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# Technical Change and Factor Shares

by

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## Abstract

The implications of technical change that directly alters factor shares are examined. Such change can lower the income of some factors of production even when it raises total output, thus offering a possible explanation for episodes of social conflict such as the Luddite uprisings in $19^{th}$ century England and the recent divergence in the U. S. between wages for skilled and unskilled labor. An explanation also why underdeveloped countries do not adopt the latest technology but continue to use outmoded production methods. Total factor productivity is shown to be a misleading measure of technical progress. Share-altering technical change brings into question the plausibility of a wide class of endogenous growth models.

## I. Introduction

This paper examines some implications of technical change that alters the relative shares of output accruing to the factors of production. This type of technical change was studied extensively in the first "golden age" of growth theory (after Solow-Swan to about 1980) but has been largely ignored in most of the second golden age (the post-Romer literature on endogenous growth theory), which generally treats technical change exclusively as an increase in total factor productivity. For example, the usual quality ladder model of endogenous growth has an aggregate production function of the form

$$Y = AL^{\alpha}\sum_{i=1}^{N}(X_i^{q_i})^{1-\alpha}$$

where L is labor, the $X_i$ are intermediate goods produced in other sectors, and the $q_i$ are indices of the quality of the $X_i$. Technical change is captured by increases in the quality indices. The factor shares $\alpha$ and 1-$\alpha$ are unchanged by quality improvements. In the real business cycle literature, productivity shocks always take the form of changes in the total factor productivity coefficient in front of the production function. Again, factor shares are assumed unchanged.

There are at least two reasons why share-neutral technical change seems likely to be the exception rather than the rule. The first is the nature of the production function itself. A production function is merely a mathematically convenient way to express the relation between output and inputs. Technical change is by definition a change in that very relation. There seems no reason whatsoever to assume *a priori* that such a change in the production function is confined to only one of the function's parameters, and changes in parameters other than the neutral TFP coefficient generally alter factor shares.[1]

The second and more important reason for doubting share-neutrality of technical change is that the data show systematic changes in factor shares over time, a fact that has been known for several decades. Table 1, taken from Sato (1970), reports capital's share in the private non-farm sector of the US economy over the period 1909-1960. Figure 1 plots the data and shows quite clearly that physical capital's share of output fluctuated considerably from year to year and followed an overall downward trend over the entire period. Capital's share has a maximum of 0.397 and a minimum of 0.301, nearly a 33 percent increase from the min to the max, a large swing for a number that goes into the exponent of the Cobb-Douglas production function. The time trend is statistically significant. Regressing the log of capital's share on time yields

$$\alpha_t = \underset{(0.96785)}{1.56048} - \underset{(0.00050)}{0.00137} \log t$$

where $\alpha$ is capital's share and numbers in parentheses are standard errors. The p-value of time's coefficient is 0.0085. In addition, there is a great deal of fluctuation about the trend, as indicated by the adjusted R-squared value of only 0.113. These data belie the oft-heard assertion that "capital's share is roughly constant at about thirty percent"; capital's share quite clearly is not constant over either the short run or the long run. Sato and Hoffman (1968) present data for Japan over the period 1930-1960. The regression coefficient of capital's share on the log of time is -0.039 with a standard error of 0.014 (p-value of 0.0094) and an adjusted R-squared of 0.189. Kahn and Lim (1998) show that the shares of equipment, production workers, and non-

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[1]For example, in the CES production function

$$Y = A[aK^{\psi} + (1-a)L^{\psi}]^{1/\psi}$$

changes in either $a$ or $\psi$ alter factor shares.

production workers in the U. S. all have trends and variation about those trends over the more recent period 1959-91. Blanchard (1997, 1998) finds that capital's share in Europe has been similarly variable, falling before the mid-1980s and then rising sharply thereafter in many European countries. We have more than just simple statistics showing changes in capital's share. A wide range of formal econometric investigations of biased technical progress finds capital and labor shares changing over time. Sato and Kendrick (1963), David and Klundert (1965), Sato (1970), Sato and Hoffman (1968), to name a few, do so by estimating production functions; Binswanger (1974), Stevenson (1980), and Kumbhakar and Lozano-Vivas (2000) reach the same conclusions by estimating cost functions. Finally, the past thirty years or so in the US have seen a large increase in the share of output going to skilled labor at the expense of the share going to unskilled labor (Bound and Johnson, 1995).[2] So the income shares of subsets of the labor force also have shown movements over time. Factor shares are not constant, even approximately.

The present paper takes biased technical change as given and analyzes some of its macroeconomic implications without explaining where such change comes from.[3] A number of

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[2]Katz and Murphy (1992), among others, document that the wages of skilled workers have increased relative to those of unskilled workers. Bound and Johnson (1995) show that the supply of skilled workers also has increased relative to that of unskilled workers, thus establishing that skilled workers' income share has risen relative to that of unskilled workers.

[3]An enormous early literature examined the origins and extent of biased technical progress, beginning with Kennedy (1964). The recent shift in relative wages for skilled and unskilled labor in the U. S. has revived research into biased change. See Caselli (1999) and Acemoglu (2000) for two recent theoretical investigations. Some investigators, such as Borjas and Ramey (1994) and Dinopoulos, Syropoulos, and Xu (1999), argue that changes in the pattern of trade explain the relative wage shift between skilled and unskilled workers. Although possibly true, that explanation begs the question of why trade patterns have shifted. Presumably, the ultimate explanation is changes in technology, unless one wants to invoke changes in tastes. Baldwin and Cain (1997), Harrigan and Balaban (1999), and Bartel and Sicherman (1999) are among those providing evidence that technical change explains the relative wage shift.

results are obtained. The theory offers an explanation for the social conflict between owners of capital and labor, such as the famous Luddite uprisings in England in the early 19th century. It also offers an explanation for why some countries, notably underdeveloped ones, do not adopt the latest technological breakthroughs but continue to use apparently outdated production methods. A large fraction of endogenous growth models are called into question by the implications of technical change that alters factor shares. Finally, it is shown that total factor productivity can fall (rise) as the result of technical change that raises (lowers) aggregate output, suggesting that total factor productivity is a rather poor indicator of technical progress.

## II. Two Factors of Production

We start with the case of only two factors of production, capital and labor. We consider later a three factor model that separates physical and human capital. The two factor case is much easier analytically, permitting detailed dynamic analysis that cannot be done in the three factor framework. Moreover, the essence of most macroeconomics is aggregation, so the two factor model with capital of all forms treated as a single entity may have some methodological appeal, at least to those tending to be lumpers rather than splitters. In all that we do, production is Cobb-Douglas:

$$Y = F(K,L) = AK^{\alpha}L^{1-\alpha} \tag{1}$$

where Y is output, K is capital, L is labor, A is total factor productivity, and $\alpha$ is capital's share. The marginal products of capital and labor are

(2) $$F_K = \alpha A(K/L)^{\alpha-1}$$

$$\equiv \text{MPK}$$

(3) $$F_L = (1-\alpha)A(K/L)^{\alpha}$$

$$\equiv \text{MPL}$$

The use of Cobb-Douglas production to discuss variable factor shares may seem surprising; the usual assumption with Cobb-Douglas is that shares are constant. Samuelson (1965), for example, begins his discussion of induced innovation with the Cobb-Douglas case and imposes without discussion the restriction that factor shares are constant. However, Sato and Beckmann (1968) propose fourteen definitions of neutral technical progress, three of which permit Cobb-Douglas production with changing factor shares. They then do some empirical work with data from Germany, Japan, and the United States and show that such a production function is acceptable for all three countries and for Japan even offers the best fit of the various types considered. So the use of Cobb-Douglas production with changing factor shares in the following theoretical discussion is not at all unreasonable.[4]

II.A. *Simple Technical Change With Fixed K and L.*

We begin by examining the situation where a technical invention alters only the factor shares, leaving total factor productivity *A* the same. We also assume for now that both capital and labor are fixed. This simple case allows us to see the essence of what is going on.

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[4]A number of early studies (e.g., Sato, 1970) present evidence that the elasticity of substitution is less than one, which would rule out the Cobb-Douglas function. However, those studies all fail to distinguish between raw labor time and human capital, attributing all labor income to man-hours. If human capital were lumped with physical capital rather than with man-hours, the estimated elasticity of substitution would rise substantially. It thus is not clear that the Cobb-Douglas function is ruled out.

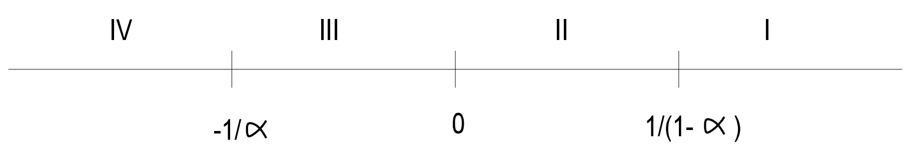
Suppose an invention is discovered that alters $\alpha$; for expository ease, suppose that the invention raises $\alpha$.[5] Will the invention be adopted? To decide, we examine the effect of an increase in $\alpha$ on the incomes of the total economy and the two factors of production. The derivatives of Y, MPK, and MPL with respect to $\alpha$ are

(4)
$$\partial Y/\partial\alpha \;=\; AK^{\alpha}L^{1-\alpha}\ln(K/L)$$
$$\gtrless 0 \quad \text{as} \quad \ln(K/L) \gtrless 0$$

(5)
$$\partial MPK/\partial\alpha \;=\; A(K/L)^{\alpha-1}[1+\alpha\ln(K/L)]$$
$$\gtrless 0 \quad \text{as} \quad \ln(K/L) \gtrless -1/\alpha$$

(6)
$$\partial \mathrm{MPL}/\partial\alpha \;=\; AK^{\alpha}L^{-\alpha}[(1-\alpha)\ln(K/L)-1]$$
$$\gtrless 0 \quad \text{as} \quad \ln(K/L) \gtrless 1/(1-\alpha)$$

We thus have four phases, depending on the value of ln(K/L)




which have the following characteristics:

I: $0 < \partial Y/\partial\alpha, \partial MPK/\partial\alpha, \partial MPL/\partial\alpha$

II. $\partial MPL/\partial\alpha < 0 < \partial Y/\partial\alpha, \partial MPK/\partial\alpha$

III. $\partial Y/\partial\alpha, \partial MPL/\partial\alpha < 0 < \partial MPK/\partial\alpha$

IV. $\partial Y/\partial\alpha, \partial MPK/\partial\alpha, \partial MPL/\partial\alpha < 0$

Recall that for the moment we are treating K and L, and therefore K/L, as constant; consequently,

[5] Results for the case where the invention reduces $\alpha$ are completely symmetric to those discussed here.

each factor's total income moves in the same direction as its marginal product.

The pre-invention values of $\alpha$ and K/L determine which phase the economy is in. A large capital/labor ratio places the economy farther to the right on the phase line (i.e., in a lower numbered phase), given an initial value of $\alpha$. The initial value of $\alpha$ affects the positions of the boundaries between phases I and II and between phases III and IV. An increase in the initial value of $\alpha$ moves both boundaries to the right, reducing the size of phase III and increasing the size of phase II (phases I and IV both remain infinite in size). The boundary between phases II and III is anchored at zero and so does not depend on the initial value of $\alpha$.

The intuition for why the economy's phase depends on the capital/labor ratio is not difficult. An increase in $\alpha$ increases the exponent of capital and reduces the exponent of labor in the production function. If capital is relatively abundant, this shift in exponent magnitude from labor to capital will raise output; if capital is relatively scarce, it will reduce output. A similar intuition underlies the changes in factor incomes.

If the economy is in phase I, adopting the invention is Pareto optimal; the income of both factors of production rises. Although labor's share of output falls with the increase in $\alpha$, total income rises by more than enough to offset this effect, as shown by the increase in MPL and thus labor income. Phase IV is the symmetric case, in which adoption reduces social welfare because it would make everyone worse off.

Phases II and III are more difficult. In phase II, total output rises, capital's income also rises, but labor's income falls. In phase III, only capital's income rises; both total income and labor's income fall. Adoption is not Pareto optimal in either case. Adoption is net benefit optimal in phase II and net benefit suboptimal in phase III because adoption raises total output in phase II and lowers it in phase III. If agents are identical, however, we can analyze the economy with the representative agent model; Pareto optimality then coincides with net benefit optimality. In that case, adoption is Pareto optimal if the economy is in phase II; it is Pareto suboptimal if the economy is in phase III (at least for now; we will see that adoption can be socially optimal even if the economy is in phase III once the economy's dynamic response is taken into account).

A general conclusion that emerges is that technical change that alters only factor shares will have increasingly broad social benefits the more abundant is the factor whose share increases. In particular, for a given capital/labor ratio, this kind of technical change can raise total output only if it favors the relatively abundant factor. As we will see below, this latter implication does not necessarily hold if the capital/labor ratio can change in response to the technical change or if the technical change alters total factor productivity as well as factor shares; nonetheless, even in these more general settings, share-altering technical change tends to favor the abundant factor. This tendency may help explain Berman's (2000) recent finding from a three-dimensional panel (industry, country, time) that, in the decade of the 1980s covered by the panel, technical change was strongly biased against less-skilled workers and in favor of both physical and human capital. If research and development is conducted largely by industries in developed countries for the benefit of those same industries, it will tend to favor technical advances that increase the income shares of the relatively abundant factors - that is, of physical and human capital. We will return to this line of argument below, when we consider which countries are likely to adopt a given type of technical change.

As interesting as the normative side of adoption is the positive side. It is clear that the invention will be adopted if the economy is in phase I and will not be adopted in phase IV. The economy's competitive equilibrium can be obtained as the solution to a central planning problem. Given that adoption makes everyone better off in phase I, a central planner would adopt in phase I.  Consequently, the competitive equilibrium solution for an economy in phase I must be adoption.  Similarly, the planner would not adopt in phase IV because adoption makes everyone worse off.  The competitive equilibrium in phase IV therefore is not to adopt.  If the economy is in phase II or III, we cannot say whether adoption will occur.  The outcome depends on who makes the adoption decision.  If owners of capital decide, then the invention will be adopted if the economy is in either phase II or phase III because captial's income is raised in both cases.  Similarly, if labor makes the decision, the invention may not be adopted in either phase II or phase III because labor loses in either case.  We thus have two interesting possibilities.  First, an invention that raises aggregate output may not be adopted because it is opposed by one of the factors of production.  Second, an invention that lowers aggregate output may be adopted anyway because it is favored by one of the factors of production.  Recall that in all this discussion we are assuming the invention raises capital's share $\alpha$.  Symmetric results obtain if the invention raises labor's share $1-\alpha$ instead.

The first of the foregoing possibilities may explain the famous episode of the Luddites, who sporadically destroyed textile machinery in various English towns over the period 1811-1816.  The Luddites blamed the machines for the prevailing unemployment and low wages.  The Luddites are usually written off in the history books as misguided fanatics, but in fact they may have been right about the cause of low wages, at least in their industries.  If the textile machines (or perhaps "new and improved" machines that replaced older ones) raised capital's share, and if England at the time was in phase II, then the introduction of the machines would have simultaneously raised total output and capital income on the one hand and reduced labor's income on the other hand.  Indeed, the Luddite uprisings occurred fairly early in the Industrial Revolution, when the capital/labor ratio was low compared to the levels it subsequently reached.

A low capital/labor ratio could have put England in a phase below phase I and thus could have created conditions favoring uprisings by the workers against technical change that raised capital's share. The subject merits further examination by economic historians. We will see a similar phenomenon at work later when we introduce human capital and discuss a possible explanation for the recent shift in income shares among low-skill and high-skill workers in the United States.

The analysis also has an interesting implication for the adoption of new technology by different countries. Suppose there are two countries, A and B, that use the same production technology (i.e., the same production function) but have different capital/labor ratios, with country A's ratio exceeding that of country B. Suppose also that the capital/labor ratios are such that country A is in phase I (relatively abundant capital) and country B is in phase IV (relatively scarce capital). An invention that raises $\alpha$ will be adopted by country A and will not be adopted by country B. Once country A has adopted the invention, it no longer has the same production technology as country B. The usual argument that all countries should have the same production functions because technology is freely transferrable therefore may be incorrect, suggesting that cross-country estimation and testing based on such an assumption may be misspecified.

Adoption of the invention alters the conditions that determine the desirability of subsequent inventions. In the example just discussed, after country A adopts the invention, its value of $\alpha$ is higher than before. This change moves the boundary between phases I and II to the right, increasing the range of capital/labor values that fall in phase II and thus tending to move country A out of phase I into phase II. This effect tends to reduce the acceptability of subsequent inventions that raise $\alpha$.[6] However, as we will see below, the change in $\alpha$ brought about by adoption also increases the steady state value of the capital/labor ratio, which puts country A deeper in phase I. It therefore is unclear whether adoption moves country A away from or toward the boundary between phases I and II.[7] We return to this issue after we discuss the economy's dynamic response to the invention. Before we study system dynamics, we examine mixed technical change, in which both total factor productivity *A* and capital's share $\alpha$ are altered by an invention.

II.B. *Mixed Technical Change.*

The previous discussion has been useful for explaining the effects of changes in factor shares brought about by an invention. However, production functions are just convenient mathematical representations of reality, and reality is very complex. One therefore should expect that a real-world invention generally would alter all the parameters of the production function. We can call this mixed technical change. In our simple Cobb-Douglas case, there are only two parameters to consider: total factor productivity *A* and capital's share $\alpha$. We now examine the effects of mixed technical change that alters both of these parameters.

The total differentials for Y, MPK, and MPL are

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[6]This discussion recalls and is related to the concept of the technology frontier, introduced by Kennedy (1964), subsequently discussed and expanded by Samuelson (1965) and others, and recently rediscovered by Caselli and Coleman (2000).

[7]The mechanism here differs fundamentally from that discussed by Basu and Weil (1998), even though the capital/labor ratio is central in both treatments. Basu and Weil assume a Cobb-Douglas production function with a fixed capital share but also assume that total factor productivity depends on the capital/labor ratio:

$$y = A(k,t)k^{\alpha}$$

where y=Y/L is output per person and k=K/L is the capital/labor ratio. In addition, they assume that for each k there is a maximum possible value of A, denoted A*(k). In contrast, the present treatment takes A as independent of k but allows capital's share $\alpha$ to change over time.

(7)
$$dY = K^{\alpha}L^{1-\alpha}[dA + A\ln(K/L)d\alpha]$$
$$\gtreqless 0 \quad \text{as} \quad dA + A\ln(K/L)d\alpha \gtreqless 0$$

(8)
$$dMPK = K^{\alpha-1}L^{1-\alpha}\{\alpha dA + A[1 + \alpha\ln(K/L)]d\alpha\}$$
$$\gtreqless 0 \quad \text{as} \quad \alpha dA + A[1 + \alpha\ln(K/L)]d\alpha \gtreqless 0$$
$$\Leftrightarrow \text{ as } \quad dA + A\ln(K/L)d\alpha \gtreqless -\frac{A}{\alpha}d\alpha$$

(9)
$$dMPL = K^{\alpha}L^{-\alpha}\{(1-\alpha)dA + A[(1-\alpha)\ln(K/L) - 1]d\alpha\}$$
$$\gtreqless 0 \quad \text{as} \quad (1-\alpha)dA + A[(1-\alpha)\ln(K/L) - 1]d\alpha \gtreqless 0$$
$$\Leftrightarrow \text{ as } \quad dA + A\ln(K/L)d\alpha \gtreqless -\frac{A}{(1-\alpha)}d\alpha$$

Denote dA + A ln (K/L)dα by Z. Then the three foregoing conditions can be written as

(10)
$$dY \gtreqless 0 \quad \text{as} \quad Z \gtreqless 0$$

(11)
$$dMPK \gtreqless 0 \quad \text{as} \quad Z \gtreqless -\frac{A}{\alpha}d\alpha$$

(12)
$$dMPL \gtreqless 0 \quad \text{as} \quad Z \gtreqless \frac{A}{1-\alpha}d\alpha$$

We can write a chain inequality showing the relation among the three terms on the far right sides of the foregoing expressions:

(13)
$$\frac{A}{1-\alpha}d\alpha \gtreqless 0 \gtreqless -\frac{A}{\alpha}d\alpha \quad \text{as} \quad d\alpha \gtreqless 0$$

Because of the symmetry of K and L in the production function, we can once again simplify discussion by restricting attention to the case where d$\alpha$ > 0. We then obtain the same results as in the previous section with only slight modification. There are four phases the economy can be in, depending on the value of Z and as shown in the following diagram:


The discussion of when adoption will be socially optimal and when it will occur in competitive equilibrium parallels that of the previous section, with the capital/labor ratio replaced by Z. We therefore need not pursue that discussion in any further detail here, except to note that technical change generally will involve changes in both A and $\alpha$ so that, in contrast to the analysis of the preceding section, it no longer is true that only technical change that favors the abundant factor will raise total output Y.

This analysis suggests that total factor productivity *A* may not be a reliable indicator of technical progress. It is possible for Y, MPK, and MPL all to increase in response to a technical change even though the change reduces total factor productivity. For Y, MPK, and MPL to increase in response to the technical change, the economy must be in phase I. The required condition is

(14)
$$dA + A\ln(K/L)d\alpha - \frac{A}{1-\alpha}d\alpha \; > \; 0$$

$$\Leftrightarrow \quad dA \; > \; [\frac{A}{1-\alpha} - A\ln(K/L)]d\alpha$$

Recall that for this discussion we are assuming d$\alpha$ > 0, so the economy can be in phase I and still have d*A* negative if the quantity in brackets is negative, that is, if ln(K/L) > 1/(1-$\alpha$). We thus could see situations in which technical progress raises total output, capital's income, and labor's income but is nonetheless associated with a *decline* in total factor productivity. Symmetrically, we could have situations in which Y, MPK, and MPL all fall even though A rises. The usefulness of total factor productivity as a measure of technical progress thus is questionable.

II.C. *Dynamics.*

We now examine the dynamic response of the economy to an episode of technical progress that alters factor shares. To make the analysis possible, we must assume the existence of a representative agent so that we can posit and solve a social planning problem. We also make three simplifying assumptions. First, labor supply L is taken as fixed throughout the discussion. Labor supply can vary either because the population grows or because hours per worker change. Allowing exogenous population growth adds nothing to the analysis, so we assume population growth is zero. Allowing hours of labor to be set as part of the optimization complicates the analysis in the usual way without providing any useful insights, so we suppose that everyone works a fixed amount. We thus are in the framework of the standard Cass growth model with population growth set to zero. Second, we assume $dF_K > 0$ because otherwise the economy would be in Phase IV, so that the invention would benefit no one and would not be adopted. Assuming $dF_K > 0$ is equivalent to assuming that $\ln(K/L) > -1/\alpha$ (see section II.A above). Third, we suppose through most of the discussion that only $\alpha$ is changed with total factor productivity *A* unaffected. As we already have seen, the conditions for the economy to be in its various phases are very similar for the cases when *A* does and does not change. So for simplicity, we hold A fixed for most of the analysis; at the end, we examine briefly the effect of allowing it to change.

The social planner's problem is to maximize the representative agent's lifetime utility subject to the aggregate budget constraint:[8]

(15)
$$\max_{C} \int U(C) e^{-\rho t} dt$$

$$\text{s.t.} \quad dK/dt = F(K,L) - C - \delta K$$

where U is a concave utility function, C is consumption, ρ is the rate of time preference, and δ is the depreciation rate. The current value Hamiltonian is

(16)
$$H = U(C) + \psi[F(K,L) - C - \delta K]$$

where ψ is the costate variable. The necessary conditions are

(17)
$$dK/dt = \partial H/\partial \psi = F(K,L) - C - \delta K$$

(18)
$$d\psi/dt = -\partial H/\partial K + \rho\psi = -\psi[\alpha A K^{\alpha-1} L^{1-\alpha} - (\rho+\delta)]$$

(19)
$$\partial H/\partial C = 0 = U_C - \psi$$

(20)
$$K_0 \text{ given}$$

(21)
$$\lim_{t\to\infty} \psi_t K_t e^{-\rho t} = 0$$

We obtain the equilibrium loci from these conditions in the usual way and so do not dwell on the details of their derivation. The equilibrium locus for capital (dK/dt = 0) is given by

(22)
$$\begin{aligned} 0 &= A K^{\alpha} L^{1-\alpha} - C(\psi) - \delta K \\ &= F(K,L) - C(\psi) - \delta K \end{aligned}$$

where C(ψ), obtained from the first-order condition (19), is the function relating consumption to the costate variable ψ and having a negative first derivative: C'(ψ) < 0. The slope of the

[8]The specification of the budget constraint is correct only for interior solutions, that is, for solutions where 0 < C < F(K, L). We ignore corner solutions throughout because they add nothing to the analysis.

dK/dt=0 locus is

(23)
$$\left.\frac{d\psi}{dK}\right|_{dK/dt=0} = \frac{\alpha A K^{\alpha-1} L^{1-\alpha} - \delta}{C'(\psi)}$$

The equilibrium locus for the costate variable (d$\psi$/dt = 0) is given by

(24)
$$\rho + \delta = \alpha A K^{\alpha-1} L^{1-\alpha}$$
$$= F_K$$
$$\Rightarrow \quad K = K^* \equiv \left[\left(\frac{\alpha A}{\rho + \delta}\right)\right]^{\frac{1}{1-\alpha}} L$$

The d$\psi$/dt=0 locus is vertical at K*. We thus have the usual phase diagram, shown in Figure 2. Note that K* is the steady state stock of capital.

Suppose now that an invention is discovered that raises $\alpha$ and leaves A unchanged. Because $F_K > 0$ (by assumption), it is immediately apparent from (24) that K* increases, causing the d$\psi$/dt=0 locus to shift to the right. Whether the dK/dt=0 locus shifts up or down depends on whether the economy is initially in phase II or III, which depends on the sign of $\partial Y/\partial\alpha$. If $\partial Y/\partial\alpha > 0$, the economy begins in phase II, and it is straightforward to see from (22) that the dK/dt=0 locus shifts down. We have the situation shown in Figure 3. The steady state moves from point $E_0$ to point $E_1$, at which the capital stock is higher and $\psi$ is lower (and therefore consumption C is higher) than originally. The exact dynamic adjustment path that will be taken to the new steady state is unclear. It could be that, immediately after the invention is adopted, $\psi$ jumps up © jumps down) and the upper path $P_U$ is followed, or it could be that $\psi$ jumps down © jumps up) and the lower path $P_L$ is followed. Which path is taken depends on the magnitudes of the production and utility function derivatives and of the other system parameters.

Irrespective of which dynamic adjustment path is optimal, adoption of the invention is optimal. To see this, first suppose path $P_L$ is the optimal adjustment path. Along that path, consumption and therefore utility is higher at all times than if the invention is not adopted and the economy remains at point $E_0$. Unambiguously, adoption is optimal. Now suppose that path $P_U$ is the optimal path. Recall that optimality here is a conditional notion; *if* the invention is adopted, then the optimal path to follow is $P_U$. Along this path, initial consumption and therefore utility is lower with adoption than without it. It therefore is not immediately clear that adoption is optimal. However, notice that a path like $P_L$ always is feasible even if not optimal. The impact effect of the invention is to raise output, by assumption. Some of that increase in output could be used to raise consumption immediately and the rest used to increase the capital stock K. The increase in K would increase output further as time passed, allowing ever more consumption. Path $P_L$ thus dominates the alternative of not adopting the invention and remaining at point $E_0$. In the case under consideration, however, path $P_L$ is itself dominated by path $P_U$, so adoption is optimal *a fortiori.*

Suppose now that the economy initially is in phase III, where $\partial Y/\partial \alpha < 0$. In that case, the dK/dt=0 locus shift up, and we have the situation shown in Figures 4 and 5. Figure 4 is drawn with the new steady state value $\psi_1$ of the costate variable above the initial value $\psi_0$. The value of $\psi$ is above $\psi_0$ along any possible dynamic adjustment path, meaning that consumption and thus utility are always lower than they were before adoption. Adoption is unambiguously suboptimal. An interesting possibility, however, is shown in Figure 5, where $\psi_1$ is below $\psi_0$, implying that consumption and utility eventually exceed their pre-adoption levels. If the eventual increase in utility is great enough to dominate the decline early on the adjustment path, adoption will be optimal. The condition to be satisfied is

$$\int_0^\infty U[C(\psi_t)]e^{-\rho t}dt > \int_0^\infty U[C(\psi_0)]e^{-\rho t}dt$$

$$= \frac{U[C(\psi_0)]}{\rho}$$

On the right side, we can use (19) to solve for $\psi_0$ in terms of A, $\rho$, $\delta$, L, and $\alpha_0$ (the value of $\alpha$ before adoption of the invention). However, on the left side, we must know the entire path of $\psi$, which requires an explicit forms for the utility function U. Even then, an analytic solution usually is not possible. In principle, though, the path of $\psi$ on the left side will be determined by A, $\rho$, $\delta$, $\lambda$, $\alpha_0$, $\alpha_1$ (the value of $\alpha$ after adoption of the invention), and the levels and derivatives of U and F. In addition to satisfying the foregoing inequality, the parameters A, $\rho$, $\delta$ and $\alpha_0$ must have values that satisfy the two inequalities required for the economy to be in phase III:

$$0 > \ln(K/L) > -\frac{1}{\alpha_0}$$

$$\Leftrightarrow \quad e^{\frac{1-\alpha_0}{\alpha_0}} > \frac{\rho+\delta}{\alpha}A > 1$$

where we have used (24) to eliminate K/L in passing from the first line to the second. We thus have a total of three inequalities to satisfy and many more than three parameters to vary. In general, we should be able to find combinations that satisfy all three inequalities, thus leading to the conclusion that adoption of the invention can be socially optimal even though it initially reduces output, consumption, and utility.

## III. Three Factors of Production

We now move to a model in which physical and human capital enter separately in the production function. Many of the results are straightforward generalizations of those obtained for the two-factor model, but the three-factor model still is worth examining because it has important implications for the recent divergence between incomes of skilled and unskilled labor and also for the failure of many underdeveloped economies to adopt technological advances, an issue already touched on above.

The production function now is a three-factor Cobb-Douglas:

(25)
$$\begin{aligned} Y &= F(K,L,H) \\ &= AK^{\alpha}H^{\beta}L^{1-\alpha-\beta} \end{aligned}$$

where H is human capital. With human capital entering as a separate factor of production, we now should interpret L as (man hours of) raw or unskilled labor. The marginal products of the three factors are

(26)
$$\begin{aligned} F_K &= \alpha AK^{\alpha-1}H^{\beta}L^{1-\alpha-\beta} \\ &\equiv MPK \end{aligned}$$

(27)
$$\begin{aligned} F_H &= \beta AK^{\alpha}H^{\beta-1}L^{1-\alpha-\beta} \\ &\equiv MPH \end{aligned}$$

(28)
$$\begin{aligned} F_L &= (1-\alpha-\beta)AK^{\alpha}H^{\beta}L^{-\alpha-\beta} \\ &\equiv MPL \end{aligned}$$

III.A. *Simple Technical Change with K, H, and L Fixed.*

As in the two-factor case, we start by examining the effect of a technical change while holding all factors of production fixed. We thus are examining the impact effect of the change. With an additional factor of production, we also have an additional factor share to consider. The number of possible combinations of parameter changes is now considerably larger than previously; to keep the discussion tractable, and to concentrate on the most interesting case, we restrict attention to a technical change that alters the relative shares of human capital and unskilled labor while leaving physical capital's share unchanged. In particular, we suppose the change raises human capital's share $\beta$, lowers unskilled labor's share $1-\alpha-\beta$, and leaves physical capital's share $\alpha$ unchanged. For now, we also simplify the discussion by supposing total factor productivity $A$ is unchanged. We thus have a situation that parallels that of section II.A above, with human capital taking the place of physical capital in the discussion.

The economy can be in one of four possible phases, determined as in section II.A by the signs of the effects of the technical change on the level of output and the incomes of the three factors of production. The derivatives of Y, MPK, MPH, and MPL and their signs are

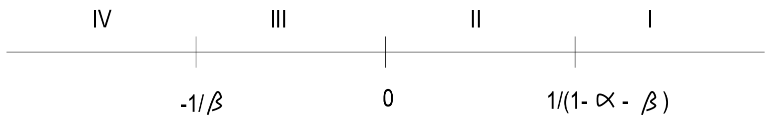
(29)
$$\begin{aligned} \partial Y/\partial\beta &= AK^{\alpha}H^{\beta}L^{1-\alpha-\beta}\ln(H/L) \\ &\gtrless 0 \quad \text{as} \quad \ln(H/L) \gtrless 0 \end{aligned}$$

(30)
$$\begin{aligned} \partial MPK/\partial\beta &= \alpha AK^{\alpha-1}H^{\beta}L^{1-\alpha-\beta}\ln(H/L) \\ &\gtrless 0 \quad \text{as} \quad \ln(H/L) \gtrless 0 \end{aligned}$$

(31)
$$\begin{aligned} \partial MPH/\partial\beta &= AK^{\alpha}H^{\beta-1}L^{1-\alpha-\beta}[1+\beta\ln(H/L)] \\ &\gtrless 0 \quad \text{as} \quad \ln(H/L) \gtrless -\frac{1}{\beta} \end{aligned}$$

(32)
$$\begin{aligned} \partial MPL/\partial\beta &= AK^{\alpha}H^{\beta}L^{-\alpha-\beta}[(1-\alpha)\ln(H/L)-1] \\ &\gtrless 0 \quad \text{as} \quad \ln(H/L) \gtrless \frac{1}{1-\alpha-\beta} \end{aligned}$$

The four phases are shown in the following diagram




and have the following characteristics:

I: $0 < \partial Y/\partial\beta, \partial MPK/\partial\beta, \partial MPH/\partial\beta, \partial MPL/\partial\beta$

II. $\partial MPL/\partial\beta < 0 < \partial Y/\partial\beta, \partial MPK/\partial\beta, \partial MPH/\partial\beta$

III. $\partial Y/\partial\beta, \partial MPK/\partial\beta, \partial MPL/\partial\beta < 0 < \partial MPH/\partial\beta$

IV. $\partial Y/\partial\beta, \partial MPK/\partial\beta, \partial MPH/\partial\beta, \partial MPL/\partial\beta < 0$

Which phase the economy is in depends on the pre-invention values of α, β, and ln(H/L).

The adoption results are the same as in section II.A, so we need dwell on the details. Adoption always is optimal and occurs in phase I, adoption never is optimal and never occurs in phase IV, and adoption may or may not be optimal and may or may not occur in phases II and III. What is worth spending some time discussing are the implications of these results for two current events.

III.A.1. *Unskilled labor's income share.*

It has been widely noted that in the United States over the last thirty years or so the real income of unskilled labor has been stagnant while total income and the incomes of physical capital and especially human capital (skilled labor) all have risen. This event is regarded by at least some people as evidence of a breakdown in "the system" that has left unskilled workers behind and that must be corrected by some sort of government intervention. The foregoing results suggest that existence of the event in question is not compelling evidence of a defect in the workings of the economy. A technical change that raises the income share of skilled labor will reduce the income of unskilled labor in all phases except phase I. In principle, we could check which phase the economy is in simply by examining the relation between ln(H/L) on the one hand and the share parameters $\alpha$ and $\beta$ on the other. In practice, this is difficult to do because data on human capital H are sketchy at best.

Of course, real-world technical advances generally will be more complex than that discussed in this section; in particular, we would expect total factor productivity *A* and probably physical capital's share $\alpha$ to change in most cases. Indeed, the data show an upward trend in *A* and a downward trend in $\alpha$. We have seen, however, that allowing *A* to change does not alter the general character of the results. Allowing $\alpha$ and $\beta$ to change simultaneously complicates the results but again does not change the main conclusion that it is quite possible that a technical advance can reduce the income of unskilled labor even though it raises total output and the incomes of other factors of production.

III.A.2. *Adoption of new technology by underdeveloped economies.*

We already have seen in section II.A that countries with different capital/labor ratios may make different adoption decisions regarding a technical advance even if they have the same technology at the time the advance is made. A similar result emerges here but now seems much more likely to occur. Suppose countries A and B have the same technology but different human capital/labor ratios, with A having the higher value of H/L. An invention that raises $\beta$ is more likely to be beneficial to country A than to country B and so more likely to be adopted by A than by B. Recent experience in the United States, just discussed in section III.A.1, suggests that important recent technical advances have raised $\beta$. We therefore should expect to see countries with higher values of H/L adopting those advances. Although we do not know how to measure H exactly, it seems likely that levels of education should have a positive correlation with H. If that is so, then the countries that will adopt the new technologies are the developed countries, for they are the countries with relatively high levels of education. Such recent technological advances may not make poor countries any poorer, but they do make the rich countries richer and increase the amount of income inequality across countries. Again, this outcome is not in any way due to a defect in "the system" but merely a consequence of the relative factor endowments of different countries. It does, however, emphasize the importance of education and government policies that encourage it.[9] It therefore may explain why some countries seem stuck in a state of underdevelopment. Basic education is not something that firms typically will invest in, simply because its general nature prevents firms from capturing the return to any investment they make in it. Rather, basic education must be undertaken by individuals themselves. In most countries, government provides or subsidizes basic education. If the government does this job badly, then it may condemn its economy to the use of backward technology, guaranteeing that the economy remains poor. This sort of failure may even instigate a vicious circle. In the foregoing discussion, education may bring little reward if the available technology is suited for a low level of education. People therefore may be discouraged from acquiring education in the first place, thus guaranteeing that the technology will remain backward and continue to not reward education. In addition, low levels of education hold down the marginal product of physical capital, making business fixed investment unattractive. We will further pursue this issue in section III.C below.

This last prediction resembles Caselli and Coleman's (2000) empirical finding that countries with a relative abundance of unskilled labor are relatively inefficient at using skilled labor and capital. Caselli and Coleman define a factor's productive efficiency as its coefficient

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[9]Education plays a decisive role in Caselli and Coleman's (forthcoming) discussion of structural transformation and regional convergence in the US economy.

in the following variant of the CES production function:

$$Y = \left\{(AL)^{\sigma} + [(BH)^{\rho} + (CK)^{\rho}]^{\sigma/\rho}\right\}^{1/\sigma}$$

Unfortunately, the limit of this function as σ and ρ go to zero is not Cobb-Douglas but rather

$$Y = (AL)(BH)(CK)$$

The factor efficiencies do not become factor shares (all of which are 1 in this case) when the elasticity of substitution goes to one, as they do for the standard CES function

$$Y = \left[AL^{\sigma} + BH^{\sigma} + (1-A-B)K^{\sigma}\right]^{1/\sigma} \qquad 0 < A,B; \quad A+B < 1$$

Thus Caselli and Coleman's estimates of factor efficiencies cannot be applied to the Cobb-Douglas functions used throughout the present analysis; nonetheless, they are consistent with the predictions obtained above.

III.B. *Mixed Technical Change.*

Allowing total factor productivity $A$ to change leads to the same conclusions as in section II.B above. The main effect is to change the expressions that define the boundaries of the four phases. The details are left to the reader.

III.C. *International Capital Flows in a Dynamic Model.*

Because there now are two state variables, K and H, a full dynamic analysis like that performed for the two-factor model is not be feasible. It is possible, however, to work out the steady state solutions. The results for a closed economy are not much different from what was obtained in section II.C above. In contrast, the results for in a model with international capital flows provides interesting insights into economic development. Discussion will be brief with mathematical details relegated to the Appendix.

We suppose there are two countries D (domestic) and E (external). Country D is highly developed and exports physical capital to country E, which for simplicity is assumed to produce no capital of its own. All markets in both countries are competitive. The two countries produce the same final good with the aggregate Cobb-Douglas production function

$$Y_i = A_i K_i^{\alpha_i} H_i^{\beta_i} L_i^{1-\alpha_i-\beta_i}$$

where i = D or E. Each country takes what the other country does as given.

Country D's planning problem is to maximize the representative agent's utility over domestic consumption $C_D$ , investment $I_E$ in capital located abroad, and investment $I_H$ in domestic human capital

(33) $$\max_{\{C_D, I_E, I_H\}} \int_0^\infty u(C_D) e^{-\rho t} dt$$

subject to the dynamic equations

(34) $$\begin{aligned} \dot{K}_D &= GNP_D - C_D - I_E - I_H - \delta K_D \\ &= A_D K_D^{\alpha_D} H_D^{\beta_D} L_D^{1-\alpha_D-\beta_D} + \alpha_E A_E K_E^{\alpha_E} H_E^{\beta_E} L_E^{1-\alpha_E-\beta_E} - C_D - I_E - I_H - \delta K_D \end{aligned}$$

(35) $$\dot{K}_E = I_E - \delta K_E$$

(36) $$\dot{K}_H = I_H - \delta K_H$$

The second term on the right side of the second line in (34) is income from capital located in country E, equal to the marginal product of $K_E$ multiplied by $K_E$ itself. Solving this problem with the maximum principle leads to the following steady state solution for $K_E$:

(37) $$K_E = \left( \frac{\alpha_E^2 H_E^{\beta_E} L_E^{1-\alpha E-\beta_E}}{\delta + \rho} \right)^{\frac{1}{1-\alpha_E}}$$

Note that $K_E$, which is chosen by country D, depends positively on the value of human capital $H_E$

chosen by country E.

Country E's problem is simpler because it does not build physical capital either at home or abroad. It therefore chooses only its consumption $C_E$ to maximize representative agent utility:

(38)
$$\max_{\{C_E\}} \int_0^\infty u(C_E) e^{-\rho t} dt$$

subject to

$$\dot{H}_E = (1-\alpha_E) A_E K_E^{\alpha_E} H_E^{\beta_E} L_E^{1-\alpha_E-\beta_E} - C_E - \delta H_E$$

where the first term on the right side is GDP less the income from capital, all of which goes to country D. The steady state value for $H_E$ is

(39)
$$H_E = \left[ \frac{(1-\alpha_E)\beta_E A_E}{\delta + \rho} K_E^{\alpha_E} L_E^{1-\alpha_E-\beta_E} \right]^{\frac{1}{1-\beta_E}}$$

Note that $H_E$, which is chosen by country E, depends positively on the value of $K_E$, which is chosen by country E.

Equations (37) and (39) can be solved simultaneously for the Nash equilibrium values in terms of the underlying parameters of the two economies. Those values then can be substituted into country E's production function to obtain the equilibrium solution for output. What interests us here is the response of country E to an increase in $\beta_E$. The solution for output in country E, conditional on country D's choice of $K_E$, is obtained by substituting (39) into country E's production function, which gives

$$
\text{(40)} \quad
\begin{aligned}
Y_E &= A_E K_E^{\alpha_E}\left[\frac{(1-\alpha_E)\beta_E A_E}{\delta+\rho} K_E^{\alpha_E} L_E^{1-\alpha_E-\beta_E}\right]^{\frac{\beta_E}{1-\beta_E}} L_E^{1-\alpha_E-\beta_E} \\
&= A_E\left[\frac{(1-\alpha_E)\beta_E A_E}{\delta+\rho}\right]^{\frac{\beta_E}{1-\beta_E}} L_E \left(\frac{K_E}{L_E}\right)^{\frac{\alpha_E}{1-\beta_E}}
\end{aligned}
$$

The decision by Country E's planner on whether to adopt a technical change that increases human capital's share at the expense of unskilled labor, while leaving physical capital's share unchanged, depends on the sign of the derivative of output with respect to $\beta_E$, which determines whether adoption raises or lowers total output. The derivative is

$$
\text{(41)} \quad \frac{\partial Y_E}{\partial \beta_E} = \text{term in } \beta_E + Y_E\left[(1-\beta_E)^2\left(\frac{K_E}{L_E}\right)^{\frac{\alpha_E}{1-\beta_E}} \alpha_E \ln\left(\frac{K_E}{L_E}\right)\right]
$$

The sign of the first term on the right side is ambiguous in general. The second term on the right side, however, is positive or negative as $K_E$ is greater or less than $L_E$. Therefore, the higher the capital/labor ratio, the more likely country E is to adopt a technical change that raises $\beta_E$. Intuitively, adoption is increasingly beneficial the higher is the level of human capital $H_E$, but the level of $H_E$ depends positively on $K_E$. When a new technology arises that favors human capital, country E is less likely to adopt if it is relatively scarce in human capital. It is likely to be scarce in human capital if it is scarce in foreign investment. Foreign investment is likely to be low if human capital is low, as can be seen from the solution for $K_E$ given by (37). But human capital is likely to be low if $\beta_E$ is low, because human capital's marginal product (return to human capital) depends positively on $\beta_E$. This is the vicious circle referred to earlier: low foreign investment holds down human capital, which makes adoption of technical changes favoring human capital unlikely, which keeps human capital low, which holds down foreign investment. This dynamic seems especially relevant in recent years, when, judging from the U. S. experience, there have been substantial technical advances favoring human capital at the expense of unskilled labor. These advances may be of little value to countries with low levels of human capital, which may remain stuck in a poverty trap relative to the developed nations.

## IV. Share Changes in Endogenous Growth Models

To this point, we have considered only models in which growth is exogenous (and, to keep matters simple, have assumed there was no exogenous growth). We now look at a few examples of endogenous growth models and study the effect of technical change that alters factor shares. The general result is that the economy's growth rate changes, with the direction of change depending on the magnitudes of some of the economy's parameters. However, a serious question also arises concerning the plausibility of many endogenous growth models.

IV.A. *AK-Type Models.*

The simplest AK model, with the production function

$$Y_t = AK_t \tag{42}$$

has only one factor of production with a fixed share of one, so technical change that alters factor shares is ruled out by implicit assumption. Slightly more elaborate AK-type models admit such change, however. In particular, consider the model of learning-by-doing with knowledge spillovers.[10] We suppose the production function is

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{43}$$

and assume that

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[10]See Barro and Sala-i-Martin, Chapter 4.

(44) $$A_t = A(K_t^a)^{1-\alpha}$$

where $K^a$ is the average aggregate capital stock. The presence of $K^a$ reflects the knowledge spillovers. If we assume identical firms, the average aggregate capital stock equals each firm's capital stock, allowing us to substitute K for $K^a$ and write the production function as

(45) $$Y_t = A\,K_t L_t^{1-\alpha}$$

If we then assume that the representative agent has the constant relative risk aversion utility function[11]

(46) $$U(c) = \frac{c^{1-\theta}-1}{1-\theta}$$

we can derive the balanced growth rate

(47) $$\gamma = \frac{1}{\theta}(A\alpha L^{1-\alpha} - \delta - \rho)$$

where δ is the rate of depreciation and ρ is the rate of time preference. Technical change that alters factor shares changes this growth rate. For general technical change that alters both total factor productivity A and capital's share α, the change in the growth rate is

(48) $$d\gamma = \left(\frac{\alpha}{\theta}\right)L^{1-\alpha}dA + \left(\frac{A}{\theta}\right)L^{1-\alpha}(1-\alpha lnL)d\alpha$$

As before, A is shown to be a misleading measure of technical progress, for the foregoing expression can be positive even if A is negative. The condition is that

(49) $$dA > -\left(\frac{A}{\alpha}\right)(1-\alpha\ln L)d\alpha$$

[11]This form has been shown by King, Plosser, and Rebelo (1988) to be necessary for balanced growth in an endogenous growth model.

The right side will be negative if 1 > $\alpha$lnL, in which case dA can be negative.

Another AK-type model is the one-sector model with physical capital K and human capital H. The production function is

(50) $$Y = AK^{\alpha}H^{1-\alpha}$$

and the accumulation equations for K and H are

(51) $$\dot{K} = I_K - \delta K$$

(52) $$\dot{H} = I_H - \delta H$$

The balanced growth rate is[12]

(53) $$\gamma = \frac{1}{\theta}\left[A\alpha\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} - \delta - \rho\right]$$

The change in this growth rate brought about by a change in $\alpha$ is

(54) $$d\gamma/d\alpha = -\frac{A}{\theta}\alpha\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\ln\left(\frac{1-\alpha}{\alpha}\right) \gtrless 0 \quad \text{as} \quad \frac{1-\alpha}{\alpha} \lessgtr 1$$

IV.B. *Variety and Quality Ladder Models.*

A simple variety model specifies the production function as

(55) $$Y_i = AL_i^{1-\alpha}\sum_{j=1}^{N}(X_{ij})^{\alpha}$$

where $X_{ij}$ is the employment of intermediate good j by firm i and N is the number of intermediate goods. Providers of intermediate goods are monopolistically competitive and conduct research

[12]See Barro and Sala-i-Martin, Chapter 4.

to invent new varieties of intermediate goods. The growth rate for this model is[13]

$$(56) \qquad \gamma = \frac{1}{\theta}\left[(L/\eta)A^{1/(1-\alpha)}\left(\frac{1-\alpha}{\alpha}1\right)\alpha^{2/1-\alpha} - \rho\right]$$

where $\eta$ is the cost to create a new type of product, measured in terms of final output Y. This growth rate is a function of $\alpha$, so technical change that alters $\alpha$ also alters the growth rate $\gamma$. This kind of technical change usually is ignored in a variety model, where technical progress is assumed to arise solely through creation of new varieties. However, a more realistic model would allow technical change that alters factor shares through changes in $\alpha$.

A simple quality ladder model has the same production function as the variety model except that N is assumed fixed. Also, the quantity of the intermediate good $X_{ij}$ in the production function is the *effective* quantity, equal to the physical quantity $X^*_{ij}$ multiplied by a quality indicator $q^k$, where q is the quality step size and k is the current quality step. In this model, monopolistically competitive suppliers of intermediate goods compete to discover ways to advance up the quality ladder by increasing k. The balanced growth rate is[14]

$$(57) \qquad \gamma = \frac{[q^{\alpha/(1-\alpha)} - 1][(L/\zeta)A^{1/(1-\alpha)}\left(\frac{1-\alpha}{\alpha}\right)\alpha^{2/(1-\alpha)} - \rho]}{1 + \theta\,[q^{\alpha/(1-\alpha)} - 1]}$$

which once again is a function of $\alpha$ and so is altered by any technical change that alters $\alpha$.

IV.C. *A Problem with Many Endogenous Growth Models.*

All endogenous growth models rest on a knife-edge assumption. In the simplest models, it is a straightforward assumption of constant returns to the reproducible factors, seen most easily

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[13]See Barro and Sala-i-Martin, Chapter 6.

[14]See Barro and Sala-i-Martin, Chapter 7.

in the AK model and its variants. In the knowledge spillover model, we had to assume that total factor productivity was a function of aggregate capital $K^a$ raised to the power 1-$\alpha$, that is, raised exactly to the power of labor's share. There is no particular reason to make this assumption, except that it is required if the model is to deliver balanced growth. If the exponent of $K^a$ is less than 1-$\alpha$, the model goes asymptotically to a steady state with no growth; if the exponent exceeds 1-$\alpha$, output goes to infinity in finite time (Solow, 1994).

In more complex models, such as the variety and quality ladder models, the knife-edge assumption is more difficult to spot, but it is there. For example, in the variety model discussed above, it must be assumed that the net present value of research and development equals $\eta$, the exogenous cost of R&D; otherwise, R&D either is zero or infinity. The expression for this assumption is

$$(58) \qquad \eta = LA^{1/(1-\alpha)}\left(\frac{1-\alpha}{\alpha}\right)\alpha^{2/(1-\alpha)}\int_t^{\infty} e^{-\left[\frac{1}{v-t}\int_t^{v} r(m)dm\right]\cdot(v-t)}\,dv$$

Note that this condition is not guaranteed by any market behavior; it is simply assumed to be met. In the quality ladder model, the knife-edge assumption is that the probability of success in R&D in intermediate good industry j has the form

$$(59) \qquad p_{jk_j} = Z_{jk_j}(1/\zeta)\,q^{-(k_j+1)\alpha/(1-\alpha)}$$

where Z is the quantity of resources expended on R&D in industry j. Again, there is no market behavior to guarantee that this condition is met.

These knife-edge conditions are simply assumed; no convincing reason is given to believe they are true. Jones (1995) finds them sufficiently implausible that he doubts the validity of all endogenous growth models. The implausibility is only increased by the possibility of technical change that alters factor shares. The foregoing expressions show that the knife-edge conditions in the models discussed all depend on the share parameter $\alpha$. Should $\alpha$ change, then something else in the condition must change in a way that exactly offsets the effect of the change in $\alpha$. It is hard to imagine what force would lead to such a perfectly balancing offset.

In practice, virtually all endogenous growth models use the Cobb-Douglas because otherwise solutions are difficult or impossible to obtain. In principle, however, the Cobb-Douglas production function is not required; other forms, such as the CES, can be used. It may be that an endogenous growth model with one of these alternatives would avoid the problems associated with non-share-neutral technical progress, but it is hard to say for certain because it is not clear in many cases exactly what the form of the knife-edge assumption would be and how it would be affected by changes in factor shares induced by technical change.

## V. Conclusion

The usual approach to modelling technical change assumes that it alters only total factor productivity and does not alter factor shares directly. There is no particular reason to expect technical change to be of such a restricted nature, and indeed the data suggest that factor shares do vary as a result of technical change. The foregoing analysis has examined the implications of technical change that is not share-neutral and has obtained a number of interesting implications. We have seen that non-neutral change of this type can lower the income of some factors of production even when it raises total output, thus offering a possible explanation for episodes of social conflict such as the Luddite uprisings in 19th century England, caused by textile workers who felt themselves made worse off by inventions that clearly raised total output. We also have seen that underdeveloped countries may choose not to adopt the latest technology but instead may prefer to continue using "outmoded" production methods. Non-neutral technical change, if adopted, could lower the income of those countries even though it raises the income of the developed countries. The explanation underlying both these results is that the effect of non-neutral technical change on total output and on factor incomes depends on the capital/labor or human capital/labor ratio of the economy. A change that increases the share of a particular factor of production will tend to raise total output the more of that factor the economy has relative to the other factors of production. Similarly, the response of factor incomes to the change also depends on the relevant factor ratios. Countries having the same technology but different factor ratios will experience different effects of a particular non-neutral invention, possibly leading countries to make different decisions regarding adoption of the invention. The dynamic analysis has shown that it can be socially optimal to adopt a non-neutral invention even if it initially lowers aggregate output because it may lead to enough capital accumulation to more than offset the initial negative impact effect. Also, we have seen that total factor productivity may be a misleading measure of technical progress because it can move in the opposite direction as output when technical progress is non-neutral. Finally, the possibility of technical change that alters factor shares raises doubts about the plausibility of endogenous growth models. Those models all depend on knife-edge conditions of various sorts, and all the conditions are functions of the factor share parameter. Should the share parameter change in response to non-neutral technical change, the knife-edge conditions would be violated unless some other parameter changed in exactly such a way as to offset the effect of the variation in the factor share. It is difficult to see what would cause such a serendipitous offset.

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**Table 1**
Capital's Share of Income, United States

| Year | Capital's Share | Year | Capital's Share |
|---|---|---|---|
| 1909 | 0.335 | 1935 | 0.351 |
| 1910 | 0.33 | 1936 | 0.357 |
| 1911 | 0.335 | 1937 | 0.34 |
| 1912 | 0.33 | 1938 | 0.331 |
| 1913 | 0.334 | 1939 | 0.347 |
| 1914 | 0.325 | 1940 | 0.357 |
| 1915 | 0.344 | 1941 | 0.377 |
| 1916 | 0.358 | 1942 | 0.356 |
| 1917 | 0.37 | 1943 | 0.342 |
| 1918 | 0.342 | 1944 | 0.332 |
| 1919 | 0.354 | 1945 | 0.314 |
| 1920 | 0.319 | 1946 | 0.312 |
| 1921 | 0.369 | 1947 | 0.327 |
| 1922 | 0.339 | 1948 | 0.332 |
| 1923 | 0.337 | 1949 | 0.326 |
| 1924 | 0.33 | 1950 | 0.363 |
| 1925 | 0.336 | 1951 | 0.345 |
| 1926 | 0.327 | 1952 | 0.317 |
| 1927 | 0.323 | 1953 | 0.311 |
| 1928 | 0.338 | 1954 | 0.305 |
| 1929 | 0.332 | 1955 | 0.329 |
| 1930 | 0.347 | 1956 | 0.319 |
| 1931 | 0.325 | 1957 | 0.311 |
| 1932 | 0.397 | 1958 | 0.301 |
| 1933 | 0.362 | 1959 | 0.316 |
| 1934 | 0.355 | 1960 | 0.309 |

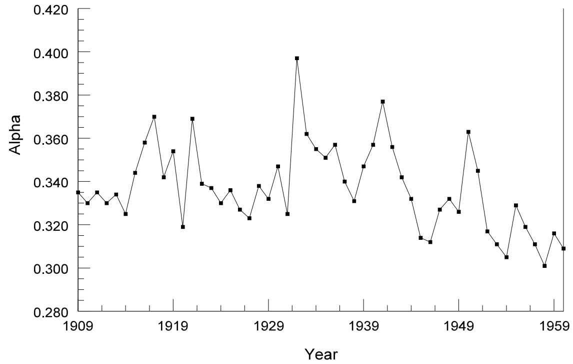


Figure 1: Plot of capital's share; Sato data

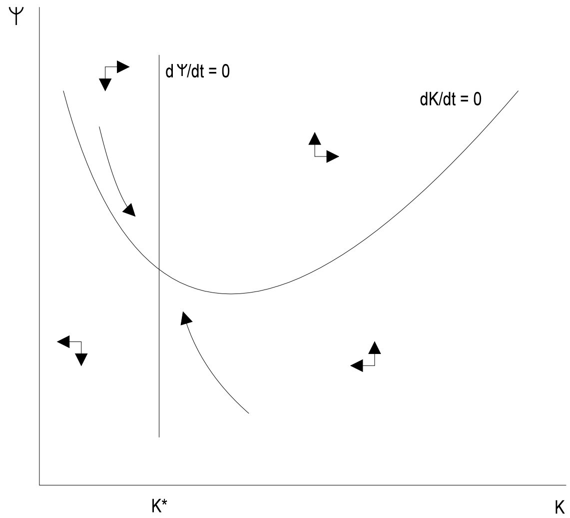


Figure 2: Generic phase diagram

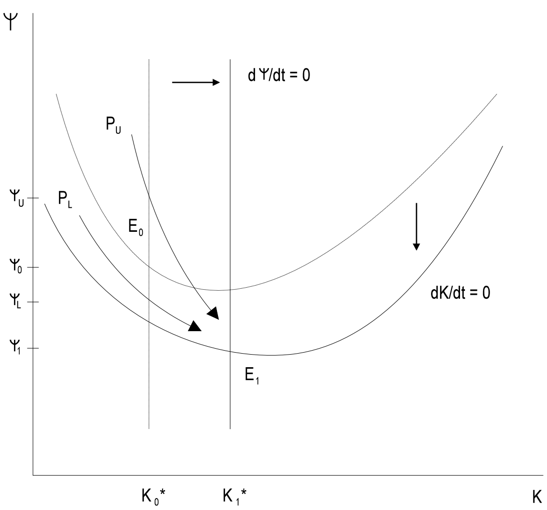


Figure 3: Technical advance with $dY/d\alpha>0$

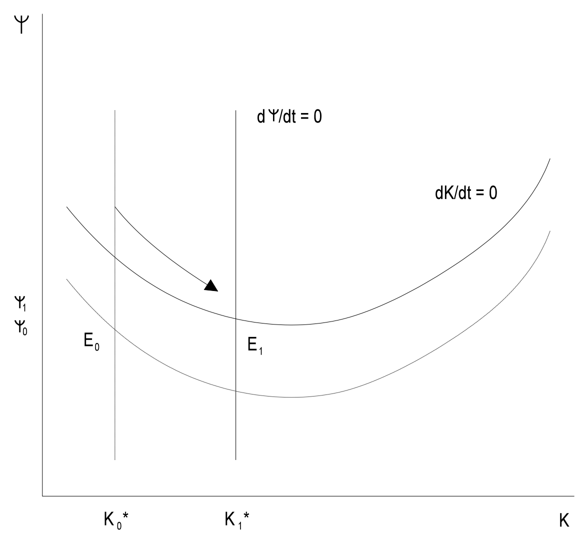


Figure 4: Suboptimal adoption in phase III

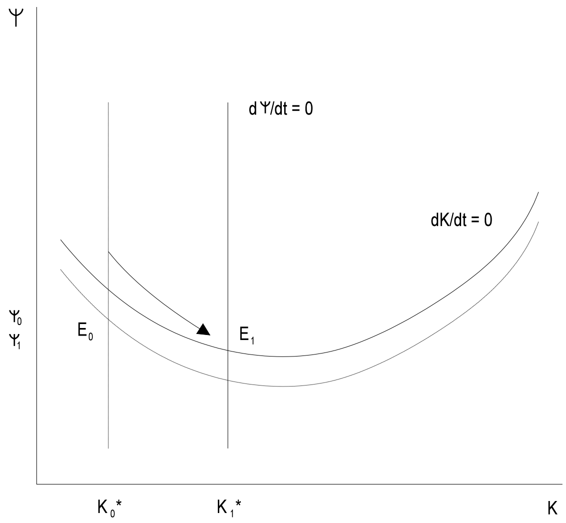


Figure 5: Optimal adoption in phase III