A RESTATEMENT OF THE WELFARE COSTS OF INFLATION. THE WASTE OF SCARCE RESOURCES IN THE MANUFACTURING, BANKING AND HOUSEHOLD SECTORS.

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ABSTRACT. Lucas (2000) estimates that the US welfare costs of inflation are around 1% of GDP. This measurement is consistent with a specific distorting channel in terms of the Bailey triangle under the demand for monetary base schedule (outside money): the displacement of resources from the production of consumption goods to the household transaction time à la Baumol. Here, we consider also several new types of distortions in the manufacturing and banking industries. Our new evidences show that both banks and firms demand specific occupational employments to avoid the inflation tax. We define the concept of "the float labor": The occupational employments that are affected by the inflation rates. More administrative workers are hired relatively to the blue-collar workers for producing consumption goods. This new phenomenon makes the manufacturing industry more roundabout. To take into account this new stylized fact and others, we redo at same time both "The model 5: A Banking Sector -2" formulated by Lucas (1993) and "The Competitive Banking System" proposed by Yoshino (1993). This modelling allows us to characterize better the new types of misallocations. We find that the maximum value of the resources wasted by the US economy happened in the years 1980-81, after the 2nd oil shock. In these years, we estimate the excess resources that are allocated for every specific distorting channel: i) The US commercial banks spent additional resources of around 2% of GDP; ii) For the purpose of the firm floating time were used between 2.4% and 4.1% of GDP); and iii) For the household transaction time were allocated between 3.1% and 4.5 % of GDP. The Bailey triangle under the demand for the monetary base schedule represented around 1% of GDP, which is consistent with Lucas (2000). We estimate that the US total welfare costs of inflation were around 10% of GDP in terms of the consumption goods foregone. The big difference between our results and Lucas (2000) are mainly due to the Harberger triangle in the market for loans (inside money) which makes part of the household transaction time, of the firm float labor and of the distortion in the banking industry. This triangle arises due to the widening interest rates spread in the presence of a distorting inflation tax and under a fractionally reserve system. The Harberger triangle can represent 80% of the total welfare costs of inflation while the remaining percentage is split almost equally between the Bailey triangle and the resources used for the bank services. Finally, we formulate several theorems in terms of the optimal nonneutral monetary policy so as to compare with the classical monetary theory.

1. Introduction

Historical episodes suggest that the welfare costs of inflation can be enormous. Remember that the chaotic German hyperinflation had a decisive role for the beginning of the II World War. By that time, the involved countries spent more than 50% of GDP as military expenditure On the other hand, in Latin America, the presidents Carlos Menen and Fernando Henrique Cardoso got reelected twice due to their main accomplishment: a successful implementation of a stabilization plan. But, so far, in this issue, the economic literature stands behind: the welfare costs of inflation are considered negligible. Keynesians call them "shoeleather costs" to justify inflationary policies to alleviate the unemployment. On the other hand, the recent works, such as, Lucas (2000), Aiyagari, Rao, Braun and Eckstein (1998), English (1999), Dotsey and Ireland (1996), and Fisher (1981) estimate that these costs are around 1% of GDP. Ironically, the monetarists are also in trouble to defend a tight money issuing: Why should one care about inflation if its social costs are small?

To be fair, their social costs are consistent with a specific channel for the misallocation of resources. For Keynesians, these costs are small because costless resources are spent to update the menu prices in an inflationary environment. On the other hand, Lucas (2000) assumes that the channel is due to resources that are displaced from the production of consumption goods to the household transaction time. Lucas (1993) considers that both currency and bank deposits save the shopping transaction time.

This work provides several new stylized facts to show that since Bailey (1956) until Lucas (1993) and Yoshino (1993), the issue did not have received the appropriate attention. According to the new stylized facts, we provide new foundations for modelling and estimating the welfare costs of inflation. We find that the US welfare costs of inflation were around 10% of GDP in the years 1980-81. Thus, we should be at least non-Keynesians. This big distortion leads us to examine more carefully the standard monetary policy such as the Friedman optimum quantity of money and the Simon proposal for 100% of reserve requirement.

This paper is organized in the following way. Section 2 shows how inflation can affect the product side and the productive factor input such as labor; Subsection 2.1 shows the effects of inflation in the industrial share of GDP; Subsection 2.2 illustrates the change in the breakdown of the PCE (personal consumption expenditure) due the loss of the consumer purchasing power; Subsection 2.3 shows the effects of inflation in the banking industry; Subsubsection 2.3.1 estimates the components of the bank value-added; Subsection 2.4 describes the concept of "firm floating labor" - occupational employments that are correlated with the inflation rates; Section 3 contains a model to take into account the new stylized facts; Section 4 provides several estimations for the components of the US welfare costs of inflation; in the Section 5, we formulate several theorems, in particular, the reason for arising the Harberger triangle in the market for loans; Finally, the section 6 contains the conclusions.

2. The Stylized Facts: Effects of Inflation in the Product Side and in the Productive Factor Input by Industry

We analyze the US economy for identifying the types of the inflationary misallocations of resources. We have selected the US economy instead of any other country due to the quality of data in terms of detailed and long time series across industries.

2.1. Effects of Inflation in the Industrial Share of GDP.

Figures 1 provides the first motivation for this paper. It illustrates how the inflation rates misallocate the gross product originating by industry as share of GDP. Inflation hurts the durable and nondurable goods industries while the commercial banking industry is benefited by inflation[1]. This figure suggests the nature of the welfare costs of inflation: the loss of the consumption goods - argument of the utility function. Thus, part of the resources are shifted from the goods producing sector to the banking industry[2].
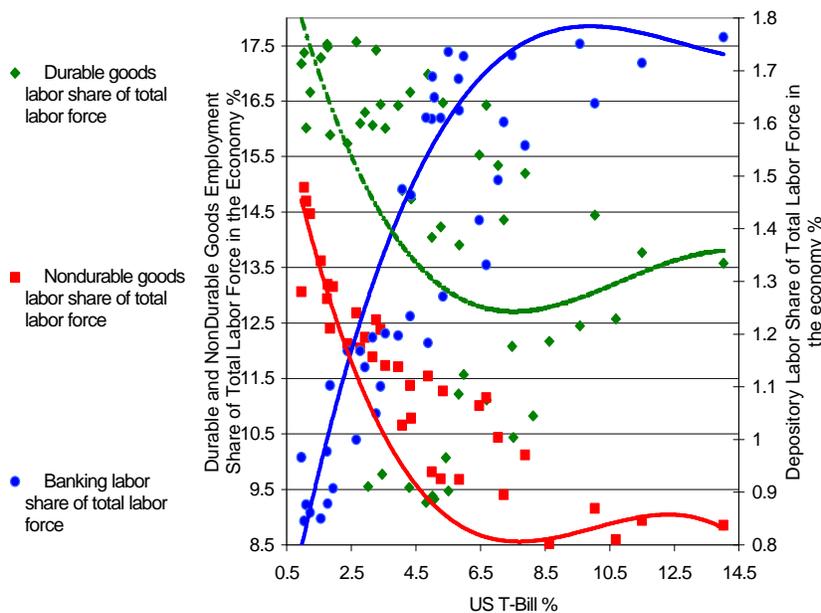

Figure 1. The Welfare Costs of Inflation: Loss of Consumption Goods and Overbanking. US: 1957-99

The problem of estimating the welfare costs of inflation according to the figure 1 is that the reported series exhibit unit roots and they are cointegrated. Thus, the first approach to calculate the welfare costs of inflation is by considering a VAR

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[1]Source of data: http://www.bea.doc.gov/bea/uguide.htm#_1_16. Period: 1947-99. Gross Domestic Product by Industry. Table NDN-0272.

Table A.1 in the appendix reports also how inflation and other variables affect the commercial banks and the goods industry.

[2]This graph uses the nominal interest rates in the x-axis because the welfare costs are usually defined in terms of the optimum quantity of money à la Friedman (1969) or Bailey (1956).

with a VEC (vector error correction) model[3]. In this sense, the figure 2 below shows the estimated welfare loss costs of inflation in terms of loss of consumption goods (arguments of the utility function) as function of the inflation rate by considering the estimated long-run relationships.
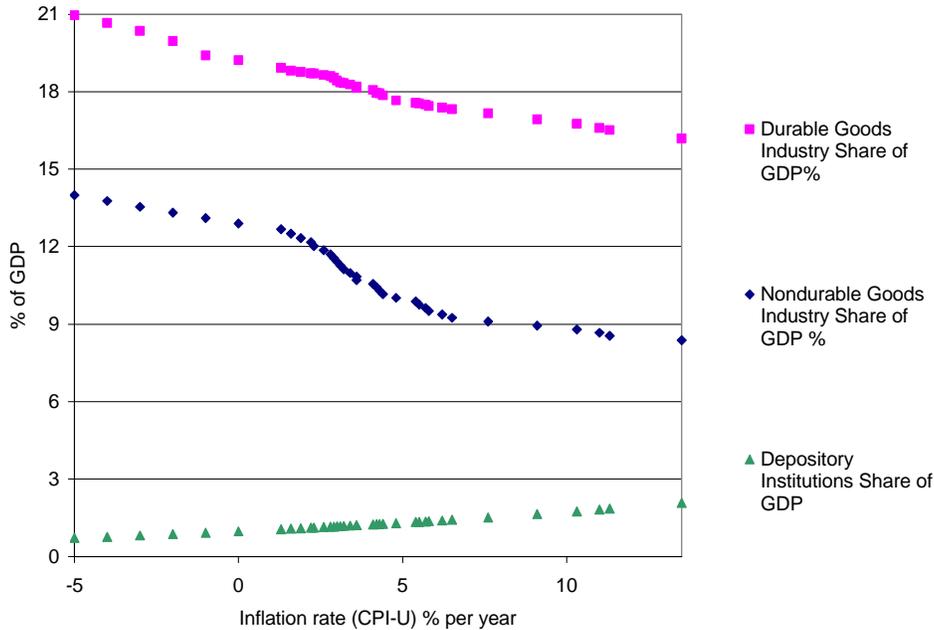

Figure 2. The Fitted Value for the Consumption Goods and Bank Value Added. US: 1947-99

To calculate the realized US welfare loss, we make the following experiment: i) First, we consider the Friedman optimum quantity of money which renders the nominal interest rate equal to zero or the deflation rate equal to the marginal rate of time preference of 5% per year. Under this optimal condition, the values addeds are 14% of GDP for the nondurable goods, 21% of GDP for the durable goods and 0.7% of GDP for the depository institutions; ii) Second, in the years 1980 (after the 2nd oil shock), the inflation rate was 13.5% per year. Under this worst condition, the value added were 8.3% of GDP for the nondurable goods, 16% of GDP for the durable goods and 2% of GDP for the depository institutions; iii) By subtracting this maximum from this minimum, we get that the total welfare loss were 5.6% of GDP for non-durable goods, and 4.7% of GDP for durable. Thus, the total welfare costs of inflation were around 10% of GDP in terms in terms of the total consumption goods (durable and nondurable goods) foregone due to inflation.

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[3]By considering a VAR-VEC model, we get the following long run relationships between the value added by industry as share of GDP as function of the inflation rate:

1. Log (Nondurable goods share of GDP) =- 0.900*log(1+Inflation rate/100) - 0.0123*(time trend) + 2.618

2. Log (durable goods share of GDP)= -2.096*log(1+Inflation rate/100) - 0.01039*(time trend) + 2.99

3. Log (bank value added as share of GDP) = 13.718*log(1+Inflation rate/100) - 0.023844

At same time, the depository institutions increased their share by 1.3% of GDP (inflationary overbanking). These sizeable welfare costs are not reported so far in the literature. For instance, Lucas (2000) gets only 1% of GDP.

Figure 2 above constitutes in our main challenges in terms of finding the appropriate and sufficient distorting channels. In another words, the current state of the art may be missing huge triangles.

2.2. **Effects of Inflation in the Composition of the PCE (Personal Consumption Expenditure).** Previous figures have shown the inflationary effects in the supply side. On the other hand, by analyzing the demand side, the figure 3 illustrates that the loss of consumer purchasing power due to inflation which leads to: i) less demand for non-durable consumption goods such as food; and ii) more demand for personal services[4].
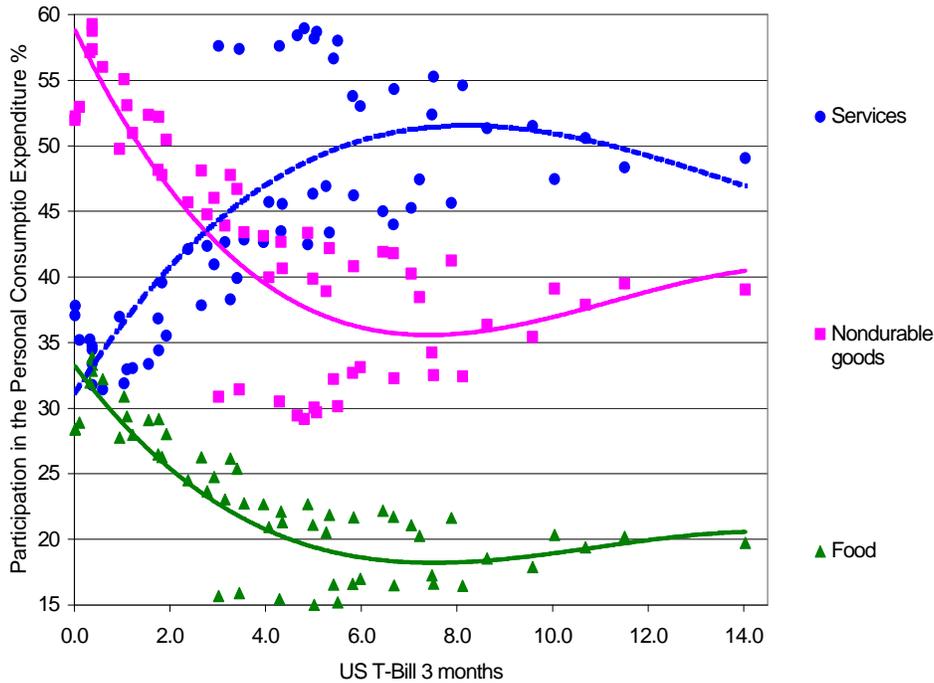
Figure 3. Effects of Inflation in the Composition of the Personal Consumption Expenditure. US: 1939-99

2.3. **Effects of Inflation in the Banking Industry.** Figure 4 shows how inflation affects the insured commercial banks in terms of the numbers of institutions,

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[4]Source: BEA. Table 2.6. PCE- Personal Consumption Expenditures by Type of Product.

This data shows also that inflation makes the households to spend relatively more on medical care and bank services.

branches, offices and bank employees.[5]   The classical inflationary overbanking is
also very clear for the US economy[6].

Figure 4. Inflation and Resources Allocated in the Commercial Banks. US:
1934-99

2.3.1. *Effects of Inflation in the Components of Bank Value-Added.* Figure 5 shows
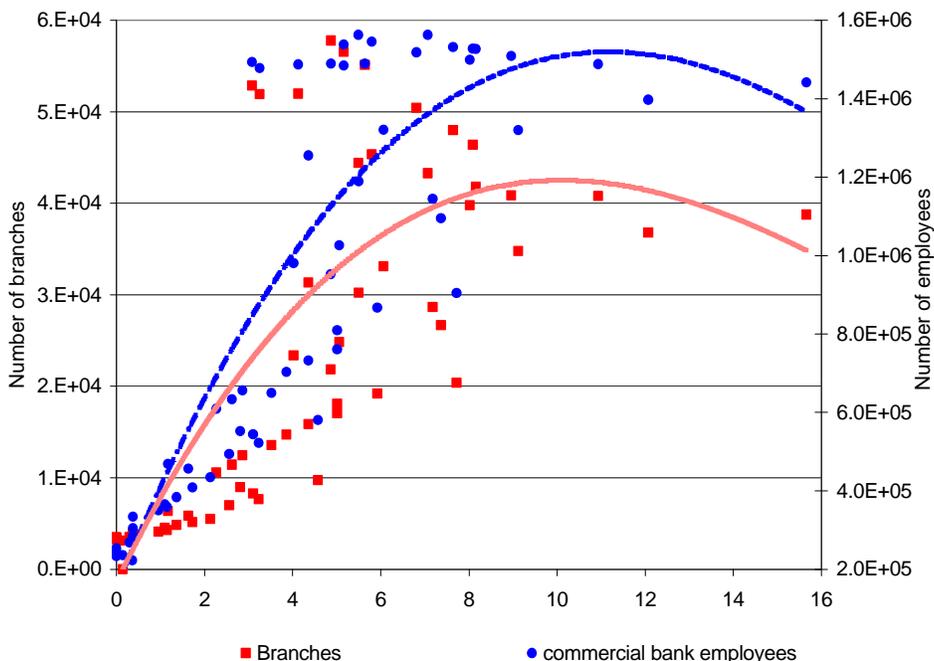that inflation induces more real resources allocated in the insured commercial
banks[7]. The inflationary overbanking happens in terms of the labor expenses, the

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[5]Series: US T-Bill 3 months. Secondary market. Serie: tbaa3m. Source: US Federal Reserve
System. Code H1.RIFSGFPIM03_N.M.

Banking data: institutions, branches, offices, employees. Source: FDIC-Insured Commercial
Banks. Tables CB01 and CB08.

[6]Bresciani-Turroni (1937) reports the hypertrophy of the German banks during the hyper-
inflation in the period 1913-1923. Daly (1967) reports similar effect for Uruguay in the period
1958-64. Yoshino (1993) provides several figures for explaining the effect of inflation in the bank-
value added and bank employment for Latin American countries. Aiyagari, Braun and Eckstein
(1998) report the cases of Israel, Argentina and Brazil.

[7]Source of data: http://www2.fdic.gov/hsob/SelectRpt.asp?EntryTyp=10. Table CB04. In-
sured Commercial Banks United States and Other Areas Year-to-Date Activity, 1934 - 1999.

The insured commercial bank value added as percentage of GDP is calculated by considering
the following bank budget constraint:

$(i_l\text{-p}).A - i._d L + \Phi S + X = w.n + \Pi + T + E$

Where:

$i_l.A$ = interest income by charging the nominal interest rate (i_l) on assets (A), such as,
loans, leases, investment securities, trading account assets, Fed. Funds sold, securities purchased,
balances due from depository institutions; p.A = percentage provision (p) for expected loss on
assets (A): loans and lease; $i._d.L$ = nominal interest rate ($i._d$) paid on liability (L) such as deposits,
Fed. Funds, Purchased Securities, Borrowed Money, Subordinated Notes & Debentures; $\Phi.S$= fee

occupancy expense (new branches), the service expenses, and all other non-interest expenses[8].
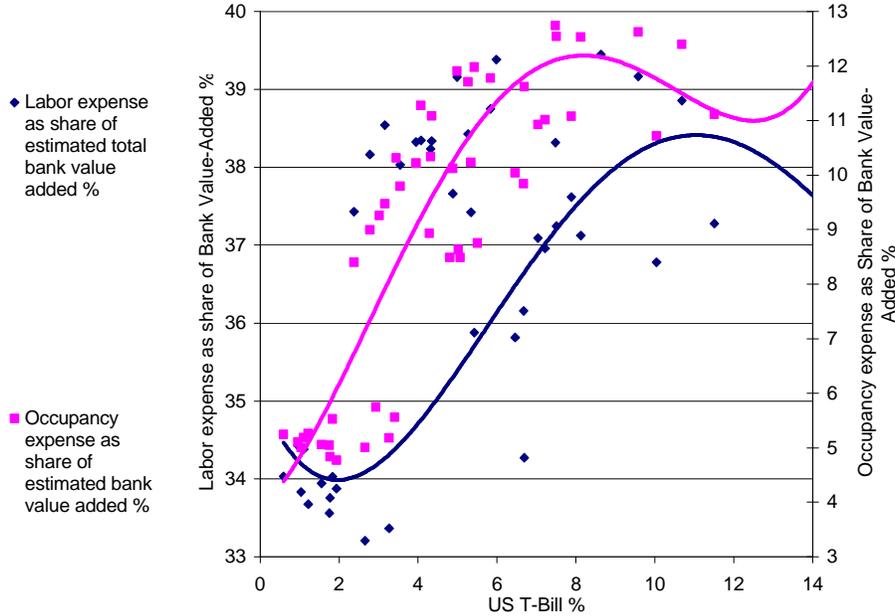

Figure 5. The Demand for Real Resources by the Commercial banks. US: 1957-98

We examine next the impact of inflation in terms of the occupational employments by both the consumption goods and banking industries to characterize better the nature of the social costs.

2.4. **Effects of Inflation in the Occupational Employment by Industry.**
According to the figure 6, when the US T-Bill rates increase from zero to 14% per year, the durable goods industry employment share of the total labor force in the economy drops by 7.6%. Still in this experiment, the nondurable goods employment share of the total labor force in the economy falls by 8.9%. On the other hand, the employment shares increase by 1.2% for the commercial banks.

income and other non-interest income. (Φ) represents the fee charged on bank services (S); X = net extra-ordinary revenue; w.n = employee salaries and benefits. With wage (w) on bank labor (n); T = pre-tax net operating income; Π profits in terms of Securities Gains/losses (-); E = services input expenditures such as capital services, educational expenses, business services, occupancy expenses and all other non-interest expenditure.

Source of data for the reported value added by the depository institutions: http://www.bea.doc.gov/bea/uguide.htm#_1_16. 1947-99 Gross Domestic Product by Industry. Table NDN-0272.

Our estimation of the value added by the insured commercial banks is close to the one reported by the BEA. We have done this imputation to estimate the cost components of value-added.

[8]We should point out that the BEA and United Nations report also the cost components of the financial sector value added. But according to the economist of BEA, there is not an exact correspondence with the U.N. structure. But, there is the following relationships between the U.N. and U.S. classifications: 1. Compensation of employees: COMP (series by BEA); 2. Capital consumption: Included in PTI (Property-type income); 3. Net operating surplus: Included in PTI; 4. Indirect taxes: IBT; 5. Subsidies received: Included in PTI. We have doubts about the series related to the capital consumption. Thus, we have made our own imputation by considering the income statment of insured US commercial banks.

Thus, the consumption goods industry employment share of the total employment in the economy falls by 16.5%. This percentage gives another alternative measure for the welfare costs of inflation in terms of the labor market[9]. Thus, the impact of inflation in the labor market looks much severe than the loss of consumption goods according to the figures 1 and 2. Just a small part of this loss is compensated by the increased employments in the depository institutions. Thus, inflation displaces only a small part of the labor from the consumption goods industry to the banking industry[10].
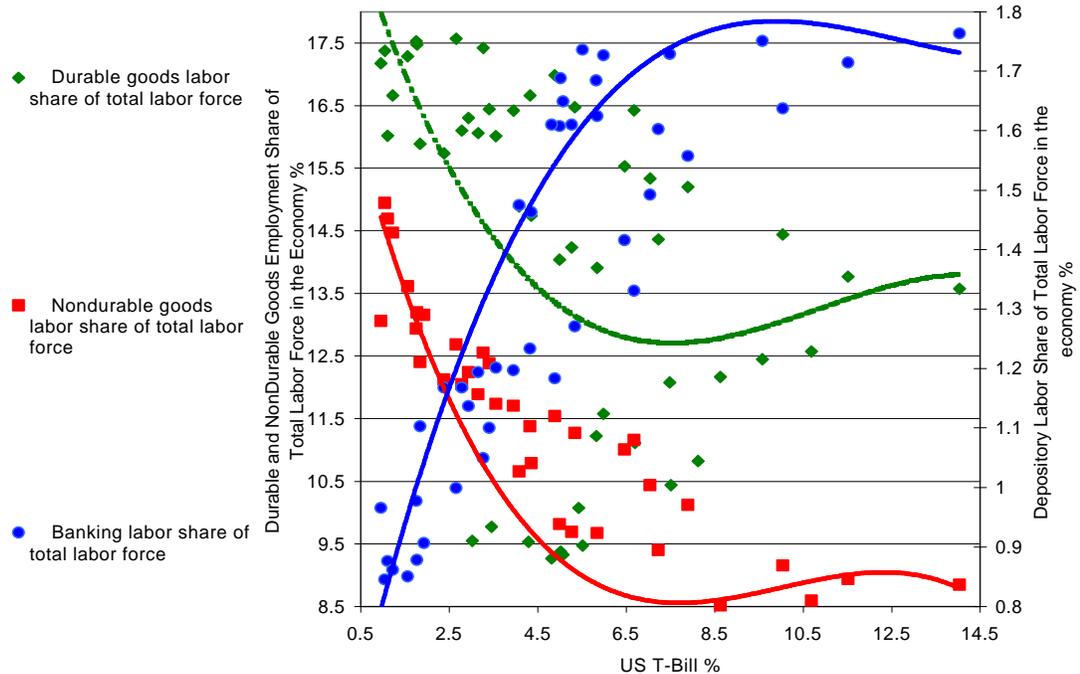

Figure 6. Effects of Inflation in the Labor Market by Private Industry. US: 1948-98.

We provide below further new details for the misallocation in terms of the occupational employments among industries[11].

Table 1 suggests that due to inflation the non-durable consumption goods industry requires more administrative support including clerical occupations[12]. These

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[9]Source of data. BEA. National Account. Full Time Equivalent Employment.

[10]See also some reported regressions in the table A.2 in the appendix.

[11]Source of Data: BLS (Bureau of Labor Statistics). Employment by occupation and industry. Household Data. Employed Persons by Industry, Sex, Race and Occupation. Period: 1972-94. Referred by the BLS as "The Old Occupational Employment". These data were received under request and they are not available in the BEA site.

[12]The Administrative support occupations including clerical are composed of: i) Records processing occupations; ii) Financial records processing occupations; iii) Bookkeeping, accounting, and auditing clerks; iv) Secretaries; v) Other clerical and administrative support workers.

occupational employments in the nondurable goods industrial are positively correlated with the inflation rates. We call this new important phenomenon as "the positive float labor".

Pearson correlations	US T-BILL	total employment	administrative support including clerical	precision production, craft and repair	other services except protective	Breakdown of total labor force Year 1994
total employment	0.36	1.00	.435(*)	.459(*)	0.05	100 %
Professional and technicial	0.05	.451(*)	-0.16	-0.01	-.456(*)	10.1
Managers and Administrators except farm	-0.21	0.24	-0.39	-.644(**)	-.681(**)	12.4
Sales	-0.18	0.18	-0.33	-.691(**)	-.607(**)	5.2
administrative support including clerical	.666(**)	.435(*)	1.00	.606(**)	.551(**)	11.3
precision production, craft and repair	.499(*)	.459(*)	.606(**)	1.00	.554(**)	14.1
machine operators, assemblers and inspectors	0.28	0.24	.450(*)	.769(**)	.612(**)	34.5
transport equipment operatives	0.40	0.21	.605(**)	.517(*)	.708(**)	3.9
handlers, equipment cleaners, helpers, and laborers	-0.33	0.19	-0.35	-.722(**)	-.609(**)	6.8
other services except protective	.431(*)	0.05	.551(**)	.554(**)	1.00	1.6

** Correlation is significant at the 0.01 level (2-tailed).
 * Correlation is significant at the 0.05 level (2-tailed).

Table 1. Inflation and Occupational Employment in the Non-Durable Goods Industry (Float Labor). US: 1972-94

Figure 7 illustrates better the meaning of the float labor in the manufacturing industry. For instance, due to the positive float labor, when the US T-Bill rates increase from 3% to 9.6% per year, there are increased employment levels by more than 20% for the secretaries, services, construction trades, financial records processing, health technicians, bookkeeping, accounting and auditing. At same time, due to the negative floating labor, inflation causes big drop of more than 30% in the employment levels for the salespersons (less sales of goods) and financial managers (boss). Thus, inflation distorts the overall composition of the labor force in the manufacturing industries (durable and non-durable consumption goods). Inflation makes the manufacturing industry more "heavy" in terms of the unskilled white-collar workers, specially financial workers, relatively to the blue-collar workers[13].

In the year 1994, the administrative employments represented 11.3% of the total labor force in the non-durable goods industry. Furthermore, these administrative employments are required to support indirectly the following occupational employments: 1) Precision production, craft and repair; 2) Machine operators, assemblers and inspectors; 3) Transport equipment operatives; and 3) Other services except protective. These indirect employments represent 54.2% of the total labor force.

For further details, see also the table A.4 in appendix which show the relationship between inflation and the types of float labor in manufacturing industry.

[13]Finally, the phenomenon of the float labor happens also inside the banking industry. See table A.4 in the appendix.
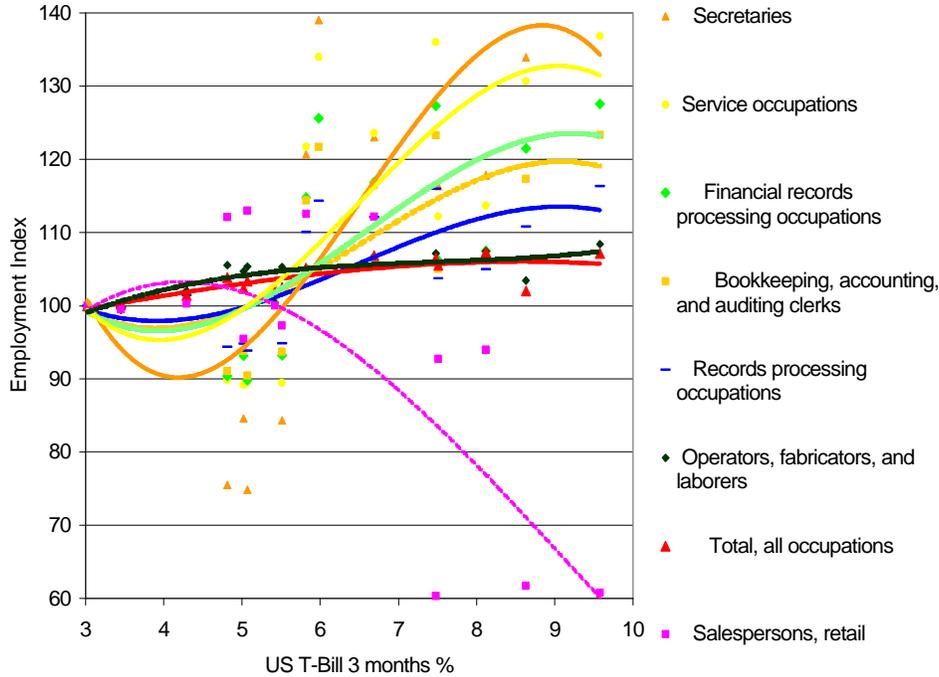
Figure 7. Inflation and Float Labor: Employment by Type of Occupation. Manufacturing Industry. US: 1983-98

3. The Model

Based on the stylized facts above, we propose a model to take into account the following phenomenons: i) the goods producing industry spend more the float labor relatively to blue-collar workers to avoid the inflation tax; ii) the households spend less time for producing consumption goods and more time for the personal services such as bank services and other banking products for saving their transaction time; and iii) the inflationary overbanking.

The economy is populated by a large number of infinitely lived households. Competitive firms produce consumption goods. For the sake of simplicity, there is no physical capital just labor as factor input. There is another sector called banks that provide both services and financial intermediation. Household supplies labor for firms and banks. Firms use labor for: i) producing consumption goods; and ii) managing their cash flow to avoid the inflation tax. According to the stylized facts above, this labor is called "positive float labor" because its occupational employment is positively correlated with inflation rates. Furthermore, the ratio of the positive float labor relative to the productive labor increases with inflation. In a simply manner, with increased inflation, the employment for white-collar workers increases relative to the employment for blue-collar workers.

Household preferences depend only on the consumption goods

$$(3.1) \qquad \sum_{t=0}^{\infty}(1+\rho)^{-t}U(C_t)$$

Each household is endowed with one unit of time that is allocated as: i) Labor input (n_t^c) for the firms to produce consumption goods; ii) Bank labor (n_t^b) to provide both bank services and financial intermediation; iii) Household transaction time or personal services (n_t^h); and iv) Industrial administrative time or firm positive float labor (n_t^f). The superscript for labor denotes the final use[14].

$$(3.2) \qquad n_t^c = 1 - n_t^b - n_t^h - n_t^f$$

The technology for producing consumption goods is given simply by

$$(3.3) \qquad C_t = n_t^c$$

Similar to McCallum and Goodfriend (1986), Lucas (1993), and Yoshino (1993), we have a transaction constraint. Households uses the following banking products: i) (m_t^h) currency; ii) loans (l_t^h); demand deposits (d_t^h); and bank services (s_t^h). These banking products save the transaction time (n_t^h) to carry out the flow of consumption expenditure (C_t)[15]. Relatively to Lucas (1993), we have added l_t^h and s_t^h.

$$(3.4) \qquad C_t = H[m_t^h, d_t^h, l_t^h, s_t^h].n_t^h$$

Banks as financial intermediary for the households attract competitively their demand deposits (d_t^h) and makes personal loans (l_t^h). If the clients deposits do not have temporarily enough balance to clear a check, banks makes automatically a consumer loan in an opened-end account. If the consumer pays goods with credit card, it is in fact a consumer loan. In this case, banks charge a interest rate in the loans and pay interests for their funding - deposits.

In this monetary economy, instead of a barter one, the banking products save the time that would be spent to get "the double coincidence of wants".

Bank services save also the transaction time because otherwise households when want to pay the bills they would have to spend bank trip time. For instance, an ATM (Automated Teller Machine) located at the workplace provides several banking products allowing workers to save bank trip time and to allocate the remaining time for the alternative uses.

Examples of bank services (s_t^h) are direct debt of bills in the household deposit accounts and a direct credit to the firm deposit account. This clearing services saves time for both the firms and households. Even though there is a mail service to collect the bills with the payments in a form of checks which allows households to save bank trip time, banks have to make the final clearing of bills and checks.

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[14]We see latter that the misallocation of labor according to equation 3.2 gives the channels for the welfare costs of inflation. The additional channel here relatively to Lucas (1993) and Yoshino (1993) is the float labor.

[15]The motivation is the survey "1999 Study of Consumer Payment Preferences", ABA (American Bankers Association). The payments methods are: Cash Carriers; Check Writers. Reward Seekers, PIN Debitors, and Signature Debitors. The last two methods are more preferred relatively to the others. See http://www.aba.com/Surveys+and+Statistics/ss dovegraphs.htm#Total .

Equation 3.4 is a more general alternative to Gilman (1992), Aiyagari, Rao, Braun and Eckstein (1998) and English (1999). They consider credit as bank services which is misleading because credit is a bank product relative to its role of financial intermediation. In our case, bank services involves mainly the clearing and processing of bills and checks in the payment system. Furthermore, they consider currency and credit as perfect substitutes. Here, they can be also complementary depending on the signal of the cross derivatives. Thus, currency, consumer credit, deposits and bank services save the household transaction time.

Based on Lucas (1993), equation 3.4 captures also the inventory model formulated by Baumol (1952) and Fama (1980).

Now, let us work on the following question: What are the signal of the partial derivatives of household transaction technology which was given by the equation 3.4 above?

By combining equations 3.3 and 3.4, we have

$$(3.5) \qquad \frac{n_t^c}{n_t^h} = H[m_t^h, d_t^h, l_t^h, s_t^h]$$

When inflation rates increase, there are less household demands for currency (m_t^h) and deposits (d_t^h). In this scenario, to replicate both the negative float labor for producing consumption goods (n_t^c) and the positive float labor for the transaction time (n_t^h)[16], we need that the ratio of these types of labors to decrease. In another words, the value of H[..] function has to decrease - H function has to be concave for the function above to have a maximum. Currency, demand deposits, loans and bank services have to save household transaction time. Therefore[17]

$$(3.6) \qquad \frac{\partial H}{\partial m_t^h} > 0; \frac{\partial H}{\partial d_t^h} > 0; \frac{\partial H}{\partial l_t^h} > 0; \frac{\partial H}{\partial s_t^h} > 0$$

In another words, for replicating the float labor when the inflation rates increase, we need that the bank products to save the transaction time. This fact is consistent with the equation 3.6 above. Thus, to avoid the inflation tax, there is a misallocation of the time from producing consumption goods to the time devoted to make transactions. Due to inflation, consumer has to spend more shopping time to search for cheaper goods. This misallocation of time characterizes the welfare costs of inflation. This is a key point in Lucas (1993), Lucas (2000) and Yoshino (1993).

To take into account the "firm positive float labor", firms have to hire administrative workers such as financial records processing occupations to manage the cash flows derived from selling goods and spending in terms of the wage bills. To avoid the inflation tax, firms have to invest any idle cash into money market accounts through hiring bankers as financial intermediary. In another words, firms have to set up their own financial and accounting department by hiring (n_t^f) float labor.

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[16]Positive or negative float labors mean the signals of correlations of these occupational employments with the inflation rates.

[17]The drawback for this type of McCallum and Goodfriend (1986) framework according to Lucas (2000) in his class notes on Econ. 331, Theory of Income, at U. of Chicago is that the transaction technology may not be concave. Doubling the arguments of H (.) function, the value of this function can increase more than twice.

If the inflation rates increase, the demand for currency, demand deposits and loans would decrease which would require more trips to banks. Services save these trips. The total effect of inflation in the ratio of labor for producing goods relative to time devoted to make transaction depend on the partial derivatives of the arguments of H(.) function with respect to the inflation rates.

Similar to households, firms have also a transaction technology that uses the float labor (n_t^f)

$$(3.7) \qquad n_t^f = F[m_t^f, d_t^f, l_t^f, s_t^f].C_t$$

For firms to sell consumption goods (C_t), they require administrative float labor (n_t^f) such as financial records processing occupational employment to manage the cash flow derived from selling goods and paying wage bills. This administrative activity involves also making deposits in a money-market accounts and taking loans. Float labor performs financial transactions by demanding all banking products to avoid the inflation tax and saving bank trip time.

Banking products such as currency (m_t^f), firm working capital loan (l_t^f), deposits (d_t^f), and banking services (s_t^f) saves the firm float time n_t^f. Thus,

$$(3.8) \qquad \frac{\partial F[m_t^f, d_t^f, l_t^f, s_t^f]}{\partial m_t^f} < 0; \frac{\partial F[m_t^f, d_t^f, l_t^f, s_t^f]}{\partial d_t^f} < 0;$$

$$\frac{\partial F[m_t^f, d_t^f, l_t^f, s_t^f]}{\partial s_t^f} < 0; \frac{\partial F[m_t^f, d_t^f, l_t^f, s_t^f]}{\partial l_t^f} < 0.$$

By combining equations 3.3 and 3.7, we have the firm floating labor given by

$$(3.9) \qquad n_t^f = F[m_t^f, d_t^f, l_t^f, s_t^f].n_t^c$$

This functional form captures the inventory theoretic model such as Miller and Orr (1966) which focus on the demand for money by the firms which is closely related to Baumol (1952).

When inflation rates increase, the demand for currency, demand deposits and loans decrease but the demand for services increases. Consumers saves money balances and makes more frequent trips to banks which require banking services. Given our stylized fact that the ratio of the positive float labor (n_t^f) relative to the negative productive float labor (n_t^c) increases with inflation, we need that the signals of the partial derivatives above to be negative according to equation 3.8. This fact characterizes another channel for misallocation of labor inside the manufacturing industry which brings about additional welfare costs of inflation.

Banks have two major roles: i) providing services for firms and individuals in terms of making transactions, clearing, receiving bill payments etc.; and ii) making financial intermediation - accepting deposits, investing in money market accounts , and granting either working capital loans for firms or consumer credit. Thus, banks earn incomes from two sources: i) As financial intermediary, the profit is derived from charging a competitive interest rate on loans that are demanded by both households and firms, paying interest rate for attracting deposits from the households and firms, and hiring bank labor to make this financial intermediation;

and ii) As producer of services, banks charge a competitive fee on services and hire bank workers.

Every check issued by either the households or the firms requires financial transactions in terms of debit and credit through a clearing house. There is a fee charged per check issued. A check can be compensated against savings account, demand deposits or against an automatic loan - if there is not temporarily sufficient balance.

We assume a bank technology for producing both financial intermediation and services. As financial intermediary banks attract demand deposits from the firms and households, make loans for them, and put aside the required bank reserves. Services are demanded by the households and firms. Bank labor (n_t^b) is necessary for producing all banking products. Thus, we have for the firms their demands for deposits (d_t^f), the respect reserve requirements (Z_t^f), their working capital loan (l_t^f), and their bank services (s_t^f). Similarly, the households demand the following bank products: d_t^h, l_t^h, Z_t^h, and s_t^h.

The full commercial bank production function is given by a constant to return to scale technology

$$(3.10) \qquad n_t^b = B[d_t, l_t^b, Z_t, s_t]$$

All partial derivatives of equation 3.10 are positive because if banks want to produce additional unit of each product, they demand more bank labor. Equation 3.10 compared to Lucas (1993) have two additional arguments: loans and services.

Banks deposits come from the households (d_t^h) and firms (d_t^f) These deposits are subject to the same exogenous non-interest accruing reserve requirement (α_t). Bank services (s_t) are demanded by the households (s_t^h) and firms (s_t^f).

$$(3.11) \qquad d_t = d_t^h + d_t^f; \quad Z_t = Z_t^h + Z_t^f$$
$$Z_t^h = \alpha_t.d_t^h; \quad Z_t^f = \alpha_t.d_t^f; \quad s_t = s_t^h + s_t^f$$

We are assuming that the reserve requirement is binding. Lucas (1993) considers also the non-binding case.

The supply of loans by banks (l_t^b) is equal to the total demand deposits net of bank reserves. This equation represents the role of banks as an intermediary of the overall financial savings to the loans demanded by the firms and households.

$$(3.12) \qquad l_t^b = (1 - \alpha_t).(d_t^h + d_t^f)$$

By considering equations 3.10 and 3.11, we have the full commercial bank production function.

$$(3.13) \qquad n_t^b = B[d_t^h + d_t^f, (1 - \alpha_t).(d_t^h + d_t^f), \alpha_t.(d_t^f + d_t^h), s_t^h + s_t^f]$$

Before defining the budget constraint of the banks, let us remember that our economy is composed of the following sectors: i) government; ii) banks; iii) firms; and iv) households. We define their budget constraints below. The exogenous real fiscal deficit χ_t is financed by the seigniorage, which is returned in a lump-sum fashion to the payers (firms and households) of the inflation tax on monetary base. Thus, we have the following budget constraint for the Central Bank.

(3.14) $\qquad \chi_t = (m_{t+1} + \alpha_{t+1}.d_{t+1}).(1 + \pi_{t+1}) - m_t - \alpha_t.d_t = \chi_t^h + \chi_t^f$

Where: $m_{t+1} = m_{t+1}^h + m_{t+1}^f; d_{t+1} = d_{t+1}^h + d_{t+1}^f$

The lump-sum transfers χ_t^h and χ_t^f are made directly to the households and the firms.

The financial system is competitive. There is a free entry and exit. All competitive commercial banks takes the nominal interest rate on loans (i_t^l), the nominal interest rate on deposits (i_t^d), the fee charged on services (Ψ_t), and the bank wages (W_t) as given. The nominal banks profit (T) is zero. We set P_t as numeräire. In terms of the timing of the model we have: in the beginning of the previous period banks attracts deposits by paying competitive interest rates. In the beginning of the next period, banks returns the principal and the interest on deposits. Similarly for the loans, we use the same timing. The budget constraint of a competitive commercial bank is given by

$$\tau_t = \frac{T_t}{P_t} = (1 + i_t^l).\frac{L_{t-1}^b}{P_t} - (1 + i_t^d).\frac{D_{t-1}}{P_t} - \frac{L_t^b}{P_t} + \frac{D_t}{P_t}$$
$$- \frac{W_t}{P_t}.n_t^b + \frac{\Psi_t}{P_t}.s_t = 0$$

(3.15) $\qquad \tau_t = (1 + r_t^l).l_{t-1}^b - (1 + r_t^d).d_{t-1} - l_t^b + d_t - w_t.n_t^b + \phi_t.s_t = 0$

Where: $(1 + i_t^l) = (1 + r_t^l).(1 + \pi_t); (1 + i_t^d) = (1 + r_t^d).(1 + \pi_t); \frac{W_t}{P_t} = w_t;$

$\phi_t = \frac{\Psi_t}{P_t}; \pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$

(r_t^l) is the real interest rate charged on loans; (r_t^d) is the real interest rate paid on deposits. $(1+i_t) = (1 + \rho) + (1 + \pi_t)$ and ρ is the marginal rate of time preference.

By substituting the equations 3.11, 3.12, 3.13 into 3.15, we have

(3.16) $\qquad \tau_t = [r_t^l.(1 - \alpha_{t-1}) - r_t^d].d_{t-1} - \alpha_{t-1}.d_{t-1} + \alpha_t.d_t$
$$- w_t.B[d_t, (1 - \alpha_t).d_t, \alpha_t.d_t, s_t] + \phi_t.(s_t^h + s_t^f)$$

The profit of the firms in real terms is

(3.17) $\qquad \xi_t = \Xi_t/P_t = n_t^c - w_t(n_t^c + n_t^f) - (1 + \pi_{t+1}).(m_{t+1}^f + \alpha d_{t+1}^f)$
$$+ (m_t^f + \alpha d_t^f) + \chi_t^f - \phi_t^f.s_t^f - (1 + r_t^l).l_{t-1}^f + (1 + r_t^d).d_{t-1}^f + (l_t^f - d_t^f)$$

Where: χ_t^f is the seigniorage transfer from the Central Bank to the firms.

According to the Walras´Law, we combine the budget constraints of the Central Bank, banks, and firms to obtain the budget constraint of the households that own the firm and the banks. Thus, we have

(3.18) $\quad n_t^h.H - w_t.(1 - n_t^h) - \tau_t - \varepsilon_t + (1 + r_t^l).l_{t-1}^h - (1 + r_t^d).d_{t-1} - l_t^h + d_t^h$
$$+ \phi_t.s_t^h - \chi_t^h + (1 + \pi_{t+1}).(m_{t+1} + \alpha_{t+1}.d_{t+1}) - m_t^h - \alpha_t.d_t^h = 0$$

Where: χ_t^h is the seigniorage transfer from the Central Bank to the household tax payer on his monetary base.

3.1. **The Problem of Competitive Banks.** Banks maximize their present values of the profits, given by equation 3.16, by choosing (d_t, s_t) The steady state first order conditions are

$$(3.19) \qquad (1-\alpha).r^l + \alpha.\rho = r^d + (1+\rho).w.\frac{\partial n^b}{\partial d}$$

$$(3.20) \qquad\qquad\qquad \phi = w.B_s$$

Where: $\frac{\partial n^b}{\partial d} = (1-\alpha).B_{l^d} + B_d + \alpha.B_z$

Equation 3.19 gives the optimal quantities either of the bank deposits or of the supply of bank loans. The optimum quantity of demand deposits are obtained when the marginal revenue of loans (real interest earned on loans including the bank reserves in the next period) derived from additional deposit is equated to: i) the marginal costs of attracting deposits (real interest rate paid on deposits in the next period); and ii) additional value of time spent in the bank labor to intermediate this additional deposit in the previous period. Lucas (1993), p. 26, shows a condition similar to this equation.

The optimum provision of services to the firms and households are given by equation 3.20. The marginal revenue of services (ϕ) has to be equated to the marginal costs in terms of the additional value of the time spent on bank labor to produce services.

The equilibrium in the market for loans requires that the bank loans (l_t^b) equal to the demand for loans by the firms (l_t^f) and households (l_t^h).

$$(3.21) \qquad\qquad\qquad l_t^b = l_t^f + l_t^h$$

3.2. **The Problem of Households.** Households own both the firms and banks. They maximize the present value of their utilities given by equation 3.1 subject to: i) the budget constraint given by equation 3.18; ii) the household transaction constraint given by condition 3.5. Thus, we have

$$(3.22) \qquad \underset{n_t^h, m_{t+1}^h, d_{t+1}^h, l_{t+1}^h, s_t^h}{Max} \sum_{t=0}^{\infty} (1+\rho)^{-t} U\{n_t^h.H\}$$

st.

$$n_t^h.H - w_t.(1-n_t^h) - \tau_t - \varepsilon_t + (1+r_t^l).l_{t-1}^h - (1+r_t^d).d_{t-1} - l_t^h + d_t^h$$

$$+\phi_t.s_t^h - \chi_t^h + (1+\pi_{t+1}).(m_{t+1} + \alpha_{t+1}.d_{t+1}) - m_t^h - \alpha_t.d_t^h = 0$$

Where: $H = H[m_t^h, d_t^h, l_t^h, s_t^h]$

We have the following steady state first order conditions

$$(3.23) \qquad\qquad \mathcal{L}_{n_t^h} : \lambda = -\frac{U_c.H}{H+w}$$

Where: λ_t is the Lagrange multiplier associated with the household budget constraint.

$$(3.24) \qquad \mathcal{L}_{m_t^h} \quad and \quad \mathcal{L}_{n_t^h} : w.\frac{H_{m^h}.n^h}{H} = i$$

$$w\frac{\frac{\partial n^c}{\partial m^h}}{\frac{\partial n^c}{\partial n^h}} = i$$

Condition 3.24 equates the marginal rate of transformation between the household holdings of currency and the time in effecting transaction to their relative price (the nominal interest rate foregone by holding currency). Lucas (1993), p. 26, equation (5.6) shows a similar condition.

$$(3.25) \qquad \mathcal{L}_{d_t^h}, \quad \mathcal{L}_{n^h} : \frac{w.H_{d^h}.n^h}{H} = i\alpha - \frac{r^d - \rho}{1 + \rho}$$

$$\frac{w\frac{\partial n^c}{\partial d^h}}{\frac{\partial n^c}{\partial n^h}} + \frac{r^d - \rho}{1 + \rho} = i\alpha$$

Equation 3.25 equates the marginal rate of transformation between the household deposits and the time in effecting transaction to their relative price (the nominal interest rate foregone in the bank reserves net of the present value of the interest earned on deposits in excess of the marginal rate of time preference)[18].

$$(3.26) \qquad \mathcal{L}_{s_t^h}, \quad \mathcal{L}_{n_t^h} : w\frac{H_{s^h}.n^h}{H} = \phi$$

$$w\frac{\frac{\partial c}{\partial s^h}}{\frac{\partial c}{\partial n^h}} = \phi$$

Equation 3.26 equates the marginal rate of transformation between the household bank services and the time in effecting transaction to their relative price (the costs of bank labor for producing additional bank services).

$$(3.27) \qquad \mathcal{L}_{l_t^h}, \mathcal{L}_{n_t^h} : \frac{w.H_{l^h}.n^h}{H} = \frac{r^l - \rho}{1 + \rho}$$

$$w\frac{\frac{\partial n^c}{\partial l^h}}{\frac{\partial c}{\partial n^h}} = \frac{r^l - \rho}{1 + \rho}$$

Equation 3.27 equates the marginal rate of transformation between the loans demanded by the households and the time in effecting transaction to their discounted relative price (the real interest rate paid on loans in excess of the marginal rate of time preference).

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[18]Deposits are made in the begining of the period. While the interest earned on deposits happens in the beginning of the next period. Thus, the present value of the interest earned on deposits tomorrow should be discounted by $\frac{1}{1+\rho}$.

3.3. **The Problem of Firms.** Firms maximize their profit (condition 3.17) subject to the firm float time constraint given by equation 3.9, Thus, we have

$$(3.28) \qquad \underset{n_t^c n_t^c, m_{t+1}^f, d_{t+1}^h, l_t^h, s_t^h}{Max} \sum_{t=0}^{\infty} (1+\rho)^{-t} \xi_t$$

$$\xi_t = n_t^c - w_t.(n_t^c + n_t^f) - (1 + \pi_{t+1}).(m_{t+1}^f + \alpha d_{t+1}^f)$$
$$+ (m_t^f + \alpha d_t^f) + \chi_t^f - \phi_t^f.s_t^f - (1 + r_t^l).l_{t-1}^f + (1 + r_t^d).d_{t-1}^f + (l_t^f - d_t^f)$$

Where: $n_t^f = F[m_t^f, d_t^f, l_t^f, s_t^f].n_t^c$

The steady first order conditions are

$$(3.29) \qquad \mathcal{L}_{n_t^c} : w = \frac{1}{1+F} \equiv \frac{1}{1 + \frac{\partial n^f}{\partial n^c}}$$

Given that $\frac{d(n^f)}{dn^c} = F \geqslant 0$, this economy gets less production of consumption goods relative to an undistorted economy. To avoid the inflation tax, the firm requires more float labor when producing consumption goods. The loss of consumption goods is a direct measure of the welfare costs due to the firm float labor.

Another nice interpretation of condition 3.29 is that inflation increase the ratio of the float labor as percentage of blue collar workers. This phenomenon makes the manufacturing industry more roundabout or inefficient. Thus, the real wage or the productivity of blue-collar workers would decrease given that other factor inputs are constant.

$$(3.30) \qquad \mathcal{L}_{m_{t+1}^f} : -w.F_{m_t^f}.c_t = i_t$$
$$-w.\frac{\partial n^f}{\partial_{m^f}} = i$$

Equation 3.30 gives the optimum quantity of money (currency) which is obtained when the value of the marginal benefit of saving the firm float time is equated to the opportunity cost of carrying currency (nominal interest foregone).

$$(3.31) \qquad \mathcal{L}_{l_{t+1}^f} : -w.F_{l^f}.c = \frac{1 + r_t^l}{1 + \rho} - 1$$
$$-w.\frac{\partial n^f}{\partial_{l^f}} = \frac{r_t^l - \rho}{1 + \rho}$$

Equation 3.31 gives the optimum quantity of loans demanded by the firms which is obtained when the marginal benefit of an additional loan in terms of saving the value of the firm floating time is equated to the present value of the marginal costs of loans (spread between the real interest rate paid on loans and the marginal rate of time preference).

$$(3.32) \qquad \mathcal{L}_{d^f_{t+1}} : -w.F_{d^f}.n^c + [\frac{1+r^d}{1+\rho} - 1] = i.\alpha$$

$$-w.\frac{\partial n^f}{\partial_{d^f}} + \frac{r^d - \rho}{1+\rho} = i.\alpha$$

Equation 3.32 gives the optimum quantity of deposits which are obtained when the marginal benefits of additional deposit in terms of saving the value of firm float time and earning the present value of the real interest rate on deposits in excess to the marginal rate of time preference is equated to the marginal costs of deposits (nominal interest foregone in the bank reserve).

$$(3.33) \qquad \mathcal{L}_{s^f_t} : -w.F_{s^f}.n^c = \phi$$

$$-w.\frac{\partial n^f}{\partial_{s^f}} = \phi$$

Equation 3.33 gives the optimum quantity of bank services demanded by firms which is obtained when the marginal benefits of additional services in terms of saving the value of firm float time is equated to the marginal costs (fee charged on services).

Some of the equations above can be simplified. By using the homogeneity of B, H and F functions permit us the application of Euler´s theorem.

$$(3.34) \qquad n^b_t = B[d, l^b, Z, s] = d.B_d + l^b.B_l + Z.B_z + s.B_s$$

Using equations 3.19 and 3.20 into condition 3.34, we have the value of scarce resources allocated in the commercial bank industry[19].

$$(3.35) \qquad w.n^b = w.[B_d + \alpha.B_z + (1-\alpha)B_l].d + w.B_s.s$$

$$= \frac{[(1-\alpha).r^l - r^d + \alpha\rho]}{(1+\rho)}.d + \phi.s$$

$$= \frac{l^b.(r^l - \rho) - d.(r^d - \rho)}{(1+\rho)} + \phi.s$$

Equation 3.35 shows that the value of the scarce resources allocated in the commercial banking industry to provide banking products for saving the household transaction time and firm float labor can be measured by: i) Harberger triangle in the market for loans (present value of the real interest revenue on loans demanded by both firms and households net of the real interest paid on all deposits plus the updated value of bank reserves); and ii) the bank services revenue.

Similarly for the H(..) function.

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[19]In our economy, the GDP is defined as $p_t.y_t = w_t.(n^c_t + n^b_t + n^h_t + n^f_t)$.

Where: p_t was set as numeraire.

Thus, for estimating the welfare costs of resources allocated in the banking industry as percentage of GDP, we have

$$\frac{w.n^b}{y} = \frac{l^b.(r^l - \rho) + d.(\rho - r^d) + \phi.s}{(1+\rho).y}$$

(3.36) $H[m^h, d^h, l^h, s^h] = m^h.H_{m^h} + d^h.H_{d^h} + l^h.H_{l^h} + s^h.H_{s^h}$

Using equations 3.24, 3.25, 3.26, 3.27 into 3.36, we have

(3.37) $w.n^h = i.(m^h + \alpha.d^h) + [l^h.\dfrac{r^l - \rho}{(1 + \rho)} - d^h.\dfrac{r^d - \rho}{(1 + \rho)}] + \phi.s^h$

The value of resources spent in the household transaction time can be measured by: i) Bailey triangle: the inflation tax paid by household on their holdings of the monetary base; ii) the Harberger triangle: the present value of the real interest rates spread paid on loans minus the present value of the real interest rate spread received on the households deposits; and iii) the household expenditure on banking services.

Similarly for F function.

(3.38) $F[m^f, d^f, l^f, s^f] = -m^f.F_{m^f} - d^f.F_{d^f} - l^f.F_{l^f} - s^f.F_{s^f}$

By using equations 3.30, 3.31, 3.32, 3.33, 3.7 and into 3.38, we get

(3.39) $w.n^f = i(m^f + \alpha.d^f) + l^f.(\dfrac{r^l - \rho}{1 + \rho}) - d^f.(\dfrac{r^d - \rho}{1 + \rho}) + \phi.s^f$

The firm float time can be measured by the sum of: i) Bailey triangle: the inflation tax on the firm holdings of the monetary base; ii) the Harberger triangle; and iii) the firm expenditure on bank services.

For solving the system, first notice that we have:
i) Twelve endogenous variables $(n_t^c, n_t^b, n_t^f, n_t^h, m_t^h, d_t^h, l_t^h, s_t^h, m_t^f, d_t^f, l_t^f, s_t^f)$;
ii) Ten equations 3.19, 3.20, 3.24, 3.25, 3.26, 3.27, 3.30, 3.31, 3.32 and 3.33;
iii) Two equilibrium conditions:
    iii.1) The society resource constraint (equilibrium in the goods market) which is given by equation 3.2.
    iii.2) The equilibrium in the market for loans: Supply equals to the demand, given by equation 3.21.
Thus, the system is determined and we obtain the optimal choices.
We define the value of the "consolidated total welfare costs of inflation" $W(i)$ as function of the nominal interest rate (i) as composed of[20]:

(3.40) $W(i) = w.(n^b(i) + n^h(i) + n^f(i)) = w.(1 - n^c)$

By using equations 3.35, 3.37 and 3.39 into 3.40, we define alternatively "the total welfare costs of inflation" as

(3.41) $W(i) = i.(m + \alpha.d) + 2.\dfrac{[l^b.(r^l - \rho) - d.(r^d - \rho)]}{(1 + \rho)} + 2.\phi.s$

(3.42) $W(i) = i(m + \alpha.d) + 2.w.n^b$

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[20]Thus, "the total welfare costs of inflation" is just a sum of the values of resources allocated in the banking industry, for the firm float time and for the household transaction time.

According to equation 3.41, the total welfare costs are composed of: 1. The first term above: the Bailey triangle under the demand for monetary base schedule; 2. The second term above: the Harberger triangle in the market for loans; and 3. Third term above: the triangle in the bank services.

Furthermore, the total welfare costs of inflation can be composed of: i) the Bailey triangle; ii) twice the value of resources allocated in the commercial banks.

This formula contain twice the value of bank resources because for every bank transaction there are resources spent by the bank and by the client. In another words, to make a withdrawals it is necessary a bank teller and the customer spending bank trip time.

Figure 8 illustrates conceptually the components of the total welfare costs of inflation. We define next the relative importance of these triangles.
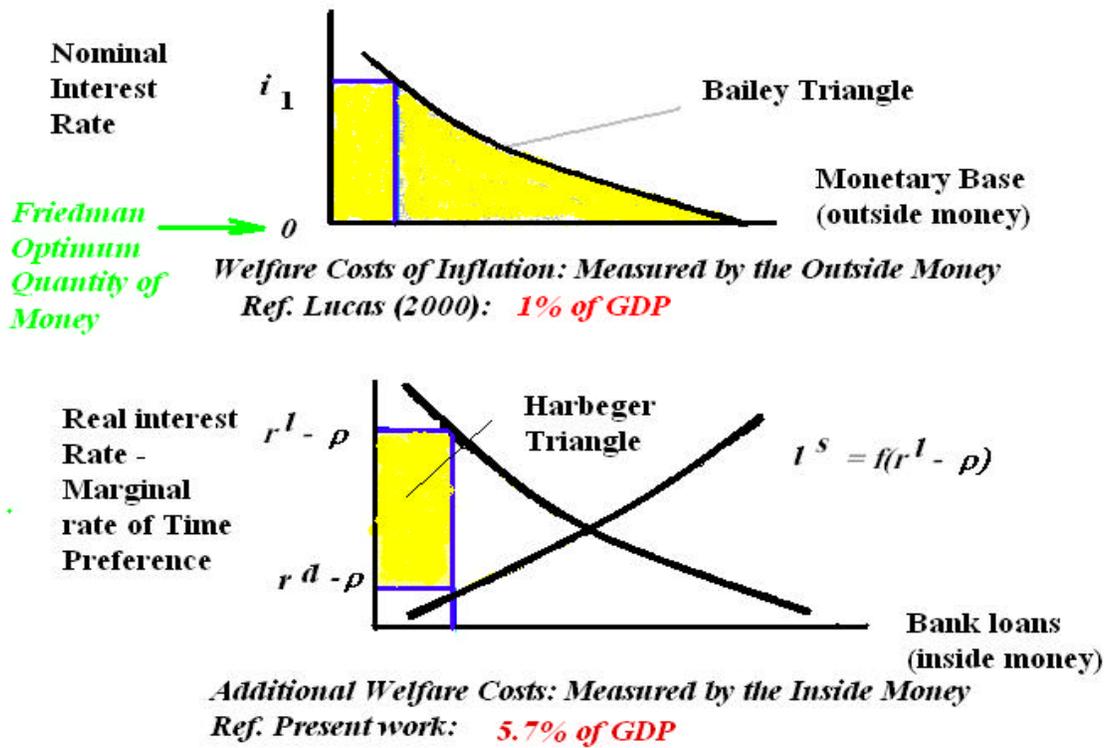
Fig 8. Monetary Aggregates for Measuring the Welfare Costs of Inflation: Outside and Inside Moneys

4. Estimation of the US Welfare Costs of Inflation

There are several approaches to estimate the welfare costs functions. For instance, we had according to the equations 3.40, 3.35, 3.37, and 3.39 :

$$(4.1) \qquad w.(1 - n^c) = w.[n^b(i) + n^h(i) + n^f(i)]$$

$$Where : w.n^b = \frac{l^b.(r^l - \rho) - d.(r^d - \rho)}{(1 + \rho)} + \phi.s$$

$$w.n^h = i.(m^h + \alpha.d^h) + [l^h.\frac{r^l - \rho}{(1 + \rho)} - d^h.\frac{r^d - \rho}{(1 + \rho)}] + \phi.s^h$$

$$w.n^f = i(m^f + \alpha.d^f) + l^f.(\frac{r^l - \rho}{1 + \rho}) - d^f.(\frac{r^d - \rho}{1 + \rho}) + \phi.s^f$$

One alternative is to consider the left hand side of the equation above. In this sense, there are two ways to measure the welfare costs of inflation: First, by estimating econometrically the figure 1 (the arguments of the utility functions in terms of consumption goods foregone); and Second, by following Lucas (1993) methodology: The idea is to make parametrization of each n^b, n^h, n^f function by assuming a Cobb-Douglas (homogenous) production functions. In the appendix, we derive the parametric forms.

Another approach is to consider the right hand side of this equation by estimating separately $w.n^b$, $w.n^h$.and $w.n^f$ This approach makes an inference of the welfare costs in terms of the Harberger and Bailey triangles by using the financial data according to the sectorial budget constraints. In this approach, we estimate the values of resources allocated in the banking industry, for the firm float time and for the household transaction time.

We develop these alternative approaches to check whether the estimations are consistents. Let us start with the first approach by examining the loss of consumption goods (arguments of the utility function) due to inflation. The result was reported in the figure 2 above, which gave us a total welfare costs of 10.3% of GDP. This figure should be roughly consistent with other results when we consider the alternative approaches. In this sense, let us work on the right hand side of the equation 4.1, that is, to make the imputation of the real resources allocated for each distorting channel by considering the financial data according to the sectorial budget constraints.

Figure 9 illustrates the estimated values for the household transaction function according to the equation 3.37 but in a naïve manner, i.e., by fitting a quadratic polynomial to the original data. It gives also an idea of the relative importance of the components of this welfare function[21]. The criticism about this procedure is

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[21]The Series and the Methods for estimating the equation 3.37 are the following:

1. To estimate the household real excess interest rate expenditure, we take the real interest rates spread as (the nominal prime rate on bank loan minus the nominal interest rate charged on the discount window) multiplied by the annual household debt stock. For the discount factor as a proxy for the marginal rate of time preference, in real terms, we have used the value of 2.18% per year due to the average real rate of the FED Fund in the period 1964-99. Lucas (1993) consider the value of 5% per year. We consider this value too excessive because we do not observe any average real interest rate bigger than this percentage. There would be no savings at all.

The debt stock is obtained by using the US Flow of Funds. Source: Federal Reserve Board, Balance Sheet of the Personal Sector. http://www.federalreserve.gov/releases/z1/current/data.htm.

the possible spurious regression because the series exhibit unit roots and they are cointegrated. Thus, we have used a VAR with VEC (vector error correction) model to retrieve the long run relationship. See figure 10 below.

The consolidated household debt is composed of the following series: Mortgage debt (Flow of Funds, series FL173165105), Other mortgage debt (FL173165205), consumer credit (FL153166000), and Other liabilities (FL173199005).

2. The real interest revenue by the household sector is calculated by estimating the stock of the household interest earning assets. This annual stock is multiplied by the excess real interest rate on CD (nominal interest rate on CD 6 mo. - minus the nominal interest rate charged on discount window). The assets held by household are composed of : open market paper (FL163069103), US savings bonds (FL313161400), Other Treasury Securities (FL173061105), Agency securities (FL153061705), Municipal Securities (FL153062005), Corporate and Foreign bonds (FL153063005).

We should emphasize that the PCE (Personal Consumption Expenditure) reports the total values in terms of the **nominal** interest received and paid by the households. But, we can not use these series because according to equation 3.37 we have to use the **real** interest expenditure and revenue.

Of course, depending on the stocks and real interest rates considered, the estimated Haberger triangle can change.

3. The personal expenditure in the bank service charges, trust services, and safe deposit box rental (included in PCE - personal consumption expenditure) comes from http://www.bea.doc.gov/bea/dn1.htm, under the headings "Selected NIPA Tables, Table 2.6, line 88

4. To estimate the household holdings of monetary base, we have first calculated the participation of the household holdings of M1 relative to the total M1. This percentage was applied to the St. Louis monetary base. This imputation was done because the balance sheet of household in the Flow of Funds reports only the M1 holdings by the households. The M1 held by the households comes from. Federal Reserve.Board, Flow of Funds. Series FL153020005.Q. The M1 held by the firm comes from. Federal Reserve.Board, Flow of Funds. Series FL143020005. Finally, to estimate the inflation tax on monetary base paid by household, we have used the nominal interest rate (US T-Bill 3 months) multiplied by the stock of monetary base held by the household. Similar method was used to estimate the inflation tax on monetary base paid by the firms.
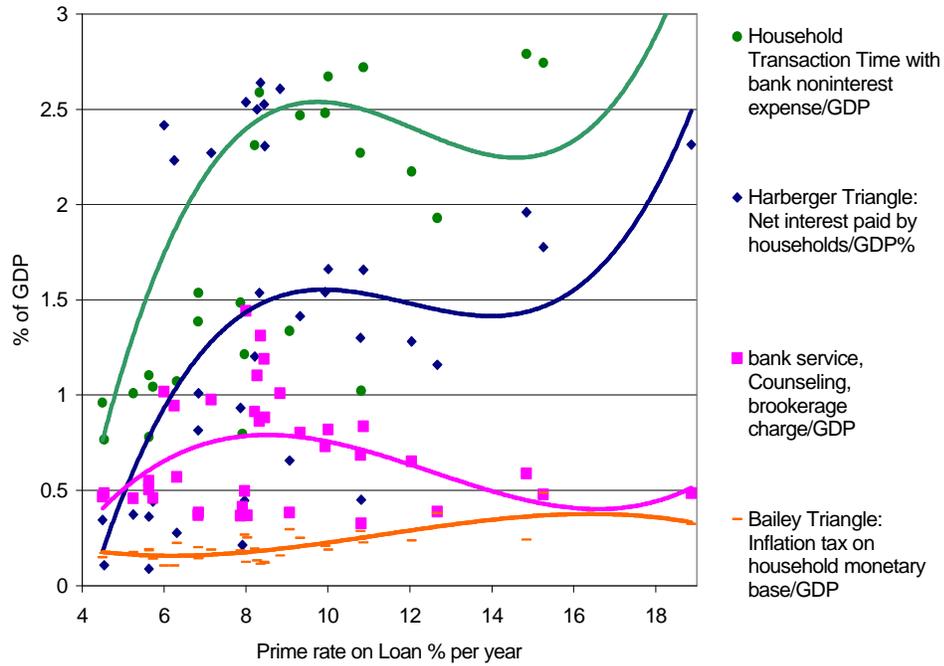
Figure 9. The Original Data for the Estimated Household Transaction Time. US: 1964-99

Figure 10 shows the fitted values for the welfare costs of inflation due to the resources allocated for the household transaction time as function of the inflation rate[22]. In the year 1980, after the 2nd oil shock, the US inflation rate reached the maximum value of 13.5% per year and the US economy spent 4.8% of GDP in terms of the real resources for the purpose of the household transaction time. On the other hand, under nominal interest rate equal to zero or deflation equal to 5% per year, the USA would have spent the minimum amount of 0.25% of GDP. Thus, the USA have wasted additional amount of 4.5% of GDP for the purpose of household transaction time.

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[22]For obtaining the fitted welfare cost function reported in the figure 10, we have used a VAR-VEC model with the following the long run relationship between the welfare costs of inflation and inflation rate (CPI-U):

Log(Household transaction time with expenditure on bank fee/GDP) =9.147936286*Log(1+Inflation)/100) + 0.06513130506*(time trend) - 1.171826093.

We have tried also the prime rate on loan as a dependent variable but it does not cointegrate with the transaction time.
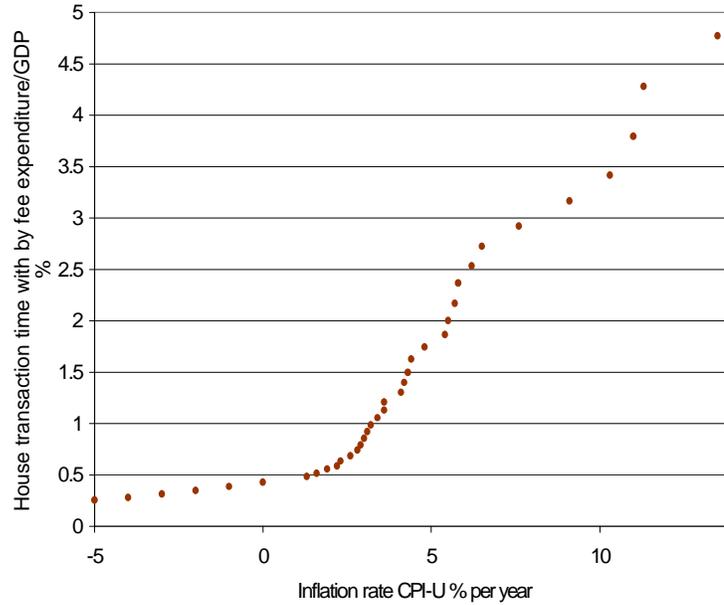
Figure 10. The Value of Resources Wasted for the Household Transaction Time. US: 1964-99

Figure 11 shows the actual values for the firm float time[23]. Again, to estimate the welfare costs we have considered the fitted values (not the actual values) according to a VAR-VEC model to obtain the figure 12 below.

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[23]Description of the method used for estimating the firm float time, according to equation 3.39:

- To estimate the real interest rate spread expenditure by the firms, we take first the nominal prime rate on loan minus the nominal interest rate charged on discount window, then, we multiplied this spread by the firm debt. The debt of firms comes from the Flow of Funds (Federal Reserve Board), series: firm credit market liability, code (FL144104005);

- The interest earning asset comes from the Flow of Funds, series firm credit market instruments asset, code (FL124004005.Q);

- The real interest rate earned comes from the nominal CD 6 mo (Federal Reserve Board) minus the nominal interest charged on discount window. The discount factor ρ is 2.18%..

Given that the BEA does not report the firm expenditure on bank services, we estimate it by considering the noninterest revenue by the insured commercial banks (source: FDIC) minus the bank services paid by households according to the PCE (source: BEA).
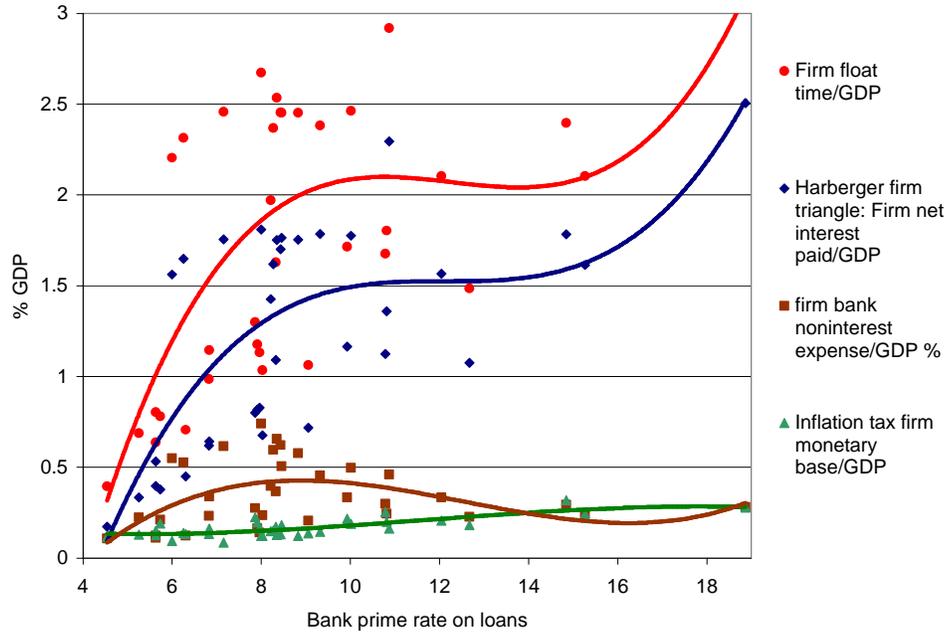
Figure 11. The Original Data for the Estimated Firm Float Time. US: 1964-99

Figure 12 provides the fitted value of the wasted resources for the firm float time[24]. The minimum welfare costs in terms of the value of resources spent for the firm float time would be 0.14% of GDP if the nominal interest rates were zero. The maximum welfare costs were 2.5% of GDP when the nominal interest rates were 18.8% in the year 1981. If the FED had reduced the nominal interest to zero, the welfare gain in terms of saving resources for the firm float time would be 2.4% of GDP.

---

[24]The VAR-VEC model gives us the following long run relationship for the firm float time as function of the prime rate on loan:

Log(1+Firm   Float   Time/GDP)=0.1970463379*Log(1+Prime   Rate   on   Loan)+ 0.02250819926*(time trend) + 0.1344847176
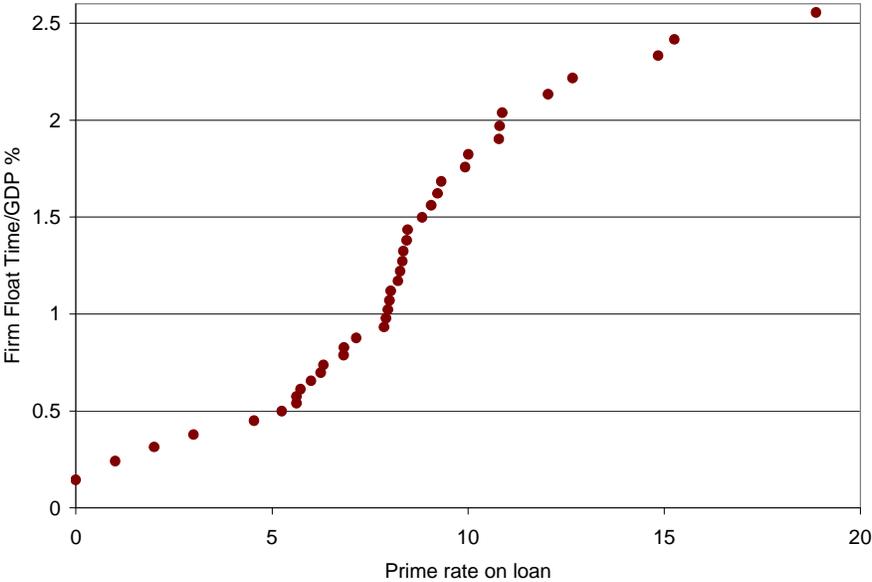
Figure 12. The Firm Float Time. US: 1964-99

Figure 13 shows the original data for the welfare costs due to the resources allocated in the insured commercial banks[25]. Figure 14 below shows the fitted values according to a VAR-VEC model[26].

[25]The value of resources used in the commercial banks is estimated according to the equation 3.35.

The real interest rates spread paid on CD is calculated as the nominal interest rate on CD 6 months minus the nominal interest rate charged on discount borrowing window.

The real interest rates spread paid on loans is its nominal interest rate minus the nominal interest rate charged on discount borrowing window. The discount factor (ρ) is equal to 2.18% per year.

The real T-Bill interest rate spread is its nominal interest rate minus the nominal interest rate charged on discount borrowing window.

The real interests rates spread on revenue by the commercial banks is the nominal interest rate on prime loans minus the nominal interest rate charged on discount borrowing window. This spread is multiplied by the total assets to get the real bank gross revenue.

The total assets is composed of bank credit (FL764005005), customer liab. on acceptance (FL293169605), and miscellaneous assets (FL763090005). See http://www.federalreserve.gov/releases/z1/current/data.htm, table L.109, Commercial Banking.

The real interest expenditure on bank liability has several components.: 1. The net interbank liability (FL764110005) is multiplied by the real interest rate spread on discount rate over the discount factor (ρ); 2. The small time and savings deposits large time deposits (series FL763131005 and FL763135005) have a cost given by their stock multiplied by the interest rates spread on CD 6 months over the interest rate charged on discount borrowing window; 3. The federal funds and security RP (FL762150005) has a cost given by its stock multiplied by the real interest rates spread on US T-Bill 3 mo.; 4. The real interest expendinture on credit market instruments (series FL764104005) is its stock multiplied by the real interest rates spread (bank prime loan rate minus the interest rate on discount window); 5. The real interest expenditure on miscellaneous liabilities (series FL763190005) is its stock multiplied by the real interest rates spread on bank prime loan rate.

The commercial bank fee income comes from FDIC. http://www2.fdic.gov/hsob/hsobRpt.asp, series fee income.

[26]We have considered only the market for credit whose size is 50% of GDP in the year 1999 for USA. We have not considered the distortions in other fixed income market which represented 150% of GDP. Thus, our welfare costs are underestimated because we have not took into account the distortions in the savings institutions, credit unions, bank personal trusts and estates, life insurance, other insurance companies, private pension funds, state and local gov. retirement funds, money market funds, mutual funds, closed-end funds, government sponsored enterprises, federal related mortgage pools, ABS issuers, finance companies, mortgage companies, REITs, brokers & dealers, funding corporations.

Remember that the funds have their own entity. In another words, funds that are administered by the banks are not their liabilities. These funds do not face the distortions caused by the interest rates spreads but they do have another type of distortions due to how the administrators are remunerated: percentage of the net asset or additional remuneration if the administrator beat the benchmark.
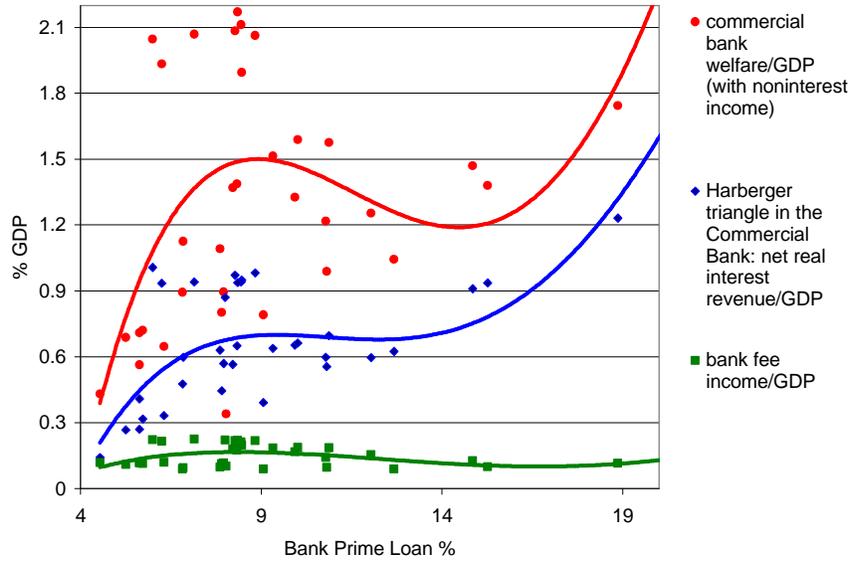
Figure 13. The Original Data for the Estimated Resources Allocated in the
Commercial Banks. US: 1964-99

Figure 14 shows the fitted values of resources wasted in the insured commercial
banks.[27] We present two estimates by considering both the bank service fee income
and the non-interest income. In fact, this last criteria is more relevant because
it includes not only the standard bank fee, but also the income from counseling,
and brokerage charge. Under the noninterest income criteria, the minimum value of
resources allocated in the commercial banks would be 0.37% of GDP if the deflation
rate is set equal to 5% (marginal rate of time preference). In the year 1980, when
the inflation rate reached 13.5% per year, the additional wasting of resources in the
commercial banks relative to this minimum were 2.5% of GDP. Under the criteria of
the bank services fee income, the additional resources allocated in the commercial
banks would be 1.9% of GDP.

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[27]We measure the values of resources allocated in comercial banks by considering either the
bank services fee income or the bank services (noninterest income) due to fee, counselling and
brokerage

By using a VAR-VEC model, we obtain the following long run relationships for the value of
resources allocated in the commercial banks as function of the inflation rates:

Log(commercial banks with noninterest income) - 3.291982335*Log(1+Inflation rate/100) -
0.04966550402*(Time Trend(65)) + 0.9065525201=0

Log(commercial banks with service fee income) - 8.197794115*Log(1+Inflation rate/100) -
0.06459487011*(Time Trend(65)) + 2.042545463=0
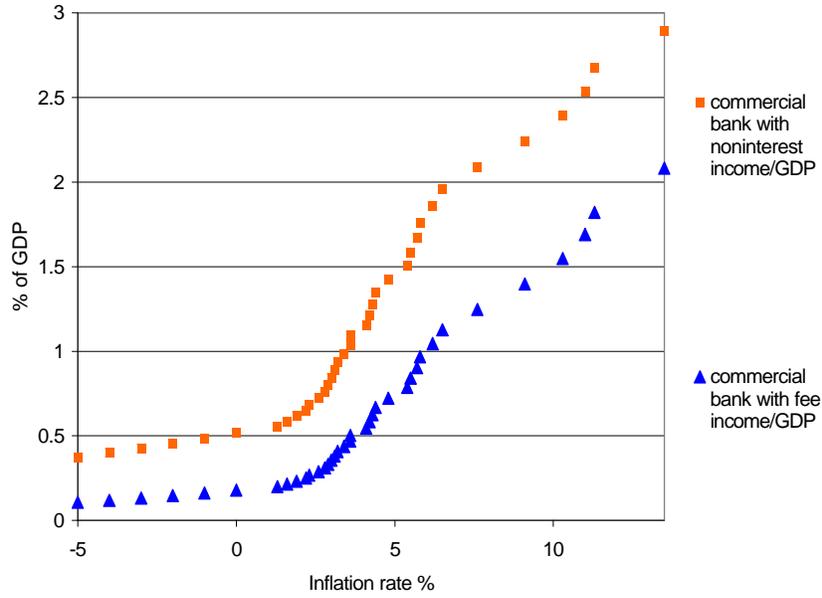
Figure 14. The Value of Resources Allocated in the Commercial Banks. US:
1964-99

Figure 15 shows the actual data for the total consolidated welfare costs of inflation which are obtained by the sum of the resources allocated for the household transaction time, for the firm floating time, and for the financial system. The firm float time is important as the household transaction time. The distorting Bailey triangle is relatively the smallest one. The state of art paid too much attention to this triangle and it has not considered the other bigger distortions presented here.
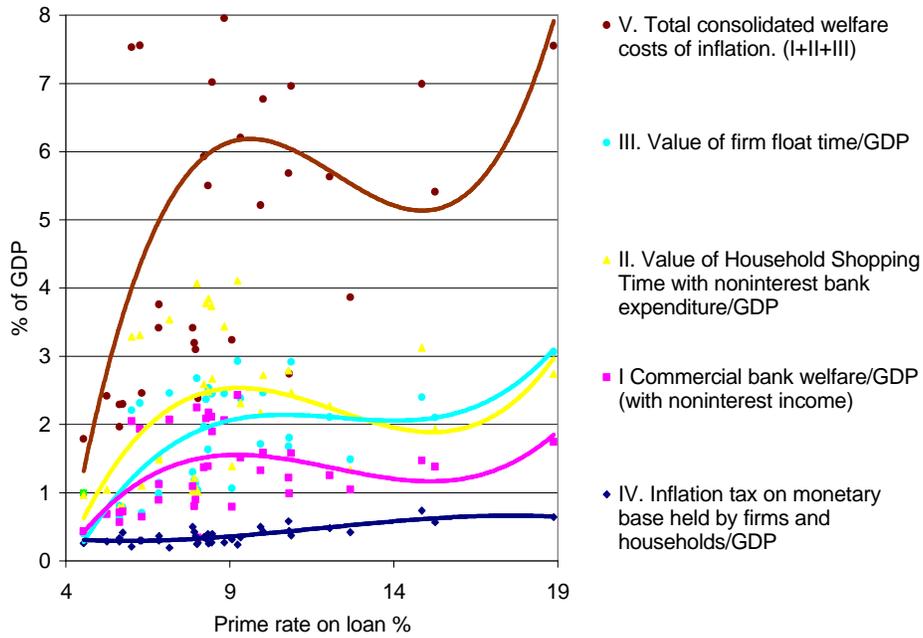
Figure 15. The Original Data for the Estimated Total Welfare Costs of Inflation
and their Components. US: 1964-99

Figure 16 shows the actual data for the estimated components of the total welfare
costs of inflation. Here, we can notice that the Bailey triangle under the demand
for the monetary base schedule.which reaches almost 1% of GDP. This figure repli-
cates Lucas (2000). The biggest source for the additional distorting channel comes
from the Harberger triangle in the market for loans. Furthermore, we have also
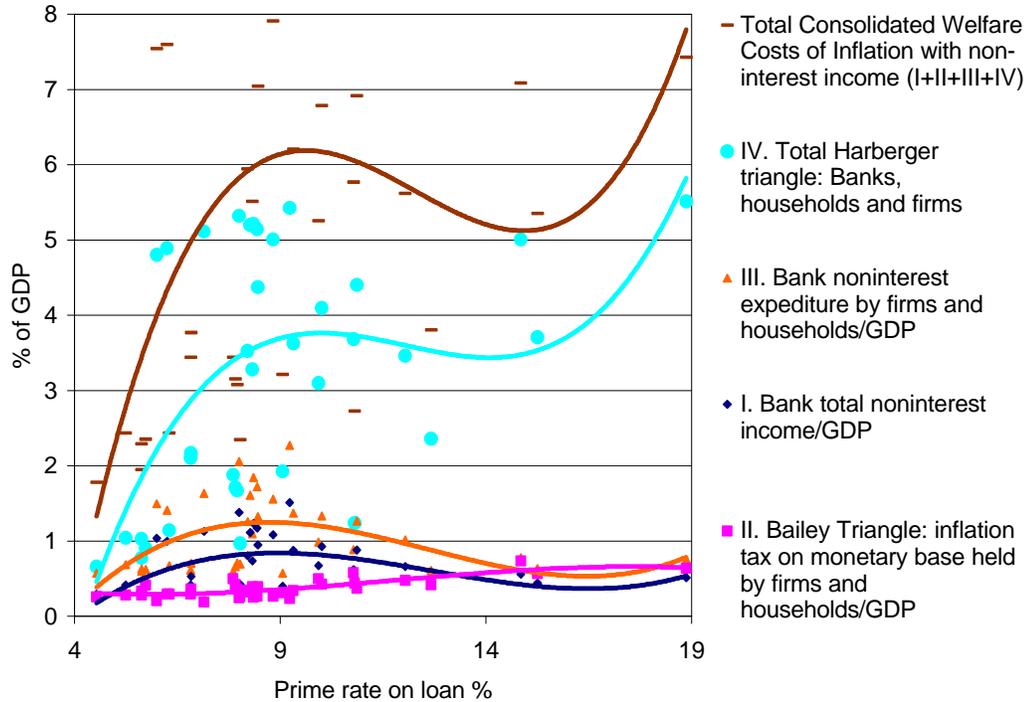considered the bank services which is sizeable as the Bailey triangle

Figure 16. The Original Data for the Estimated Economic Components of the
Total Welfare Costs of Inflation. US: 1964-99

Figure 17 shows that the Harberger triangle in the market for loan is by far
the most important distorting component of the total welfare costs of inflation[28].
On the other hand, both the Bailey triangle and the bank services expenditures
by both the households and firms represent the smallest shares of the total welfare
costs. Furthermore, when the inflation rates increase, the distortion captured by
the Harberger triangle increase their share as percentage of the total welfare costs.
The services expenditure decreases its share and the Bailey triangle is stable. One
explanation for this stylized fact is that the US market for loans was around 50%
of GDP while the monetary base was 2.4% of GDP in the year 1999. Thus, the
biggest distortion happens in the market for loan relative to the Bailey triangle
under the demand for monetary base schedule[29].

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[28]Professor Larry Sjaastad had the suitable intuition when told me a long time ago in Chicago
that Lucas (1993) does not capture the distortion in the market for loans that I call here as "the
Haberger triangle". Lucas considered only the Bailey triangle. To be fair with Lucas (1993),
p. 26, he had the bank profit function. According to his equation 5.9, page 28, Lucas (1993)
calculates the net interest rate paid by the households in the year 1990. He obtains the welfare
costs of 1.3% of GDP due to household transaction time. But he did not have noticed that he was
estimating the Haberger triangle. Lucas (2000) skips the model 5 "A Banking Sector-2" which
was presented in Lucas (1993).

[29]The reason for the increasing Haberger triangle as share of the welfare costs, according to
the theorem 6 (to be proved below) is that the distorting inflation tax increases the interest rates
spread in the market for loans. Furthermore, the higher inflation rates make the demands for
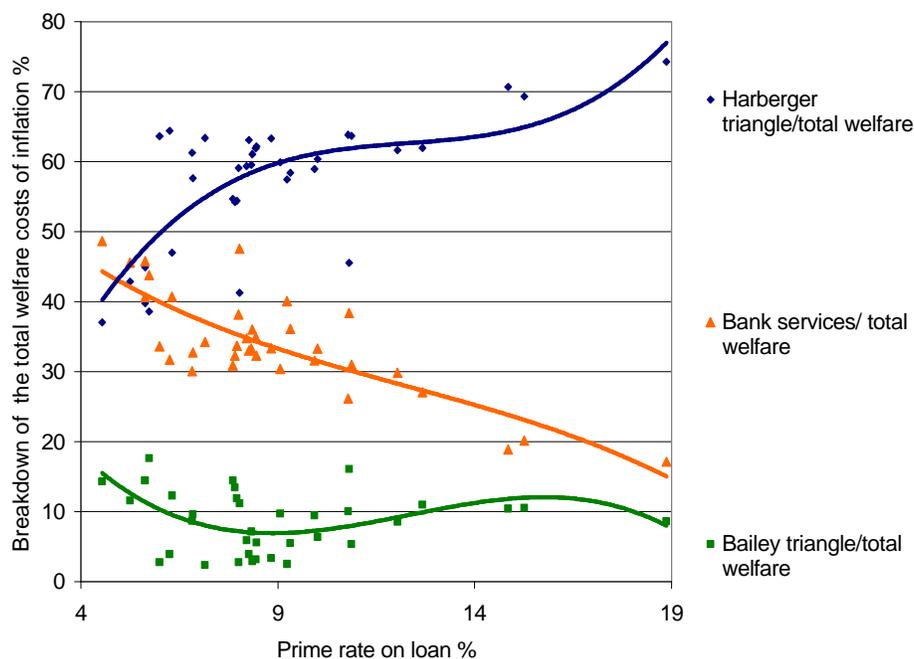
Figure 17. The Distorting Triangles of the Total Welfare Costs of Inflation. US: 1964-99

Figure 18 shows that, when the inflation rates increase, the Harberger triangles hurt much more the bank clients, specially the households, relatively to the commercial banks itselves. Thus, the distorting inflation tax in a fractionary reserve system has bigger incidence on the households and firms sectors in terms of wasting scarce resources.

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the monetary base to decrease. But the demand for bank services schedule shifts outwards with inflation (inflation induces the savings on money holdings and consumer makes more banking transaction - more trips to the banks in the Baumol sense). On the other hand, if the bank fee changes there would be movements along the demand for the bank services schedule.
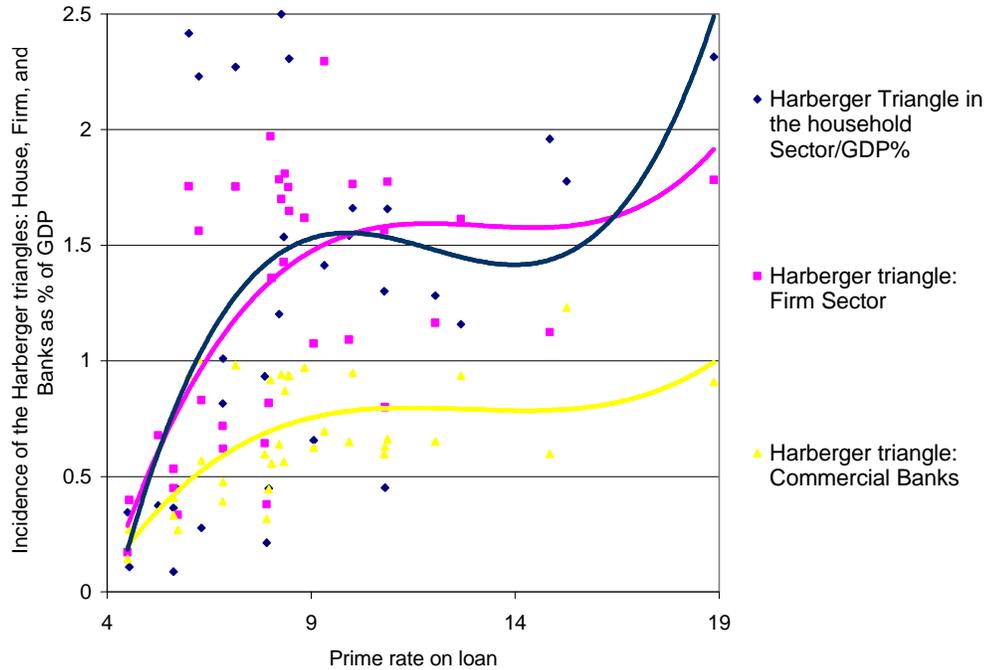
Figure 18. The Sectorial Incidence of the Harberger Triangle

Figure 19 reports the fitted values for the welfare costs of inflation under two criterias: bank noninterest income and bank fee income. In the year 1981, when the prime rate on loan reached 18.8% per year, the maximum total welfare costs of inflation were 12% of GDP. On the other hand, under the Friedman optimal zero nominal interest rate, the welfare costs would be 1.3% of GDP. Thus, the total welfare loss with noninterest income were 10.7% of GDP if we measure on the right hand side of equation (4.1)[30]. On the other hand, the total welfare costs of inflation with bank fee income were 9% of GDP.

Remember that under the alternative approach, according to the figure 2, we had a total welfare costs of 10.3% of GDP (consumption goods foregone due to inflation). Thus, we have obtained quite consistent measurements for the total welfare costs by using two different approaches.

---

[30]To get these estimations for the total welfare costs of inflation, we have used the following long run relationship according to a VAR-VEC model to incorporate the cointegration among the unit roots variables:

Log(1+total welfare costs of inflation)= 0.03910678037*Prime rate on loan+0.03380584454*(time trend)+ 0.635857874

We have tried also another dependent variables such as inflation rates and US T-Bill. But they do not cointegrate except the prime rate on loan.
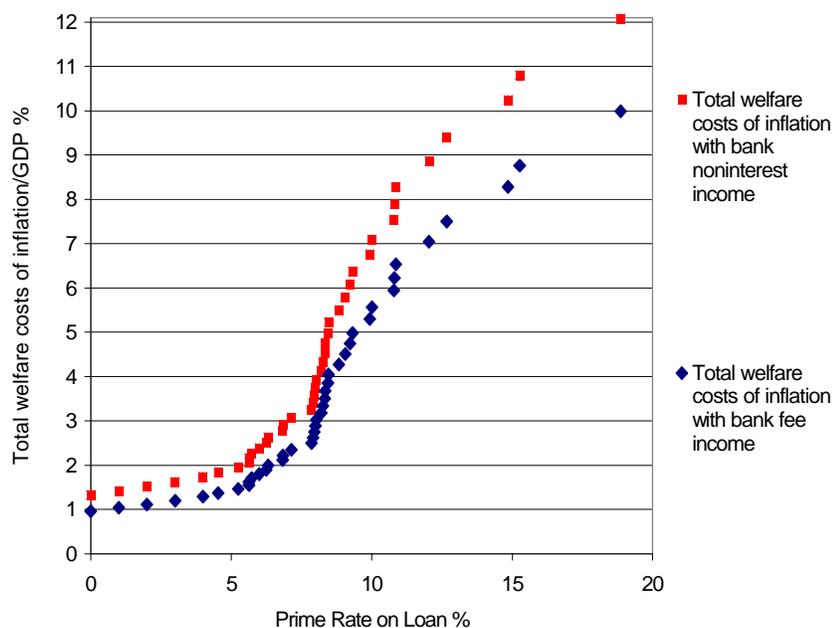
Figure 19. The Total Welfare Costs of Inflation. US: 1964-99

Another approach to check the consistency of our previous measurements is to follow Lucas (1993) in terms of making the parametrization of the wasting functions H, B and F. See appendix 7.2 for further details. In this sense, figure 20 shows the parametrized total welfare costs of inflation and their distorting channels. Here, we get that the total welfare costs of inflation are 9.3% of GDP. Under the previous approaches, we had that the total welfare costs were around 10% of GDP. The parametrized total welfare costs of inflation are 9.3% of GDP which are composed of: i) The household transaction time: 3.1% of GDP; ii) the firm float time: 4.2% of GDP; and iii) the excess resources allocated in the commercial banks: 1.9% of GDP.
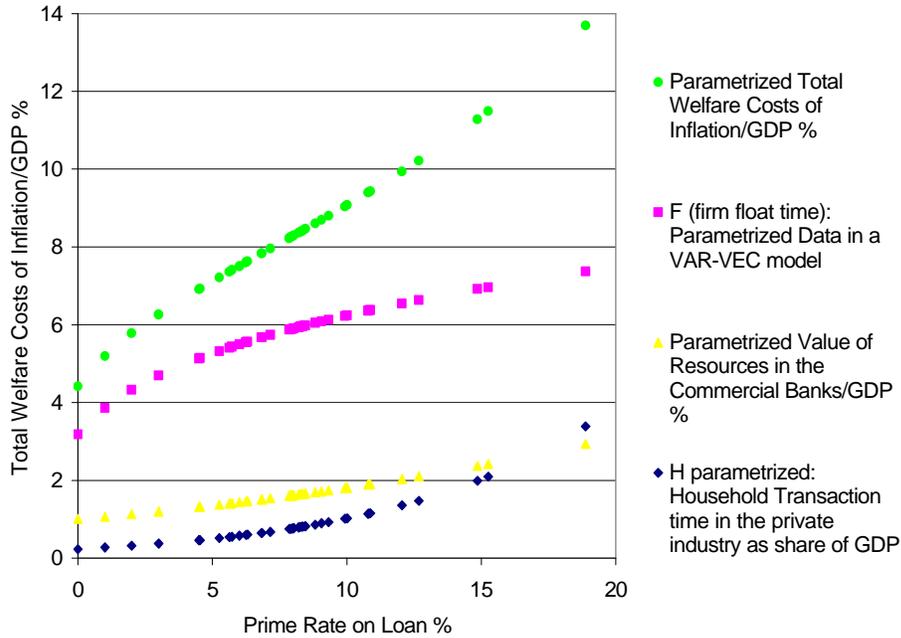
Figure 20. The Parametrized Welfare Costs of Inflation and their Components. US: 1964-99

5. Consequences for the Monetary Theory

Given our economy with the nonneutrality of money and several distorting channels for the inflation tax, we formulate next some essays in terms of the monetary policy so as to compare with the classical monetary theory in an economy with superneutral money such as Friedman (1969). We examine also whether the Simon proposal for 100% reserve requirement can reduce the welfare costs.

First, let us retake the challenge made by Lucas (2000), p. 270, the 2nd paragraph of the section 6 "Conclusions and Further Directions" about the relevant monetary aggregate. So far, our model has shown that the Bailey triangle is measured by the triangle under the demand for the monetary base (outside money). While the Harberger triangle is measured in the loan market which is provided by the commercial banks. In this sense, we propose:

Theorem 1. *The total welfare costs of inflation are composed, at least, of the resources that are misallocated in: i) the commercial banks; ii) for the firm float time; and iii) for the household transaction time. Alternatively, the total welfare costs of inflation is composed of: i) the Bailey triangle under the demand for monetary base schedule; ii) the Harberger triangle in the market for loans; and iii) the triangle in the market for the bank services.* ∎

Proof. See equation 3.40. ∎

Theorem 2. *The welfare costs of inflation due to the value of resources used in the commercial banks to avoid the inflation tax can be represented by: i) the Harberger triangle in the market for loans; and ii) the triangle in the market for banking services.*

Proof. See equation 3.35 ∎

Theorem 3. *The total welfare costs of inflation for the society gives equal weights to the several distorting channels: the firm float time, the households transaction time and the resources used in the financial system.*

Proof. This result is due to the society resource constraint. According to the equation 3.2, we have $1 = n^c + n^f + n^b + n^h$. By using the equation 3.40, the total welfare costs of inflation is given by $W(i) = w.(1 - n^c) = w(n^f + n^b + n^h)$. We can notice that the weights for n^f, n^b and n^h in the total welfare costs function are the same: equal to one. ∎

Theorem 4. *The welfare costs of inflation due to the value of resources used for the firm float time are composed of: i) the Harberger triangle in the market for loans that are demanded by the firms; ii) the triangle under the firm demand for the bank services; and iii) the Bailey triangle under the firm demand for the monetary base.*

Proof. See equation 3.39. ∎

Theorem 5. *The welfare costs of inflation due to the household transaction time are composed of: i) the Harberger triangle in the market for loans demanded by the individuals, ii) the Bailey triangle under the household demand for the monetary base schedule; and iii) the triangle under the household demand for bank services.*

Proof. See equation 3.37. ∎

Theorem 6. *When the inflation rates increase, the bigger distortion in the market for loans (Harberger triangle) arises due to the widening spread between the real interest rate charged on loans and the real interest rate paid on deposits.*

Proof. Based on equation 3.15, the nominal steady state budget constraint of the banks is given by

$$(\pi + r^l).(1 - \alpha).d - (\pi + r^d).d - w.n^b + \phi.s = 0$$

By taking the total derivative wrt. inflation rate (π), we have

(5.1)
$$\frac{d}{d\pi}[(1 - \alpha).r^l - r^d] = \alpha + \frac{d}{d\pi}(w\frac{n^b}{d}) - \frac{d}{d\pi}(\phi\frac{s}{d})$$

By using equation 3.13 and by assuming the homogeneity of the banking production function B(.), we have

$$n^b = d.B[1, (1 - \alpha), \alpha, \frac{s}{d}]$$

$$\frac{n^b}{d} = B(\frac{s}{d})$$

Taking the derivative of the equation above wrt. π, we have

(5.2)
$$\frac{d}{d\pi}(\frac{n^b}{d}) = B'(\frac{s}{d}).\frac{d}{d\pi}(\frac{s}{d})$$

Substituting equation 5.2 into equation 5.1, we get

(5.3)
$$\frac{d}{d\pi}[(1 - \alpha).r^l - r^d] = \alpha + [wB'(\frac{s}{d}) - \phi].\frac{d}{d\pi}(\frac{s}{d})$$

Under the homogeneity of B(.) function, the equation 3.20 can be stated as

(5.4) $$wB'(\frac{s}{d}) = \phi$$

Replacing condition 5.4 into 5.3, we have finally

(5.5) $$\frac{d}{d\pi}[(1-\alpha)r^l - r^d] = \alpha. > 0$$

Or,

(5.6) $$\frac{d}{d\pi}(r_t^l - r_t^d) = \alpha(1 + \frac{dr^l}{d\pi}) > 0$$

Where: $\frac{dr^l}{d\pi} > 0$; $\frac{dr^d}{d\pi} < 0$. ∎

We have also that $\alpha \approx 10\%$ (US bank reserve ratio as percentage of the checkable deposits in the year 1990). For Brazil, $50\% < \alpha < 75\%$ in the years 1990′s. Countries with higher inflation rates make worse the Harberger triangle by imposing more distorting banking regulation due to the effort to collect the seigniorage in the depressing monetary base.

Figure 21 illustrates the source for arising the Harberger triangle due to inflation.
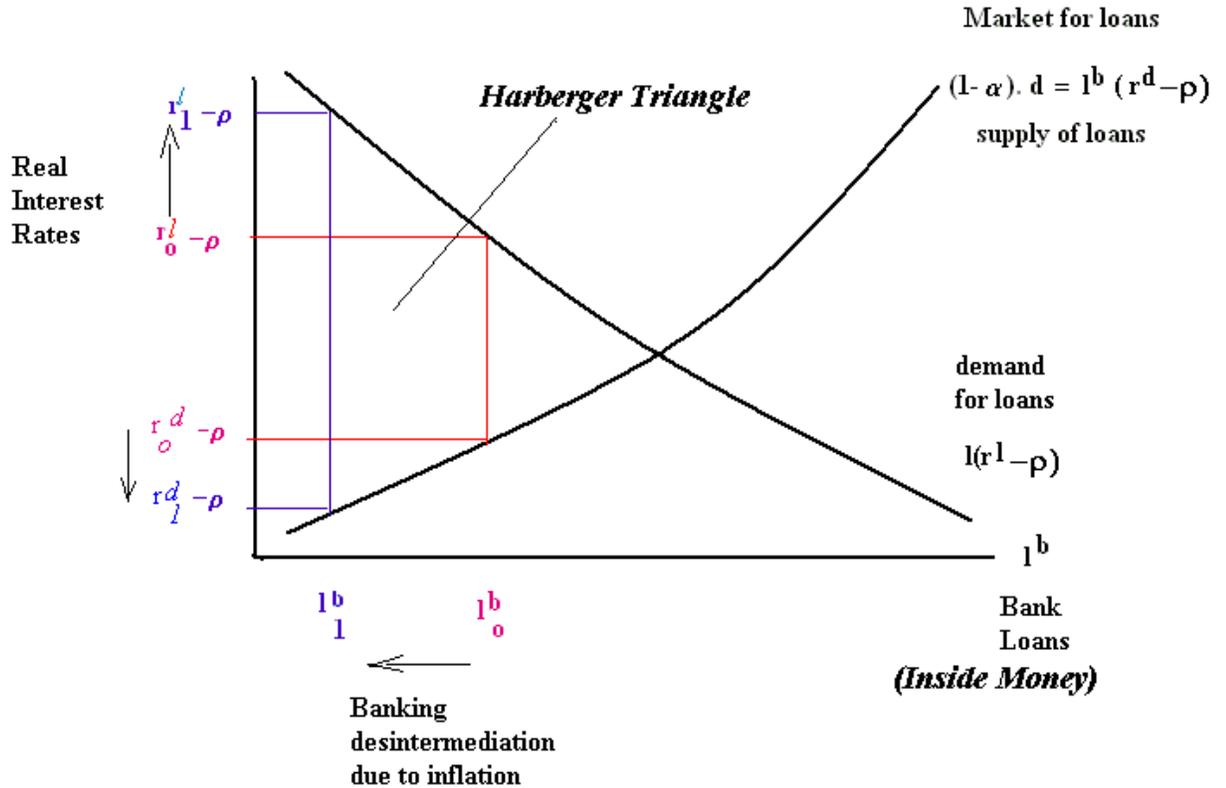

Fig21. The Source for Harberger Triangle: Widening Interest Rates Spread due to
Inflation in a Fractionary Reserve Sustem

The widening bank interest rates spread due to inflation produces also a banking disintermediation (less credit) in a fractionary reserve system.

An important question arises here: Why do the commercial banks that are subject to a fractionary reserves would not disappear in face of the financial innovations that would be made by the noncommercial financial institutions? Why would not the Harberger triangle vanish given that the market could avoid the cumbersome fractionary reserve system through financial innovations in an inflationary environment?

In practice, in the USA, in the years 1980´s, when happened unusual inflation rates, we could notice the creation of the NOW accounts in the money markets and the surge of several types of funds to avoid partially the commercial banking regulation. Thus, the distorting Harberger triangle has induced the creation of the mutual funds that are not subject to the reserve requirements[31]. But, the bank regulation imposes several types of the direct compulsory credit, in particular, for the agricultural and real estate sectors. Other type of regulation imposes that the clearing of the several financial transactions should be made against the demand deposits Therefore, there is a compulsory financial transaction through the commercial banks because, otherwise, the Central Bank would not collect the inflation tax on monetary base. Even though in a general equilibrium framework due to the bank regulation, the Harberger triangle would not vanish.

Theorem 7. *About the Float Labor.*

Inflation induces the goods producing industry to hire more white-collar workers relatively to the blue-collar workers to avoid the inflation tax. This phenomenon makes the productivity of blue-collar workers to decrease. Thus, inflation makes the manufacturing industry more roundabout (inefficient). The consequence in terms of the social loss is the lower levels of consumption goods.

Proof. According to equation 3.29 we had $w = \frac{1}{1+F}$. Where: $F[m_t^f, d_t^f, l_t^f, s_t^f] = \frac{n_t^f}{n_t^c}$

Equation 3.8 shows that the partial derivatives of F( ) function wrt. their arguments are negative. For instance, when the inflation rates increase, the lower demand for money makes the F (ratio of float labor relative to blue-collar workers) to increase. In another words, the marginal productivity of the blue-collar workers decrease. Furthermore, the equation 3.2 shows that this distorting channel brings about less blue-collar workers allocated for the production of consumption goods. ∎

Appendix 7.2.provides two estimations for the F( ) function.

Theorem 8. *In our economy with several distorting channels, the optimum reserve ratio (α^*) to be set by the Central Bank should minimize the total welfare costs of inflation which is equivalent to maximize the utility level of consumers.*

The optimum α^* renders two equivalent results:

$$(5.7) \qquad\qquad \frac{\partial}{\partial \alpha}(i\alpha d) = -2.\frac{\partial}{\partial \alpha}(w.n^b)$$

$$(5.8) \qquad\qquad (i\alpha d)(1 - \eta_{d,\alpha}) = 2.w.n^b.\eta_{n^b,\alpha}$$

---

[31]But these funds present other types of distortions. For instance, the remuneration of their administrators is based on a competitive percentage of the net assets. It is in fact another kind of distorting tax.

This condition means that the optimal α^* to be set by the Central Bank should equate the marginal revenue of the inflation tax on monetary base to the marginal costs savings of the resources spent equally by both the commercial banks for providing banking products and by the bank clients to avoid the inflation tax.

Proof. By taking the total derivative of the equation 3.42 wrt. α, we get the conditions 5.7.and 5.8. ∎

Where: $\eta_{d,\alpha} = -\frac{\partial d}{\partial \alpha}.\frac{\alpha}{d} > 0; \eta_{n^b,\alpha} = -\frac{\partial n^b}{\partial \alpha}.\frac{\alpha}{n^b} > 0$
Alternatively, we have also

$$(5.9) \qquad \frac{\partial}{\partial \alpha}(i\alpha d) = -2.\frac{\partial}{\partial \alpha}\{\frac{[l^b.(r^l - \rho) - d.(r^d - \rho)]}{(1 + \rho)}\}$$

Condition above means that the optimal reserve ratio (α^*) to be set by the Central Bank is obtained when the marginal revenue on monetary base due to an additional increase of reserve is equated to twice the marginal social costs savings in terms of the Harberger triangle in the market for loans. This savings is counted twice because both the banking industry and the bank clients (firms and households) are benefited.

The current reserve ratio on checkable deposits for the USA is set at 10%. This rate is not optimal because the FED collects at most 2% of GDP as seigniorage on monetary base but imposes a social costs due to the Harberger triangle that can reach 8% of GDP on top of the Bailye triangle.

In this context, another interesting issue is related to the Simon proposal for the 100% reserve requirement in our economy with the proposed distorting channels. This proposal is not optimal because it does not necessary satisfy the equation 5.8. In fact, Simon proposal is the worst one because it would maximize the Harberger triangle according to the Theorem 6.

Theorem 9. *In an economy with the proposed distortions, the optimum nominal interest rate (i^*) should minimize the total welfare costs of inflation[32]. Here, we obtain that the optimal nominal interest rate is zero which renders the same result as Friedman (1969) with superneutral money. Thus, even in an economy with several distortion channels for the inflation tax, the Friedman (1969) result is still valid.*

Proof. By observing the equation 3.42, we can notice that the total welfare costs of inflation function W(i) is upward slopping with respect to the nominal interest rate. The components of the welfare costs are also increasing functions wrt. inflation rates. Therefore, we should set the nominal interest rate equal to zero to minimize the resources misallocated due to inflation. ∎

6. Conclusions

We assume an economy with several distorting channels for the inflation tax such as the resources allocated for: i) the firm float time; ii) the household transaction time; and iii) the commercial banks.

---

[32]Recall Friedman (1969): The optimum quantity of money that produces a zero nominal interest rate is valid in a economy with no distorting inflation tax. This results uses the condition of making the marginal utility of money proportional to the nominal interest rate. Given that the marginal costs of producing superneutral money is zero, the nominal interest rate is set equal to zero. Superneutral money is introduced in an economy in a lump-sum fashion by using a helicopter. But, here we are dealing with the non-neutral money - distorting inflation tax.

We estimate that for the US economy, in the period 1964-1999, the total consolidated welfare costs of inflation were around 10% of GDP. This percentage is consistent under several approaches: i) the consumption goods foregone due to inflation; ii) the Bailey and Harberger triangles that are estimated by considering the sectorial budget constraints; and iii) the parametrization of the each distorting channel.

We have also estimated the sectorial misallocation due to inflation:

i) The resources allocated in the commercial banks financial system were around 2% of GDP; ii) The resources spent for the firm float time were between 2.4% and 4% of GDP; and iii) The resources allocated for the household transaction time were between 3.1% and 4.5% of GDP.

Alternatively, the total welfare costs of inflation can be decomposed in terms of the Harberger triangle, the Bailey triangle and the resources that are used for producing banking services. They represented, on average, respectively 57%, 8.6% and 34% of the total welfare costs.

Our results are astonishing as compared to Lucas (2000). His figure is 1% of GDP. But he considers only the Bailey triangle under the demand for monetary base schedule (the outside money). The big differences between our results and Lucas (2000) are mainly due to the firm float time, the Harberger triangle and the resource that are used for producing the bank services. The Harberger triangle is measured in terms of the loans provided by the commercial banks (inside money). If one aggregates the sectors, the inside money would vanish and one would not capture the Harberger triangle only the Bailey triangle in terms of the outside money.

We have also formulated several theorems in terms of: i) The appropriate measurements of the welfare costs of inflation due to the specific distorting channel, such as, the household transaction time, the firm float time and the resources that used by the commercial banks; ii) The source for arising the Harberger triangle: the widening bank interest rates spread due to inflation in a fractionary reserve system; iii) The optimal monetary policy in terms of the reserve ratio; iv) The optimal inflation tax with nonneutral money which minimizes the welfare costs of inflation. The result is the Friedman's (1969) first best optimal quantity of money[33].

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[33]**Further directions for the research on the Welfare Costs of Inflation:**

i) One might argue whether we do have considered all possible relevant distortions to estimate the total welfare costs of inflation. The answer is: Almost all.

We have shown in the stylized facts, according to the figure 9, that the total US welfare costs of inflation can be around 10% of GDP in terms of the loss in the consumption goods. Furthermore, according to the figure 6, the welfare costs of inflation in terms of the employment in the consumption goods industry as share of the total employment in the economy falls by 16.5%. Our estimated welfare costs were between 9 and 10% of GDP. But, the welfare costs in terms of the household transaction time and the firm float time differ by more than 1% depending on the approach. Thus, we may have left over some small additional distortions even though our numbers are quite consistents;

ii) We have not modelled explicitly the durable goods. In fact, we have not made any distinction at all between the durable and non-durable goods. Another challenge is to include the personal services, except banking services. There are increased demands for health services, education housing, and public utility. The data suggests that the education is in terms of training the float labor. Inflation causes hunger and diseases. These services increase their participation from 30% to 50% in the PCE with increased inflation rates;

iii) Among the stylized facts, the table A.2, in the appendix 7.1, shows that the tecnology and other variables are important as the inflation rates for explaining the sectorial allocation of the

7. Appendix

7.1. Further Stylized Facts of the Real Effects of Inflation. The welfare costs of inflation suggested by the figure 1 can be misleading: Others variables besides inflation may also affect both the consumption goods and banking industries such as economic growth, personal per capita income, and technological innovations such as the use of the ATMs (Automated Teller Machines)[34].

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resources We have not made a model to capture this fact. The serious drawback of this modelling is the lack of data in terms of the prices for the accounting and calculating machines, quantity and price indexes for the bank products. BEA has started collecting these price indexes in the year 1986 but there is no quantity index for banking products.

iv) Given that the biggest distortion comes from the Harberger triangle maybe it would be necessary to analyze better the Flow of Funds. The big problem is that there are no income statements separately for the firms and households;

v) Another challenge is to model the financial system outside the commercial banks to capture the additional resources that are used by other markets beyond the market for loans to avoid the inflation tax. We have left out at least two thirds of the US financial system. The suggestion here is to take into account the distortion in the capital market (financial instruments with maturity bigger than one year). But there are no balance sheet and aggregate income statment for the funds specially for the OTC market whose international market is around \$70 trillion in terms of the notional value. For US, the capital market is around 150% of GDP. If we assume a remuneration of 1% for the administrators, the resources used would be around 1.5% of GDP. Is this figure sizeable enough to justify further research? Are those resourches used to avoid the inflation tax (level and variance) beyond the hedging purpose?;

vi) For a better parametrization of the wasting functions F, H and B, it would be necessary to construct quantity and price indices for the whole types of banking products;

vii) We have considered only the steady state welfare costs of inflation. But one interesting task is to analyze the dynamic version. Why? A stylized fact of a stabilization plan is the sluggish adjustment process of the real resources to a new efficient equilibrium associated with low inflation rates;

viii) It would be interesting to analyze how the sector interact due to inflation. The idea here is to consider the product multiplier due to the loss of consumption goods. Yoshino (2000) made a start on it. The finding here is that the Leontieff multiplier becomes less effective. Is it due to the float labor?;

viii) The welfare costs of inflation can be modelled as an option for the benevolent Central Bank in an Arrow-Debreu world. If tomorrow turns out to be a bad state for the interest rate path, the Central Bank exercise the option to implement a stabilization plan. The social price for this option gives the welfare gain for the society. The interest rate path can follow a Markovian process with stochastic shocks. The idea is to derive the stochastic dynamic version of the welfare costs.

[34]Source of data: http://www.bea.doc.gov. National Accounts Data. The variable Savings/Income means personal savings as a percentage of the disposable personal income.

Period: 1959:1998 Annual data	Model 1a	Modelo 1b	Model 2a	Model 2b	Model 3a	Model 3b
	Log (Commercial bank industry share of GDP)	Commercial bank industry share of GDP	Log (Durable Goods Industry Share of GDP)	Durable goods share of GDP	Log (Non-Durable Goods Industry Share of GDP)	Non-Durable Goods Industry Share of GDP
constant	0.234004	-0.183649	0.235983	0.920649	0.137021	7.421026
	(1.54)	(-2.39)	(1.92)	(1.5)	(116)	(3.91)
Log (commercial bank share of GDP) (-2)	0.902244					
	(20.8)					
Commercial bank industry share of GDP (-1)		1.016901				
		(27.89)				
US T-Bill (0)						-0.036733
						(-2.55)
US T-Bill (-2)		0.025854		-0.1127		
		(3.76)		(-3.44)		
log (US T-Bill) (-1)	0.160135		-0.116067			
	(4.74)		(-4.92)			
log (US T-Bill) (-3)					-0.047146	
					(-1.956)	
Growth Rate of GDP (0)				0.204773		
				(5.39)		
Growth Rate of GDP (-1)		0.017552				
		(2.20)				
Durable goods share of GDP (-2)				0.902062		
				(25.16)		
log (Durable Goods Industry Share of GDP) (-2)			0.899066			
			(19.8)			
Non-Durable Goods Industry Share of GDP (-2)						0.469459
						(3.64)
Log (Non-Durable Goods Industry Share of GDP) (-3)					0.866046	
					(15.02)	
Personal Per Capita Income						-0.000163
						(-3.80)
Log (Savings/Personal Income) (-2)	-0.199017		0.100573		0.096945	
	(-2.78)		(2.02)		(2.24)	
Adjusted R-squared	0.95	0.97	0.942514	0.956647	0.948905	0.977703
Akaike info criterion	-2.58	-1.49	-3.253602	1.527078	-3.977691	-0.361193
Breusch-Godfrey Serial Correlation LM Test						
F-statistic:	7.33 Probab. 0.002297	5.94 Probab, 0.006	3.99 Probab. 0.030	11.36 Probab. 0.00018	6.57 Probab. 0.0041	11.11 Probab. 0.00021

Method: Least Squares
White Heteroskedasticity-Consistent Standard Errors   ( ) t-student

Table A.1 Effects of Inflation and Other Variables in the Banking and
Consumption Goods Industries

In this sense, by considering the series reported in figure 1, the table A.1 shows several regressions of how the inflation rates and others variables may affect both the commercial bank industry and the consumption goods shares of GDP. The higher nominal interest rates lead to the overbanking and less both the durable goods and non-durable goods industries shares of GDP. The higher growth rates of GDP induces more both banking and durable goods industry share of GDP. The higher the personal per capita income requires less non-durable goods industry share of GDP. This is a classical corroboration of the wealth effect. On the other hand, the higher the previous savings ratio has the following effects of: i) decreasing the commercial bank share of GDP; and ii) increasing both the durable and nondurable consumption goods industries shares of GDP in the next periods. For the commercial banking industry, the magnitude of the coefficients of others variables such as the savings ratio and the growth rate of economy are similar to the

effect of inflation. Thus, the growth of the banking industry can not be attributed only to inflation. According to the model 3b, the lower nondurable goods industry share of GDP can be attributed to more both inflation and the personal per capita income[35].

Table A.2 reports regressions showing how the inflation rates and the technology affect the employment levels in the commercial banks and in the goods producing industries.[36] Inflation leads to the banking overemployment but the automatization has an opposite effect: The coefficient of the ratio bank wage/price of calculating machines is four times the opposite effect of inflation. On the other hand, both inflation and automatization hurt the labor index for the goods producing industry. In this case, the effect of the first variable is twice bigger than the second one. For the manufacturing industry (durable and non-durable consumption goods), the technology is the relevant variable relatively to the inflation rates for saving labor.

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[35]Inflation makes the whole industrial structure more roundabout or socially inefficient because resources are moved from the final stage (production of consumption goods with biggest backward multiplier of around 2 times according to Yoshino (2000) to the intermediary stage including the banking sector.

[36]Series:

1. Number of employees in the insured commercial banks. Table CB01 Source: FDIC-Insured Commercial Banks;

2. Price of Calculating and accounting machines (including ATM). Series ID : PCU3578#. Base Date : 8506. Source: BLS.

3. Average weekly earnings of production workers. Source: Bureau of Labor Statistics National Employment, Hours, and Earnings. Series ID : EEU71602004. Industry : Commercial banks. SIC Code : 602.

4. Weekly earnings of Goods-Producing Industry. Series ID : EEU00510004

Period: 1985:06 2000:08	Model 1	Model 2	Model 3
	Number of Commercial Bank Employees (thousand)	Labor Index: Goods Producing Industry	Manufacturing labor Share of Private Industry
constant	140.1609 (3.75)	-2.945695 (-3.77)]	1.164881 (2.92)
Number of Commercial Bank Employees (-1)	1.267972 (28.06)		
Number of Commercial Bank Employees (-2)	-0.356643 (-7.60)		
Labor Index: Goods Producing Industry (-1)		1.040643 (105.4)	
Manufacturing labor Share of Private Industry (-2)			0.961619 (77.23)
US T-Bill (-1)	0.939153 (2.27)		
US T-Bill (-2)		-0.120851 (-5.28)	
US T-Bill (-5)			-0.004777 (-2.01)
Average Week Commercial Bank Wage/ Price of Calculating, Accounting Machine- include ATM (0)	-3.566682 (-2.79)		
Average Week manufacturing Wage/ Price of Calculating, Accounting Machine- include ATM (-1)			-3.484673 (-2.96)
Average week goods producing earnings/price of calculating, accounting machines (-1)		-0.06807 (-2.60)	
Adjusted R-squared	0.97	0.99	0.99
Akaike info criterion	6.54	-0.24	-3.64
Breusch-Godfrey Serial Correlation LM Test F-Statistics	12.92 Probab. 0.000006	12.6 Probab. 0.000007	37.33521 Probab. 000051
Method: Least Squares		( ) t-student	( ) t-student
White Heteroskedasticity-Consistent Standard Errors			

Table A.2. Inflation and Technology affecting both the Banking and the Goods Industries. US 1985:6-2000-8

Table A.3 shows the types of floating labor in the manufacturing industry (non-durable and durable goods consumption goods) according to the signals of the correlations between the inflation rates and the levels of the occupational employments[37].

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[37]Now, we consider the new occupational BLS (Bureau of Labor Statistics) data with much more occupations for the period 1983-98. Previous figures illustrate the old BLS data for the period 1972-94. Both data sets are not compatible according to the economists of the BLS.

| Pearson Correlations: Manufacturing | US T-Bill | | | Composition of total labor force Year: 1998 |
| | Float Labor | | | |
	Negative	Neutral	Positive	
Total, all occupations			.753(**)	100%
Executive, administrative, and managerial occupations		-0.448		8.9
Managerial and administrative occupations		-0.414		6.3
Financial managers	-.588(*)			0.5
All other managers and administrators		-0.191		5.1
Management support occupations		-0.427		2.6
Professional specialty occupations		-0.407		6.8
Engineers			.611(*)	3.5
Teachers and counselors		-0.207		0.0
Health assessment and treating occupations		0.47		0.0
Health technicians and technologists			.716(**)	0.1
Technicians, except health and engineering and science			.686(**)	0.8
Marketing and sales occupations		-0.452		3.3
Cashiers		-0.489		0.0
Salespersons, retail	-.605(*)			0.2
Administrative support occupations, including clerical			.693(**)	9.9
Records processing occupations			.669(**)	1.8
Financial records processing occupations			.689(**)	1.2
Bookkeeping, accounting, and auditing clerks			.669(**)	1.0
Secretaries			.693(**)	1.1
Other clerical and administrative support workers			.823(**)	5.4
All other clerical and administrative support workers			.847(**)	3.5
Service occupations			.718(**)	1.2
All other service workers			.688(**)	0.4
Agriculture, forestry, fishing, and related occupations			.630(**)	0.5
Precision production, craft, and repair occupations			.689(**)	21.6
Blue collar worker supervisors			.582(*)	4.2
Construction trades			.769(**)	1.2
Mechanics, installers, and repairers			.740(**)	4.2
Production occupations, precision			.642(**)	11.2
Operators, fabricators, and laborers			.726(**)	45.1
Machine setters, set-up operators, operators, and tenders			.737(**)	22.9
Hand workers, including assemblers and fabricators		-0.231		12.3
Transportation and material moving machine and vehicle operators			.708(**)	3.4
Helpers, laborers, and material movers, hand			.740(**)	6.6
All other helpers, laborers, and material movers, hand			.734(**)	6.5

** Correlation is significant at the 0.01 level (2-tailed).
 * Correlation is significant at the 0.05 level (2-tailed).

Table A.3. Types of Floating Labor. Manufacturing Industry. US: 1983-98

Table A.4 shows that inflation is not affecting the total banking employment but only certain occupations for the USA, in the period 1983-98. In another words, inflation demands more employment levels for the following occupations in the depository and non-depository institutions: i) Financial workers (Records processing occupations; Financial records processing occupations; Bookkeeping, accounting, and auditing clerks); ii) All other managers and administrators; iii) Secretaries; iv) Service occupations; v) Construction trades to build new branches; vi) Helpers, Laborers, and material movers. We call again this pattern as "positive float labor" in the financial sector. It represents a detailed picture of the banking overemployment due to inflation.[38] Inflation requires less employment levels for the following occupations in the depository and non-depository institutions: i) Executive, administrative, and managerial occupations; ii) Management support occupations; and iii) Professional specialty occupations. We call these employments as "negative float labor". Inflation is making the banking industry more roundabout: i) less managers, cashiers and professional specialty; and ii) more administrative support workers in

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[38]Now, we consider the new occupational BLS data with much more occupations. Previous figures illustrate the old BLS data.

terms of backoffice and services (unskilled labor). The inflationary overbanking is characterized by more resources allocated to provide services rather than to make financial intermediation. On the other hand, inflation increases the employment levels for financial workers (except cashiers), service occupations, back-office (secretaries, other managers and administrators). We have also found above that these types of occupations were also required by the consumption goods industry.

| **Pearson correlations** | US T-Bill | | | Breakdown of the total employment. Year 1998 |
| | Float Labor | | | |
	Negative	Neutral	Positive	
Total, all occupations		-0.414		100%
Executive, administrative, and managerial occupations	-.614(*)			27.30
All other managers and administrators			.651(**)	5.8
Management support occupations	-.701(**)			15.8
Professional specialty occupations	-.503(*)			3.1
cashiers		-0.257		0.5
Records processing occupations			.600(*)	4.8
Financial records processing occupations			.549(*)	3.6
Bookkeeping, accounting, and auditing clerks			.511(*)	2.9
Secretaries			.591(*)	2.5
Service occupations			.654(**)	0.9
All other service workers			.701(**)	0.4
Agriculture, forestry, fishing, and related occupations			.497(*)	0.04
Construction trades			.622(*)	0.02
Helpers, laborers, and material movers, hand			.708(**)	0.02

* Correlation is significant at the 0.05 level (2-tailed).

** Correlation is significant at the 0.01 level (2-tailed).

Table A.4. Types of Floating Labor. Depository and Non-Depository Institutions. US: 1983-98

7.2. **Estimations of the Wasting Functions: F, B and H**   .

By following Lucas (1993), we can make the parametrization of the wasting functions H, B and F. Another approach is to infer these functions according to our first order conditions. We develop both approaches. Let us start with the easiest one - the last one.

7.2.1. Estimation of the Firm Wasting Function $F(w(\pi))$: *Float Labor*

Figure A.1 shows the values for the indirect $F(w(\Pi))$ as function of the inflation rate $(\Pi)^{[39]}$.

For the USA, in the year 1986:3, the monthly inflation rate was -0.45% (annualized rate of -5.2%) and the value of F was 4.9%. On the other hand, in the year 1990:1, the monthly inflation rate was 1.03% (annualized rate of 13.09%), and the value of F function was 13.5%. Thus, the additional welfare costs of inflation in

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[39]The F(.) function is calculated by using the equations 3.29 and equation 3.7. Thus, we have

$F = \frac{n^f}{n^c} = \frac{1-w}{w}$

The source of data is BEA for the nominal wage in terms of hours earnings in the manufacturing industry. To calculate the real wage, we have used either the CPI-U deflator or the Price Index for the Calculating and accounting machines including ATM. This last deflator tries to captures the impact of banking automation on the firm float labor as percentage of the labor devoted for producing consumption goods. The motivation is the table A.2 in the appendix. Source for the price index: BLS. Series ID : PCU3578#.Price of computers and ATM Base Date : 1985:06=100.

Figure A.1 is based in the following long-run realationship according to a VAR-VEC model:

1. LOG(F1)= 5.009041517*LOG(1+INFLATION(-1))

2. LOG(F2) = 1.762417368*LOG(1+Inflation(-1)) +2.064871023.

terms of labor misallocated for the firm float time as percentage of the directly pro-
ductive workers were 8.6%. These values for F(w) as function of the real wage were
obtained by using the nominal wage deflated by the CPI-U index. Alternatively, if
we deflate the nominal wage by the price index of computer and ATM, we get that
the additional welfare costs were 5.8%.

The idea is that inflation induces more wasting of the float labor relatively to the
productive workers. Thus, inflation decreases the marginal productivity of labor
allocated in the manufacturing industry. Therefore, w′(Π) < 0 and F′(w) > 0.
Finally, notice also that the impact of technology has an opositte effect of inflation
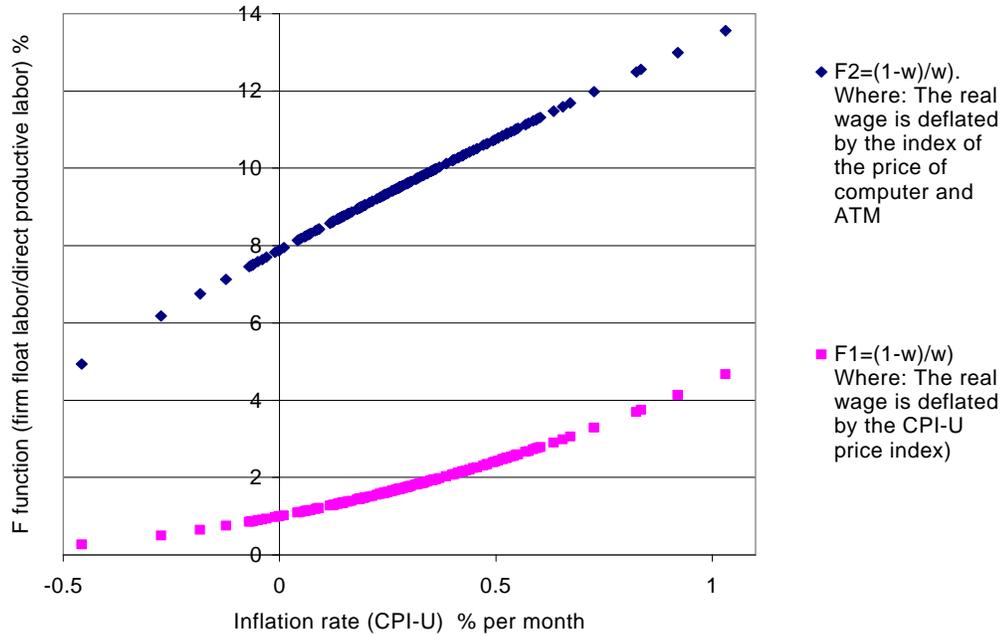by making the float labor to reduce by 2.8% (8.6%-5.8%).

Figure A1. The Fitted Values for the Float Labor/Blue-Collar Workers. US:
1964-99

Figure A.2 shows the fitted values for F(i) as function of the prime rate on
loan. The time series for F is the ratio of the administrative including clerical
employment over the blue collar employment in the manufacturing industry. We
have used a VAR-VEC model to take into account the unit root (non-stationary)
and the cointegration of the series[40]. With zero nominal interest rate, we have
F(i=0)=13.8%. In the year 1981, the prime rate on loan was 18.87% per year and
we get F(i=0.1887)=17.7%. Thus, the additional waste in the manufacturing labor
market is 3.8%.

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[40]We get the following long-run relationship:

Ratio of the Administrative and clerical employment over the blue-collar employment=Exp(
3.233912361*Log(1+prime rate/100) +2.631158095)). Here, the inflation rate does not cointegrate
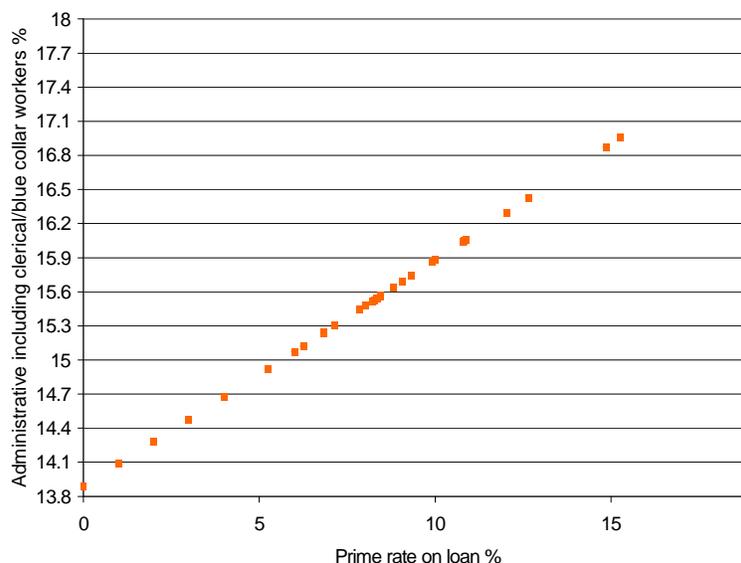with the values for F(.) function.

Figure A2. The Firm Float Labor. US: 1977-99

7.2.2. Parametrization of the functions H, B and F
Now, we follow Lucas (1993) by making the parametrization of each wasting function to obtain the total welfare costs of inflation. The purpose is to check the consistency of our previous measurements.
Estimation of F Function
The firm float time was defined by the equation 3.7 as $n_t^f = F[m_t^f, d_t^f, l_t^f, s_t^f].C_t$. This function can be parametrized as

$$(7.1) \qquad\qquad F = A(m^f)^\beta (d^f)^\gamma (l^f)^\delta (s^f)^\theta$$

Where: $\theta = 1 - \beta - \gamma - \delta$ for the Cobb-Douglas production function.

By taking the first partial derivative of this equation wrt. m^f and by using the equation 3.30, we get the parameter (β): the ratio of inflation tax paid by the firm on his currency holding over the value of firm float time:

$$(7.2) \qquad\qquad \beta \;\; = \;\; -\frac{im^f}{w.n^f}$$
$$\beta \;\; = \;\; -\frac{im^f}{\frac{F}{1+F}w.(n^c + n^f)}$$

Figure A.1 and A.2 before have reported the estimated values for F(i). We do have also the times series for the nominal interest rates (i) and the wage expenditure by the nonfinancial firms. For the year 1990, the prime rate on loan was 10% per year, and by using the figure A.2, we get F=$\frac{n^f}{n^c}$=15.8%. The value for $w.(n^c + n^f)$ = \$ 2670,7 billion (total wage expenditure by the private industry in the year 1990).

Thus, we have w.n^f = \$364.4 billion. We estimate that m^f is \$94.8 billion[41]. Thus, we get β =-0.03 and $m^f = \frac{9.48}{i}$

Similarly, by using the equation 3.32 and by making the partial derivative of equation 7.1 wrt. d^f, we get

$$(7.3) \qquad \gamma = -\frac{d^f.[-i\alpha(1+\rho) + (r^d - \rho)]}{(1+\rho).w.n^f}$$

We have that the d^f = \$169.7 billion[42], α =0.105, $\rho = 2.18\%$[43], r^d (real interest rate on CD-3 months) = 2.75% per year. Thus, we get γ =-0.00232 and $d^f = \frac{0.865542292}{[i*\alpha*1.0218-(r^d-\rho)]}$

By using the equation 3.30, and by making the partial derivative of equation 7.1 wrt. l^f, we have

$$(7.4) \qquad \delta = -\frac{l^f(r^l - \rho)}{w.n^f.(1+\rho)}$$

Given that l^f = \$3734.7 billion[44], r^l =4.61% per year[45], we have δ =-0.24 and $l^f = \frac{90.75}{r^l - \rho}$. Finally, by using the equation 3.33, and by making the partial derivative of equation 7.1 wrt. s^f, we have

$$(7.5) \qquad \theta = -\frac{\phi.s^f}{w.n^f}$$

In the year 1990, the value for $\phi.s^f$ = \$29.4 billion for the firm noninterest expenditure. Thus, we get θ =-0.15 and $s^f = \frac{54.9}{\phi}$. We have also that 1-$(\beta+\gamma+\delta)$ = 1.27 $\neq \theta = -0.15$. Therefore, the production function is not a Cobb-Douglas one. We will deal with the case $\theta = -0.15$.

From equation 7.1, and by substituting all parameters estimated above,, we have A=2.22. By substituting all estimated parameters above into equation 7.1, we get

(7.6)

$$F(i) = 2.22 * \left(\frac{9.48}{i}\right)^{-0.03} * \left(\frac{0.86}{[i * \alpha * 1.0218 - (r^d(i) - \rho)]}\right)^{-0.00232} * \left(\frac{90.75}{r^l(i) - \rho}\right)^{-0.24} * \left(\frac{54.9}{\phi}\right)^{-0.15}$$

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[41]The US Flow of Funds reports only the sum of the checkable deposits and currency. But we need the currency holdings separately by the households and the firms. We need to know also the demand deposits held by firms and households.

We consider that the aggregrate percentage of M1 hold by households and firms is valid also for them in terms of holdings of their monetary componentes: currency and demand deposits.

The "M1" are \$408.8 billion for the households and \$251.4 for the nonfinancial business. The respective percentage holdings of these sums are used to infer the percentage holdings of currency by the households and firms. Similar, assumption were made for the checkable deposits held by household and firms.

[42]Source: US Flow of Funds. Asset of the nonfinancial private business.

[43]Lucas (1993) consider that the marginal rate of time preference $\rho = 5\%$ per year. We consider this value too high because most of the real interest rates are much lower. Thus, there would be no savings at all. We take the average real interest rate on discount borrowing window as a proxy for $\rho = 2.18\%$.per year This rate is the opportunity costs of the funding for the banks.

[44]Source of data: Flow of Funds. l^f is the liability (credit market debt) of the nonfinancial business.

[45]Real interest rate on Prime Rate on loans in the year 1990.

Due to the lack of data relative to the quantity of services, we take the time series on noninterest real expenditure by the firm. For every year, we have the time series relative to the nominal interest rate $(i), (\rho - r^d)$, and ϕs^f. We plot the function $F(i)$[46] as
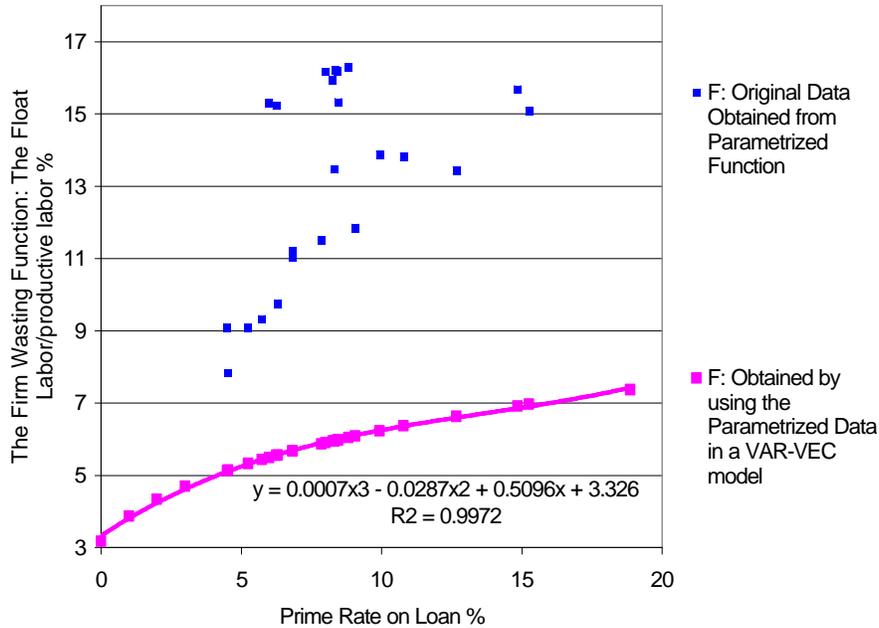

Figure A3. The Parametrized Firm Wasting Function. Float Labor. US: 1964-99

The VAR-VEC model by using the parametrized data shows that with zero nominal interest rate (i=0), we have F(i=0)=3.18% and F(i=18.87)=7.37%. Thus, the excess float labor is 4.2%. Recall that in the similar experiment, the figure A.2 gave an excess administrative workers/blue collar workers of 3.8%. On the other, according to figure A.1, the excess F(w(i)) was 5.7%.

Estimation of the B Function

According to equation 5.1, we have $B(s/d; \alpha) = \frac{n^b}{d}$., i.e., the B function represents the amount of resources that are used by the banks per deposit. Furthermore, by using the condition 5.4, we get $B'(s/d; \alpha) = \frac{\phi}{w}$. The optimal provision of services is obtained when the marginal revenue of services (tangent of B function) is equated to the bank fee deflated by the real wage.

For the year 1990, according to the United Nations, we have that the value of resources allocated in the US commercial banks was $127,5 billion in terms of compensation of employees and $27 billion for the capital consumption. Still for the year 1990, the number of bank employee was 1,5 million. The value of checkable deposits was $584 billion, the small time and savings deposits ($1.3 trillion) and large time deposits ($423 billion), which sums up to $2.3 trillion. Thus, B=1.5 million employees/$2.3 trillion=0.00065 (1/$). Usually, the B function is expressed

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[46]In fact, the reserve ratio α set by the FED changes over time.

We get the following long-run relationship based on the VAR-VEC model:

Log (F) =0.4314350785*Log (1+prime rate on loan) + 0.9555617859

as the value of bank resources per deposit which implies that: $\frac{w.n^b}{d}$=(\$127,5+27 billion)/\$2.3 trillion=6.7%. Furthermore, we have also

$$(7.7) \qquad wn^b = wB = w \quad .d^{\varkappa}.[(1-\alpha).d]^{\xi}.(\alpha d)^{\varphi}(s)^{\varrho}$$
$$wn^b = w \quad .d^{\varkappa+\xi+\varphi}(1-\alpha)^{\xi}\alpha^{\varphi}(s)^{\varrho}$$
$$wn^b = Kd^{\varkappa+\xi+\varphi}(s)^{\varrho}$$

Where: $K = (1-\alpha)^{\xi}\alpha^{\varphi}$. For the Cobb-Douglas case, $\varrho = 1 - (\varkappa + \xi + \varphi)$.
By using equation (3.20) and taking the partial derivative of equation 7.7 wrt. (s), we get

$$(7.8) \qquad \varrho = \frac{\phi.S}{W.n^b}$$

$\phi.S$ (value of the bank noninterest income)= \$54.9 billion in the year 1990. While, $W.n^b = \$154.5$ billion (bank labor expenditure and capital consumption). Thus, ϱ =0.355.and

$$(7.9) \qquad s = \frac{\varrho * W.n^b}{\phi} = \frac{0.355 * 154.5}{\phi} = \frac{54.84}{\phi}$$

By using equation (3.19) and taking the partial derivative of equation 7.7 wrt. (d), we get

$$(7.10) \qquad \chi + \xi + \varphi = \frac{d.[(1-\alpha).r^l - r^d + \alpha.\rho]}{w.n^b(1+\rho)}$$

By using, $d^f + d^h = \$2071.2$ billion, $\alpha = 0.105$; $r^l = 0.0461$; $r^d = 0,0275$; $\rho = 0.0218$, $W.n^b = \$1514.5$ billion, we get $\chi + \xi + \varphi$ =0.21. Thus 1-$\chi + \xi + \varphi$=0.79 $\neq \varrho$ =0.355. Thus, we do not have a Cobb-Douglas production function.

$$(7.11) \qquad d = \frac{33.24}{[(1-\alpha).r^l - r^d + \alpha.\rho]}$$

From 7.7, we have $K = wn^b * d^{-(\varkappa+\xi+\varphi)} * s^{-\varrho} = wn^b * d^{-0.21} * s^{-0.355}$. We have the values: :$W.n^b = \$154.5$ billion, d=\$2071.2 billion and $\phi.s$ (value of the bank noninterest income)= \$54.9 billion. Since we do not have the quantity for bank services, we deflate the bank services revenue by the general price index because we do not have the price index for the bank services[47]. Thus, in the year 1990, we obtain K=7.45.

By pugging this value of K, $\chi + \xi + \varphi$ =0.21, ϱ =0.355 and by using equations 7.9 and 7.11 into equation 7.7, we obtain

$$(7.12) \qquad \frac{w.n^b(\pi)}{GDP} = 7.45 * \left(\frac{33.24}{[(1-\alpha).r^l - r^d + \alpha.\rho]}\right)^{0.21} * \left(\frac{54.84}{\phi}\right)^{0.355}$$

---

[47]BLS reports the series Personal financial services since 1986 but we need this series since 1964. Series ID : MWUR0000SE6802 Not Seasonally Adjusted Area : U.S. City Average Base Period : DECEMBER 1986=100

Given that we do not have ϕ (the bank service price index except after the year 1986), we take the CPI-U index as a proxy for ϕ. For every year, we have the time series for i, r^l, α and r^d. Thus, we get $\frac{w.n^b(\pi)}{GDP}$. for every year.

Figure A.4 shows the estimated values for the resources allocated in the US commercial banks as percentage of GDP. Under Friedman optimum interest rate (deflation equal to 5% per year), we have $\frac{w.n^b(\pi=-5\%)}{GDP} = 0.72$. In the year 1980, the inflation rate was 13.5% per year and we had $\frac{w.n^b(\pi=13.5\%)}{GDP} = 2.2$.Thus the additional resources allocated in the commercial banks was 1.47% of GDP. Recall that in the figure 15, we have obtained that this percentage was between 1.9% and 2.5%.[48]
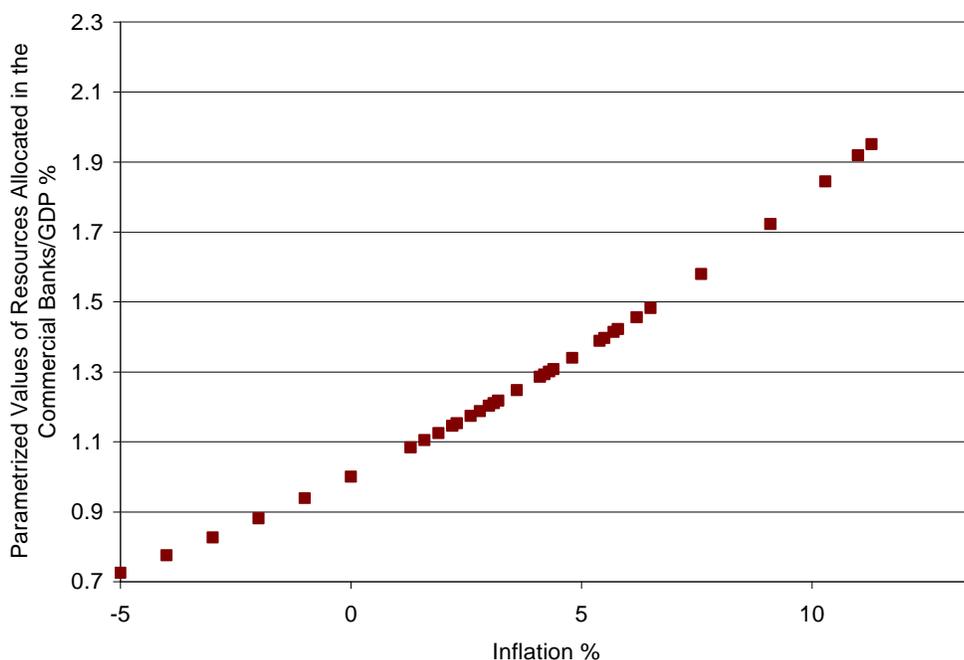

Figure A4. The Parametrized Commercial Bank Production Function. US: 1964-99

The H Function   .

$$(7.13) \qquad \frac{n^c}{n^h} = H = \Gamma.(m^h)^\zeta.(d^h)^\eta.(l^h)^\mu.(s^h)^\upsilon$$

By using equation 3.24 and the partial derivatives of equation 7.8 wrt.m^h, we have:

$$(7.14) \qquad \zeta = \frac{i.m^h}{w.n^h}$$

---

[48]The estimated long run relationship $\frac{w.n^b(\pi)}{GDP}$ according to a VAR-VEC model is:

LOG($\frac{w.n^b(\pi)}{GDP}$) =14.37907637*Log (1+inflation rate/100)

Similarly to the firm currency above, in the year 1990, we have: $m^h = \$151,7$ billion; $w.n^h = \$154.9$ billion, i.e., by using $w.n^h/GDP=0.0267$; and π $(Inflation)=0.054$. Thus, $\zeta =0.098$.

$$(7.15) \qquad\qquad m^h = \frac{15.18}{i}$$

By using equation 3.25 and the partial derivative of equation 7.9 wrt. d^h, we have

$$(7.16) \qquad\qquad \eta = \frac{d^h[i\alpha - \frac{r^d-\rho}{1+\rho}]}{w.n^h}$$

For the year 1990, we have $d^h = \$1961.6^{[49]}$; $\alpha =0.1057$; $r^l = 0.0461$; $r^d = 0,0275$; $\rho = 0.0218$. Thus, $\eta =0.0633$ and

$$(7.17) \qquad\qquad d^h = \frac{9.81}{i\alpha - \frac{r^d-\rho}{1+\rho}}$$

By using equation 3.27 and the partial derivative of equation 7.10 wrt.l^h, we have

$$(7.18) \qquad\qquad \mu = \frac{l^h(r^l - \rho)}{w.n^h(1 + \rho)}$$

We have that $l^h = \$5123.billion^{[50]}$. Thus $\mu =0.786$

$$(7.19) \qquad\qquad l^h = \frac{124.49}{(r^l - \rho)}.$$

By using equation 3.26 and the partial derivative of equation 7.11 wrt. s^h, we have

$$(7.20) \qquad\qquad \upsilon = \frac{\phi.s^h}{w.n^h}$$

In the year 1990, $\phi.s^h = \$47$ billion. Thus, $\upsilon =0.303.\Longrightarrow 1-\zeta-\eta-\mu. = 0.052 \neq \upsilon$. Therefore, we have do not have the Cobb-Douglas case.

$$(7.21) \qquad\qquad s^h = \frac{47}{\phi}$$

We have w$(n^c+n^f)=\$$ 2670.7 billion, w.n^f =$\$364.39$, w.n^c = $\$$ 2306.3 billion. Thus, H= $\frac{w.n^c}{w.n^f}$ =14.88. By using the estimated parameters into equation 7.13, we get $\Gamma =0.00212^{[51]}$

---

[49]Source: US Flow of Funds.

[50]Source: Flow of Funds.

[51]$\Gamma = H.(\frac{15.18}{i})^{-0.098}.(\frac{9.81}{i\alpha-\frac{r^d-\rho}{1+\rho}})^{-0.0633}.(\frac{124.49}{(r^l-\rho)}.)^{-0.786}.(\frac{47}{\phi})^{-0.303}$

(7.22)

$$\frac{n^c}{n^h} = H = 0.00212 * \left(\frac{15.18}{i}\right)^{0.098} \cdot \left(\frac{9.81}{i\alpha - \frac{r^d - \rho}{1+\rho}}\right)^{0.0633} \cdot \left(\frac{124.49}{(r^l - \rho)}\cdot\right)^{0.786} \cdot \left(\frac{47}{\phi}\right)^{0.303}$$

First, we estimate the H(i) $= \frac{n^c}{n^h}$ function. See figure A.5.

After, we estimate $\frac{w.n^h}{GDP} = \frac{C}{H.GDP}$ See figure A.6 below. Where: (C) is the output either by the private industry or by the manufacturing industry. The values for H is estimated according to equation 7.22 and by considering the following long-run relationship in a VAR-VEC model[52]. Thus, the parametrized welfare costs of inflation due to the household transaction time is 3.15% of GDP while the figure 10 reported that these costs are 4.5% of GDP.
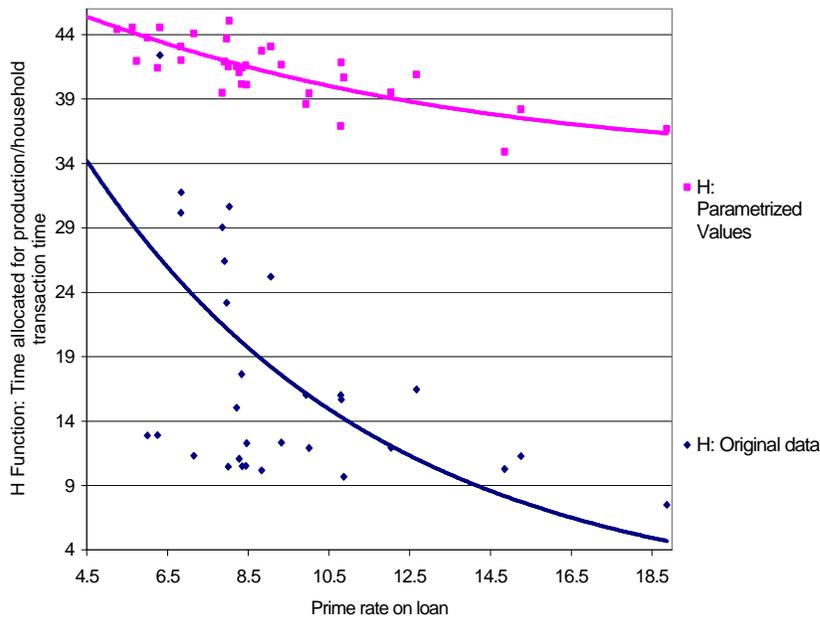

Figure A.5 The Paramanetrized H Function: Household Transaction Time

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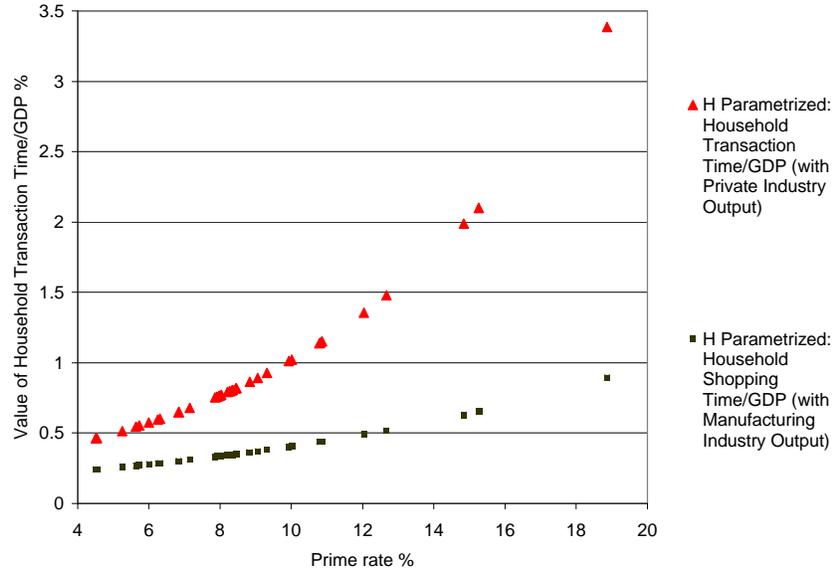[52]H=-20.21931559*LOG(1+prime) + 61.1.

Figure A.6. The Household Transaction Time/GDP. US: 1964-99

Finally, figure 20 in the main text shows the total welfare costs of inflation by making the sum of the all components of the wasting functions B, F and H that were parametrized above.

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