schwartz-Smith					
	$\alpha=\gamma=1.49$	$\alpha=\gamma=2.71$	$\alpha=\gamma=5.62$	$\alpha=\gamma=9$	$\alpha=\gamma=12$	$\alpha=\gamma=15$
Mean (in $)	-25.52	-7.92	2.70	5.56	6.39	6.78
St. dev. (in $)	11.51	4.27	4.02	4.88	5.15	5.30
Minimum (in $)	-70.83	-26.72	-15.83	-16.86	-17.39	-17.86
25% Quantile (in $)	-33.39	-10.77	-0.03	2.23	2.86	3.15
50% Quantile (in $)	-25.59	-7.91	2.64	5.51	6.34	6.74
75% Quantile (in $)	-17.69	-5.03	5.38	8.81	9.86	10.34
Maximum (in $)	21.33	10.36	19.55	25.23	27.36	28.25
Loss Probability	98.66%	97.01%	25.28%	12.70%	10.52%	9.77%
Mean Loss (in $)	-25.57	-7.97	-0.59	-0.29	-0.25	-0.24